

Quantum Optics Laboratory
Department of Physics
University of Maryland, Baltimore County
Baltimore, MD 21250

Experiment: Yanhua Shih

Theory: Morton H. Rubin

Graduate Research Assistants

H. Chen, S. Karmaker, J. Simon,
V. Tarmma, Yu Zhou

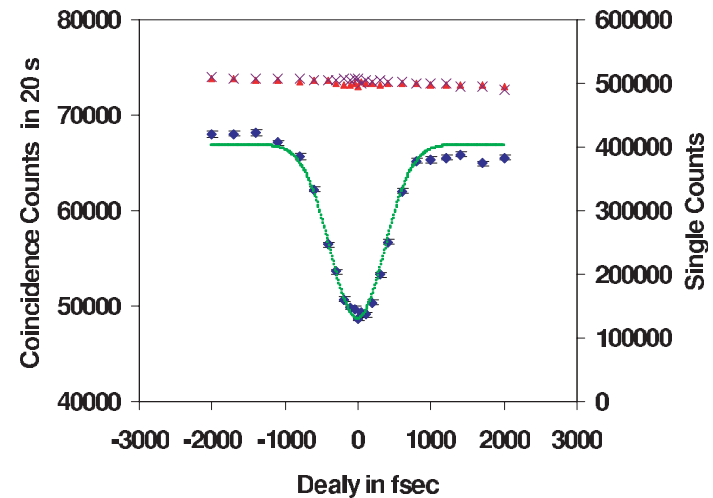
Visiting Research Scientists:

V. Berardi, M.C. Chekhova, M. D'Angelo, A. Garuccio,
X.H. He, S.P. Kulik, J.B. Liu, J.M. Wen, Z.D. Xie, H.Y. Zhang

Quantum Imaging - UMBC

Objective

- Study the physics of multi-photon imaging for entangled state, coherent state and chaotic thermal state: distinguish their quantum and classical nature, in particular, the necessary and/or unnecessary role of quantum entanglement in quantum imaging and lithography.
- Study the “magic mirror” for “ghost” imaging.
- Multi-photon sources and measurement devices.



Two-photon Interference “Dip” with Thermal Light

Approach

- Using entangled two-photon and three-photon states created via optical nonlinear interaction in spontaneous and stimulated modes for multi-photon spatial correlation study and imaging;
- Using chaotic light source, coherent light source for two-photon spatial correlation study and ghost imaging;
- Using photon counting and current-current correlation circuit to explore the nature of two-photon correlation.

Accomplishments

The physics: The nonlocal quantum interference nature of thermal light ghost imaging has been successfully studied and explored theoretically and experimentally.

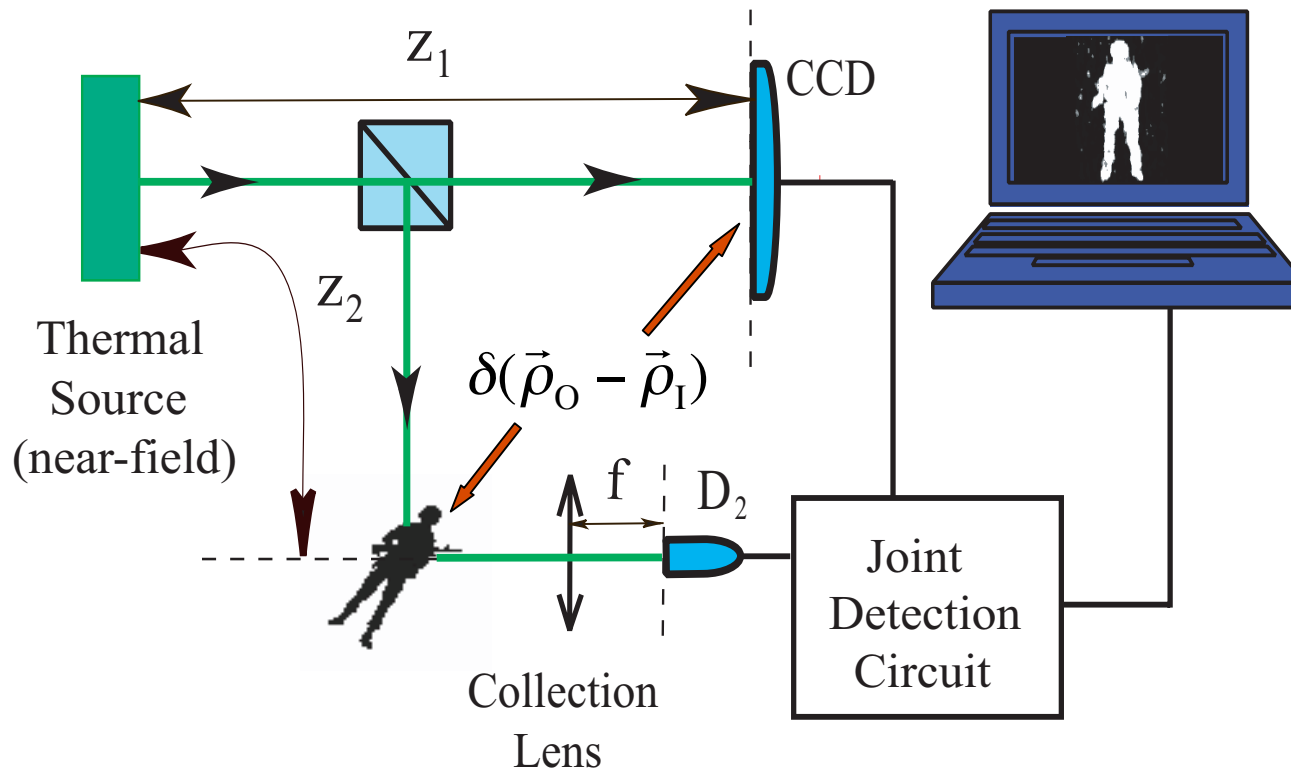
The experiment: (1) Demonstrated and published a new type of ghost imaging experiment by measuring scattered photons from the object target, a successful collaboration with ARL. (2) Observed the two-photon interference “dip” with thermal light, supporting the nonlocal two-photon interference theory of thermal light ghost imaging. (3) Obtained the three-photon temporal correlation of thermal light, which raised a question: Can N-photon correlation be considered as photon bunching?

The theory: (1) Developed the nonlocal multi-photon interference concept and formulism for Nth-order ($N > 1$) coherence of thermal light. (2) Developed a new class of $N+1$ -photon entangled states for ghost imaging with enhanced spatial resolution beyond classical limit.

Part-I

Ghost Imaging with Thermal Light

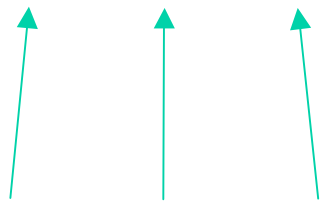
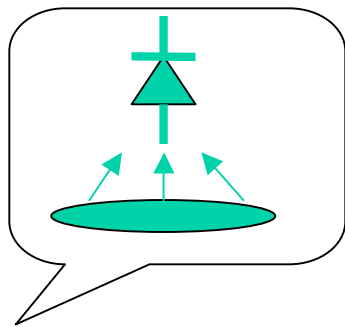
The concept and the advantages



A successful collaboration with ARL

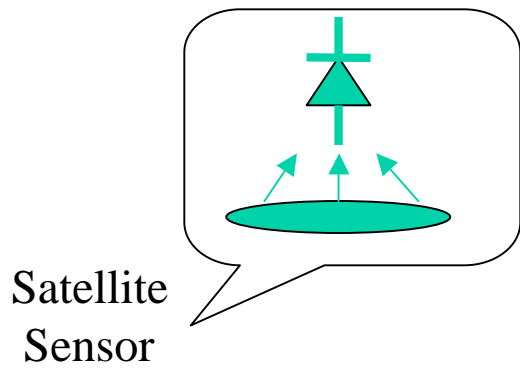
A photon counting detector, D_1 , is used to collect and to count all the photons that are randomly scattered-reflected from the soldier. A CCD array (2D) was facing the light source instead of the object. An image of the soldier was observed in the joint-detection of D_1 and the CCD. ([Near-field lensless ghost imaging.](#))

Satellite
Sensor



Using a point-like photodiode or a “bucket” detector as the sensor. The sensor cannot “see” any details of the field.

Spatial resolution: none

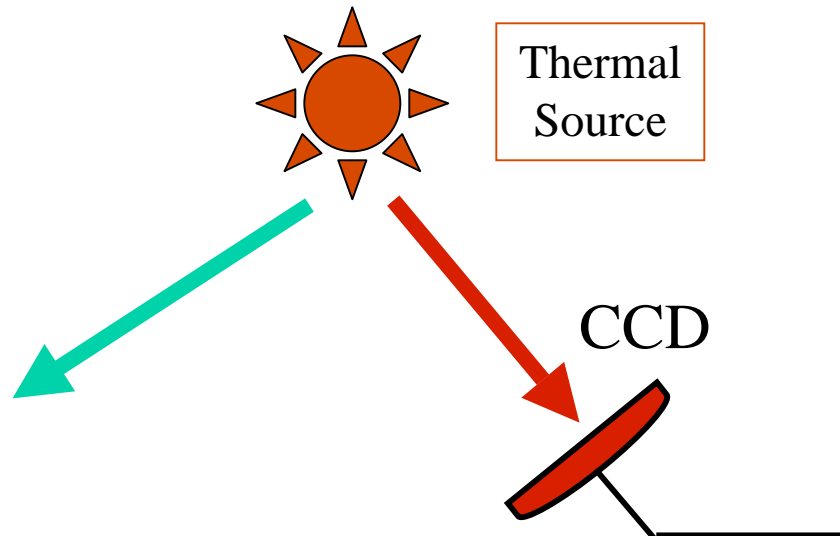
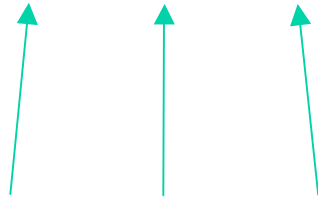


Satellite
Sensor



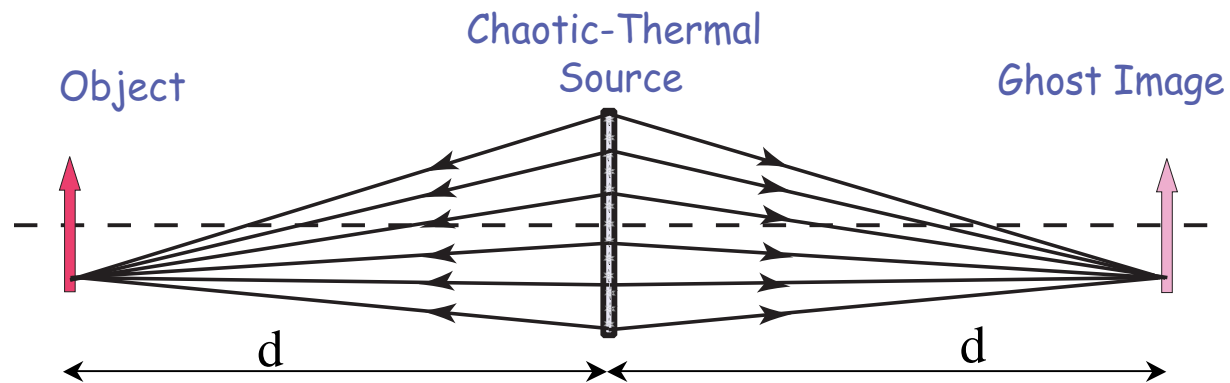
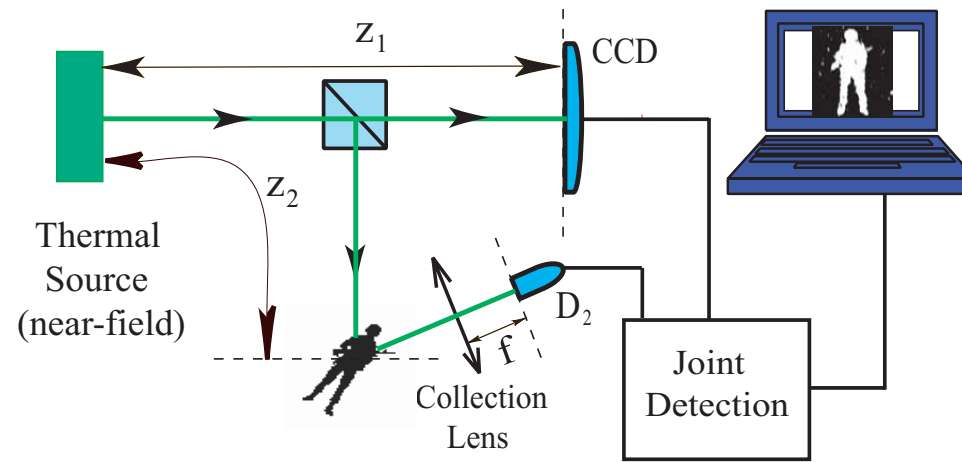
Joint-detection

Ghost Imaging turns a bucket sensor into a CCD camera

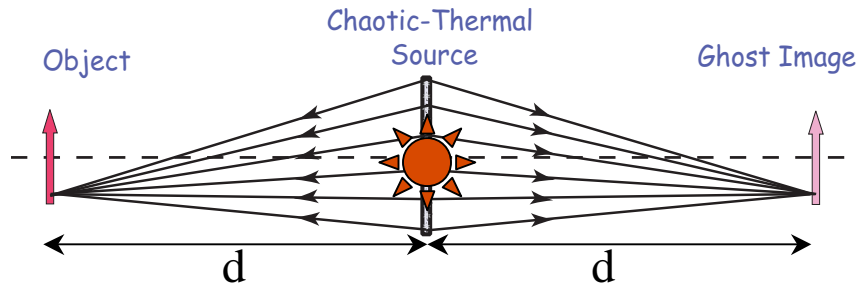


Due to its nonlocal quantum interference nature:

- (1) Nonlocal imaging (useful for certain applications).
- (2) Enhanced spatial resolution (useful for all applications).
- (3) Robust (useful for all applications).



Klyshko's picture of the near-field lensless ghost imaging.
It is easier to see the physics.



Sun: $\frac{D}{d} \cong \Delta\theta \cong 0.53^\circ$

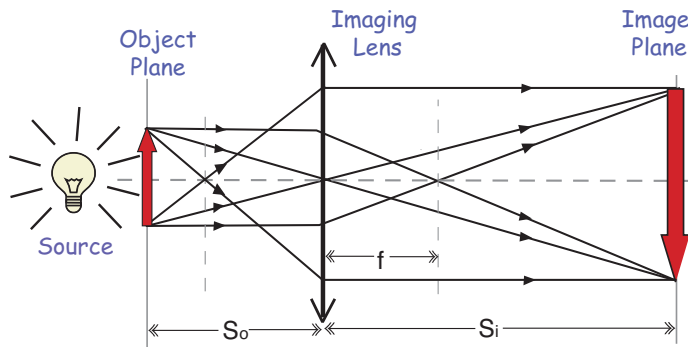
$$R_C(\vec{\rho}_I) \propto \int d\vec{\rho}_O A^2(\vec{\rho}_O) [1 + \text{somb}^2[\frac{D}{d} \frac{\pi}{\lambda} |\vec{\rho}_O - \vec{\rho}_I|]]$$

Lensless ghost imaging:

Spatial Resolution

$$\text{somb}(x) = \frac{2J_1(x)}{x}$$

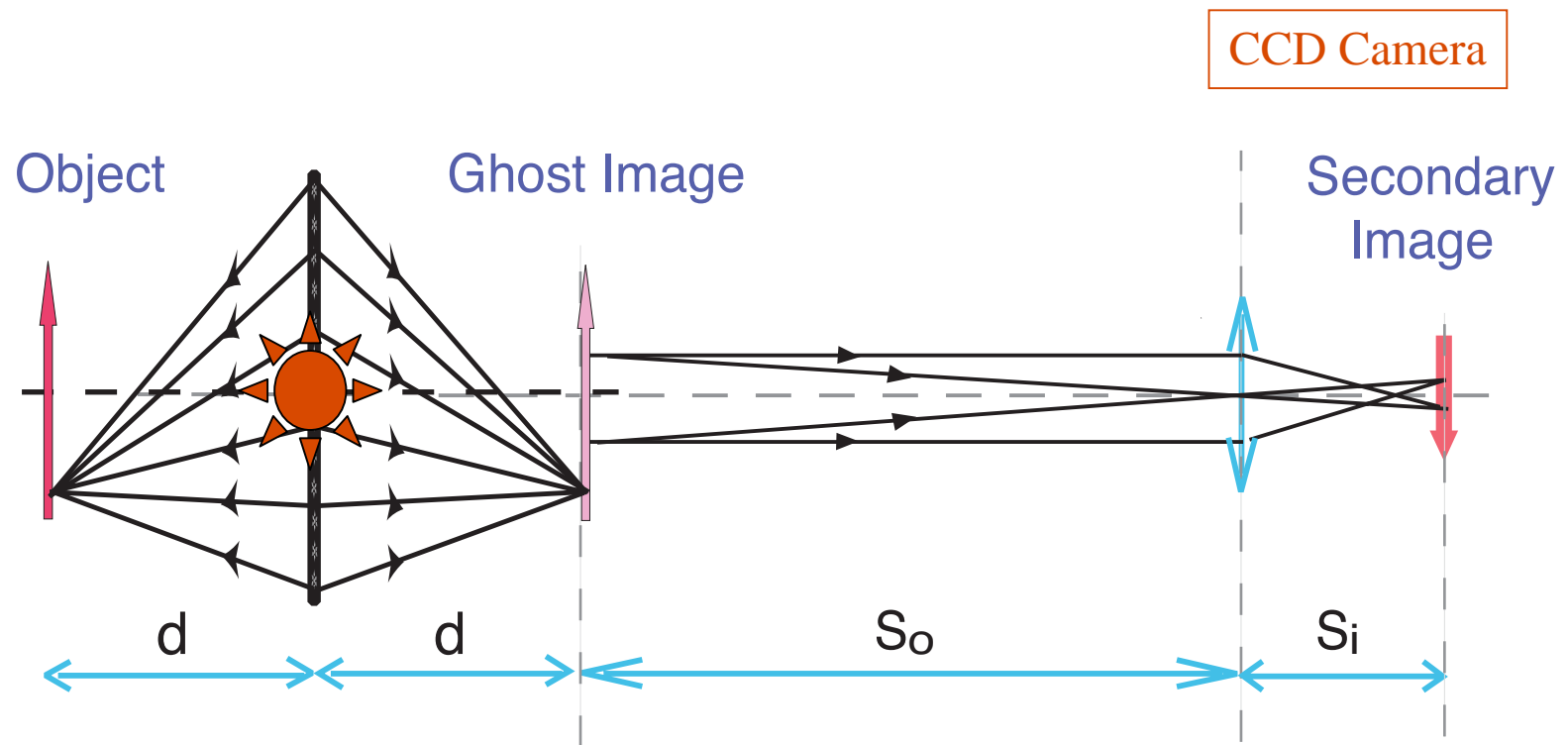
Equivalent: a classical camera with 92 meter lens taking pictures at 10 kilometers.



$$\frac{92 \text{ meters}}{10 \text{ k meters}} = \frac{D}{S_o} \cong \Delta\theta \cong 0.53^\circ$$

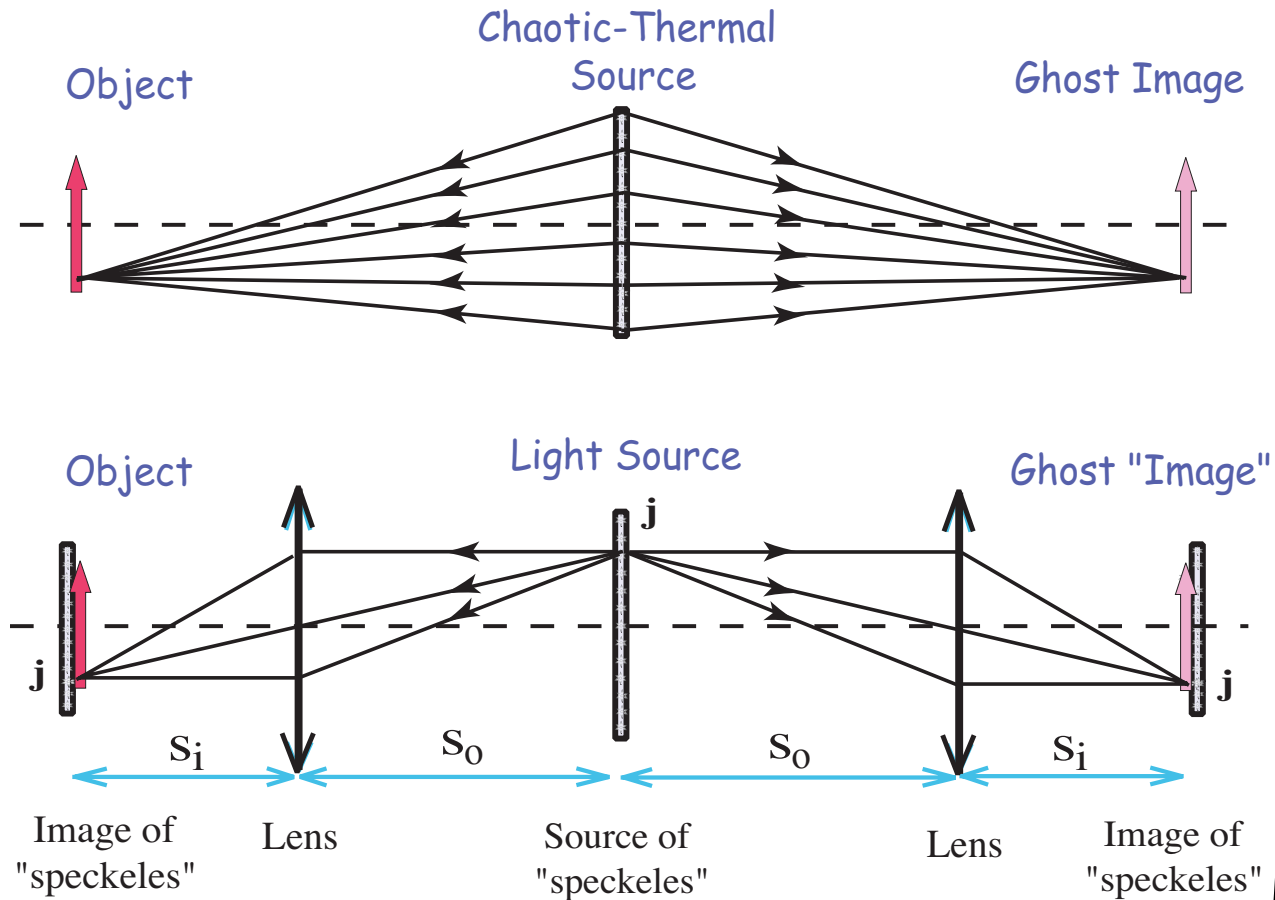
Classical imaging: $I(\vec{\rho}_i) = \int d\vec{\rho}_o A^2(\vec{\rho}_o) \text{somb}^2 \frac{D}{S_o} \frac{\pi}{\lambda} (\vec{\rho}_o - \vec{\rho}_i / m)$

A Conceptual Sun Light Ghost Imaging System



- * Enhanced spatial resolution of the ghost image.
- * Enhanced controllable field of view.

Quantum ghost imaging vs. classical ghost shadow (imager)



The spatial resolution is determined by the size of the “speckles”.

These two types of experiments are very different, and thus explore different physics. No surprise to have different interpretations.

Ghost imaging with thermal light,
for a **large** angular sized **bright** thermal source:

$I(\vec{\rho}_0) \sim \text{constant}$, $I(\vec{\rho}_1) \sim \text{constant}$ **No observable speckles!**

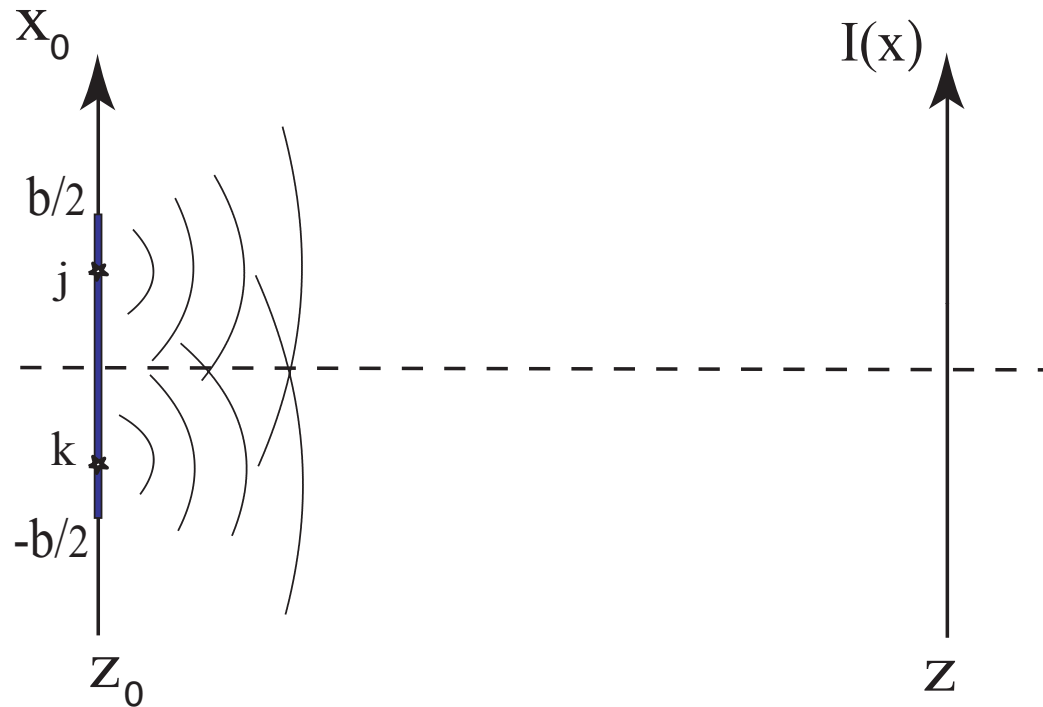
However, the constant intensity distributions do not prevent a nontrivial point-to-point image-forming correlation:

$$g^{(2)}(\vec{\rho}_0, \vec{\rho}_1) = 1 + \text{somb}^2 \left[\frac{D}{d} \frac{\pi}{\lambda} |\vec{\rho}_0 - \vec{\rho}_1| \right]$$



Where it comes from?
It is thermal light!

For a large sized bright thermal source: $\Delta I_j / I_j \sim 0$

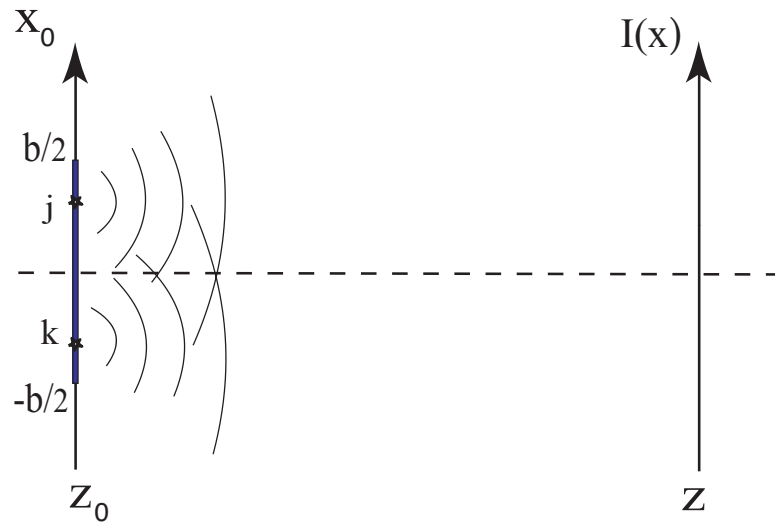


$$I_j = \langle I_j \rangle + \Delta I_j$$

Negligible

$$\langle I_1 I_2 \rangle = \langle I_1 \rangle \langle I_2 \rangle + \langle \Delta I_1 \Delta I_2 \rangle$$

Other classical ideas about ghost imaging with thermal light



A naive idea: “the source produces a pair of photon at the same point of the source, and the photons propagate to the same direction.” – **Thermal state is not entangled state !!!**

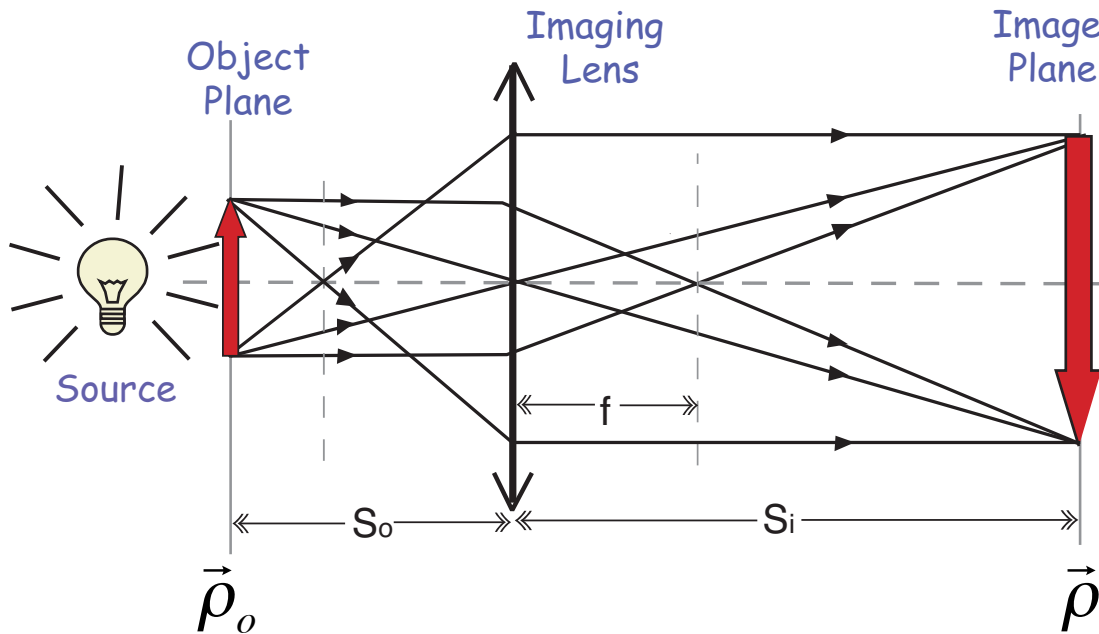
Another naive idea: “The imaging is done one source point at a time, if one takes a “picture” with the object and without the object and subtract them, then if a sufficient number of source points are used one gets an image.” - This is related to the so called “computational ghost imaging”. There are no timing requirements in this case in the sense that each picture can be taken individually and only one detector is required. – **It has nothing to do with ghost imaging !**

(1) The source is thermal, it never radiates from one point at a time; (2) Two photons are required to get an image. This seems obvious since two detectors are necessary and there are timing requirements (the joint detection has to be within the coherence time of the radiation); (3) The jointly measured pair of photons are independent and are created randomly from any one or two sub-sources.



The unique point-to-point image-forming correlation between the object and image planes in ghost imaging is the result of a constructive-destructive **interference** which involves the nonlocal superposition of two-photon amplitudes, a nonclassical entity corresponding to different yet indistinguishable alternative ways of triggering a joint-detection event.

Diffraction-limited Classical Imaging



$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Gaussian thin lens equation

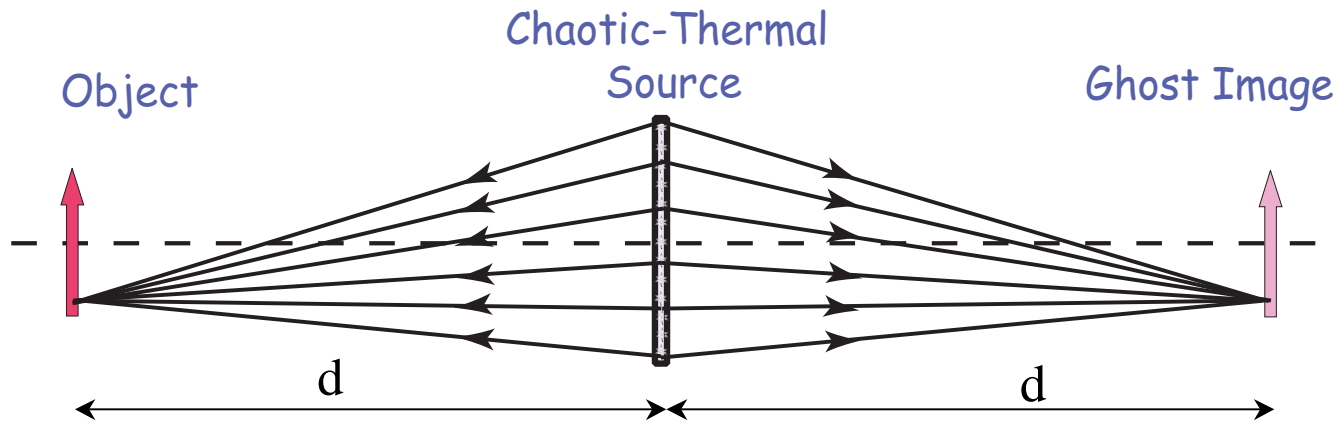
$$I(\vec{\rho}_i) = \int d\vec{\rho}_o A(\vec{\rho}_o) \text{somb}(\vec{\rho}_o - \vec{\rho}_i / m)$$

$$m = s_i / s_o$$

$$\text{somb}(x) = 2J_1(x) / x$$

Point-“spot” relationship between the **object plane** and the **image plane**: result of constructive-destructive **interferences**.

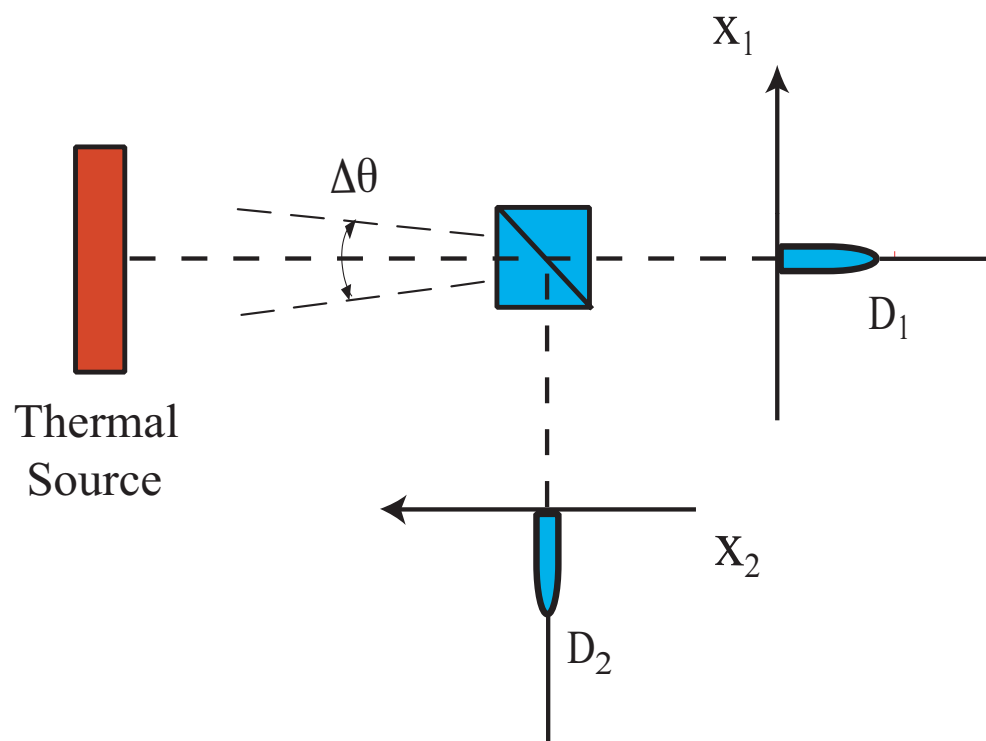
Ghost Imaging with Thermal light



$$R_C(\vec{\rho}_I) \propto \int d\vec{\rho}_O A^2(\vec{\rho}_O) [1 + \text{somb}^2[\frac{D}{d} \frac{\pi}{\lambda} |\vec{\rho}_O - \vec{\rho}_I|]]$$

Point-“spot” correlation between the **object plane** and the **image plane**: result of two-photon constructive-destructive interferences.

Quantum Theory



Glauber's theory of
Photo-detection:

$$\langle E^{(-)}(r,t)E^{(+)}(r,t) \rangle$$

The *probability* to observe a photodetection event at space-time point (r,t) .

$$\langle E^{(-)}(r_1,t_1)E^{(-)}(r_2,t_2)E^{(+)}(r_2,t_2)E^{(+)}(r_1,t_1) \rangle$$

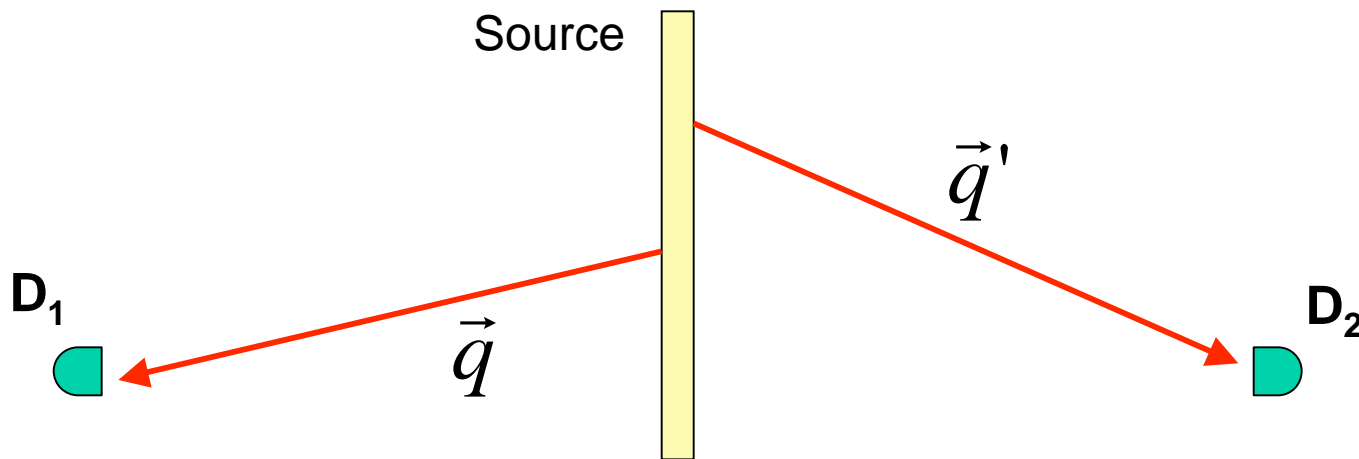
The *probability* to observe a joint photodetection event at space-time points (r_1,t_1) and (r_2,t_2) .

Glauber's theory of photo-detection

The *probability* of observing a photodetection event at space-time point (r,t):

$$G^{(1)}(\vec{\rho}_1, t_1) = \text{tr}\{\hat{\rho} E_1^{(-)} E_1^{(+)}\} = \sum_{\vec{q}} |g_1(\vec{\rho}_1, \vec{q})|^2 = \text{constant}$$

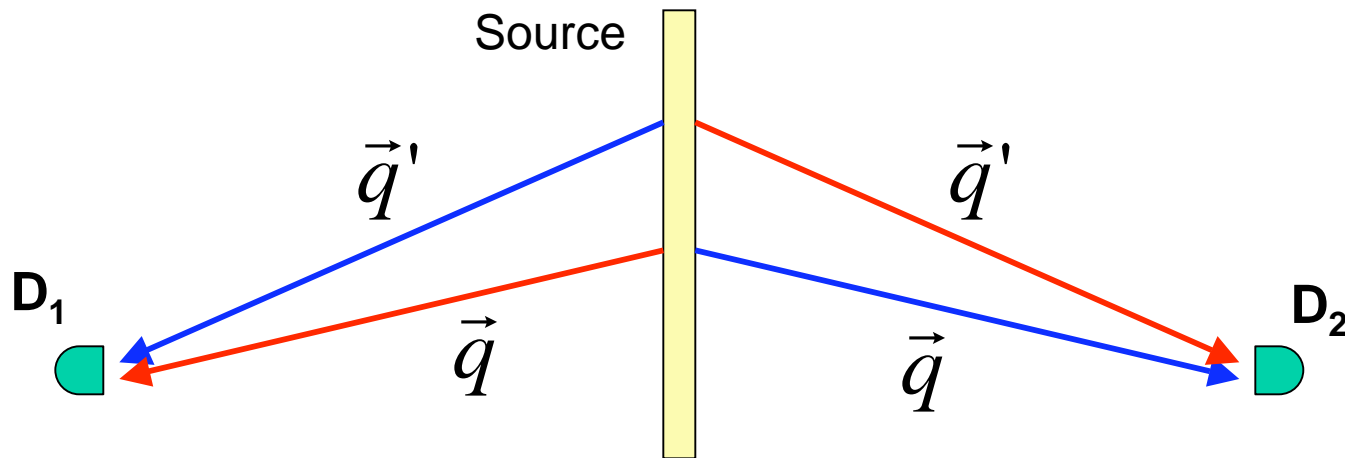
$$G^{(1)}(\vec{\rho}_2, t_2) = \text{tr}\{\hat{\rho} E_2^{(-)} E_2^{(+)}\} = \sum_{\vec{q}'} |g_2(\vec{\rho}_2, \vec{q}')|^2 = \text{constant}$$



Glauber's theory of photo-detection

The *probability* of observing a joint photodetection event at space-time points (r_1, t_1) and (r_2, t_2) :

$$G^{(2)}(\vec{\rho}_1, \vec{\rho}_2) = \text{tr} \{ \hat{\rho} E_1^{(-)} E_2^{(-)} E_2^{(+)} E_1^{(+)} \}$$
$$= \sum_{\vec{q}, \vec{q}'} \left| \frac{1}{\sqrt{2}} \left[\underline{\underline{g_2(\vec{\rho}_2, \vec{q}) g_1(\vec{\rho}_1, \vec{q}')}} + \underline{\underline{g_2(\vec{\rho}_2, \vec{q}') g_1(\vec{\rho}_1, \vec{q})}} \right] \right|^2$$



It is a two-photon interference phenomenon!

$$\begin{aligned}
G^{(2)}(\vec{\rho}_1, \vec{\rho}_2) &= \text{tr} \{ \hat{\rho} E_1^{(-)} E_2^{(-)} E_2^{(+)} E_1^{(+)} \} \\
&= \sum_{\vec{q}, \vec{q}'} \left| \frac{1}{\sqrt{2}} \left[\underline{\underline{g_2(\vec{\rho}_2, \vec{q}) g_1(\vec{\rho}_1, \vec{q}')}} + \underline{\underline{g_2(\vec{\rho}_2, \vec{q}') g_1(\vec{\rho}_1, \vec{q})}} \right] \right|^2 \\
&\propto 1 + \text{somb}^2 \left[\frac{D}{d} \frac{\pi}{\lambda} |\vec{\rho}_0 - \vec{\rho}_1| \right]
\end{aligned}$$

In Dirac's language: a pair of photons only interferes with itself, interference between two different pairs never occurs.

In Glauber's theory of photo-detection, the second-order correlation of thermal light is the result of two-photon interference. The interference occurs at the single-photon level, involves a jointly measured pair of photons and the superposition of two probability amplitudes of the measured pair, corresponding to two different yet indistinguishable alternative ways for the pair to produce a joint-detection event. The two superposed amplitudes in each individual superposition belong to one pair of jointly measured photons. In Dirac's language: a pair of photons only interferes with itself, interference between two different pairs never occurs.

$$1 + \delta(\vec{\rho}_1 - \vec{\rho}_2) = \sum_{\vec{q}, \vec{q}'} \left| \frac{1}{\sqrt{2}} [g_2(\vec{\rho}_2, \vec{q})g_1(\vec{\rho}_1, \vec{q}') + g_2(\vec{\rho}_2, \vec{q}')g_1(\vec{\rho}_1, \vec{q})] \right|^2$$

Under certain experimental condition, each individual superposition achieves constructive interference at certain space-time points. The sum of these individuals thus achieves its maximum value. In other space-time points, however, each individual superposition may achieve a different constructive-destructive interference condition and results in an averaged value of the sum.

What can we gain from the quantum interference ?

* Spatial resolution: the spatial resolution is a function of the transverse size of the thermal source:

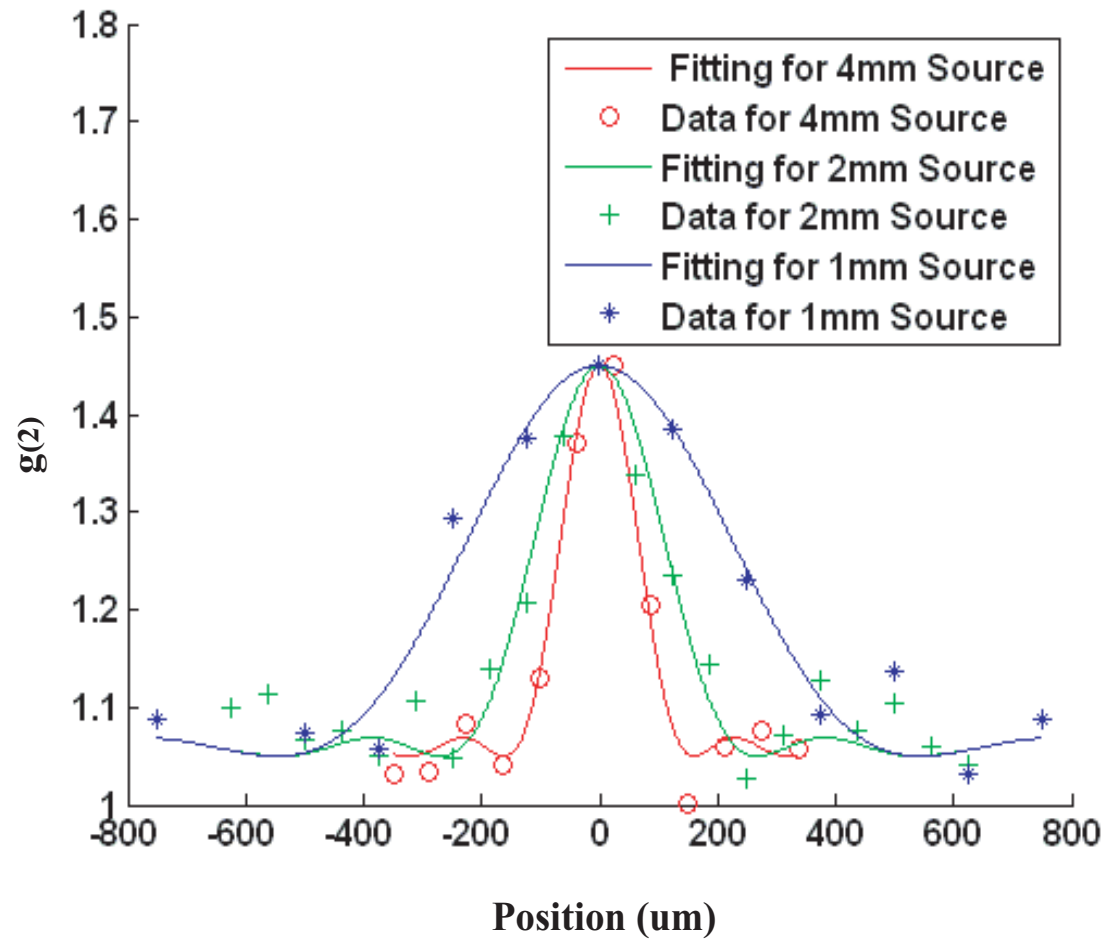
$$\text{somb}^2 \left\{ \frac{D}{d} \frac{\pi}{\lambda} |\vec{\rho}_0 - \vec{\rho}_1| \right\}$$

If $D/d \sim 0.53^\circ \dots$ 

* Robust: any “loss” has no affect on the ghost imaging, except a longer data taking time. A photon counting joint-detection coincidence circuit can cover from the single-photon level to bright light condition by the use of ND filters.

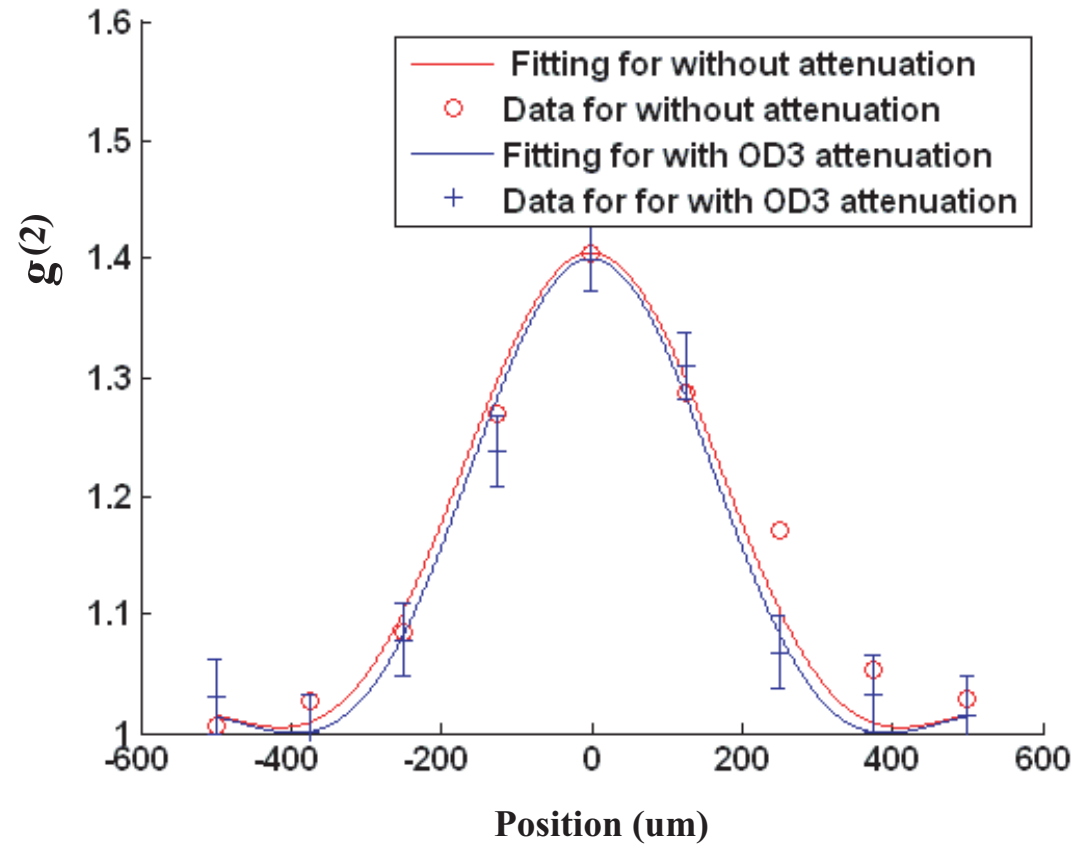
J.B. Liu and Y.H. Shih, to be published.

Experiment 1: Spatial resolution



$$g^{(2)}(x_1 - x_2) = 1 + \text{sinc}^2\left[\frac{D}{d} \frac{\pi}{\lambda} |x_1 - x_2|\right]$$

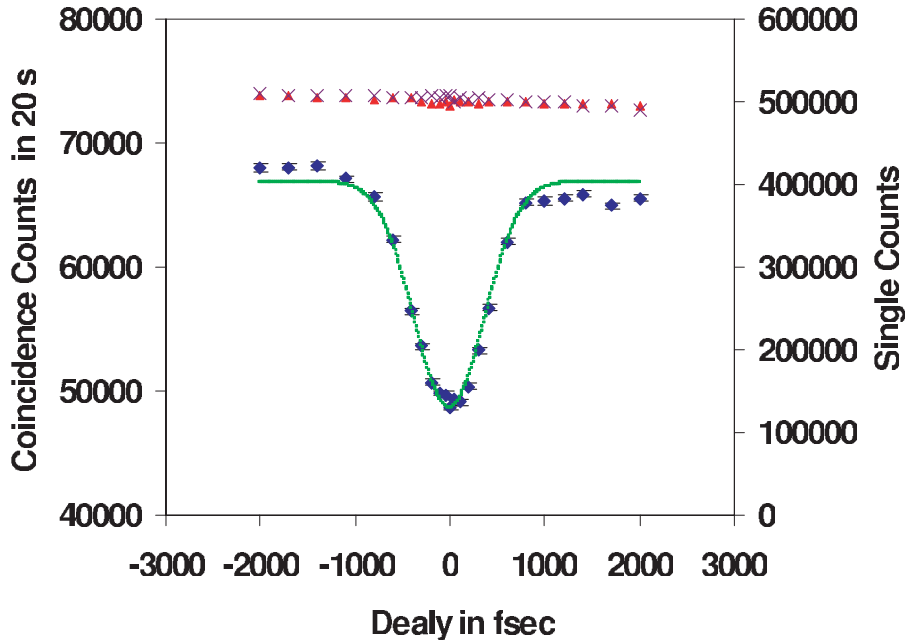
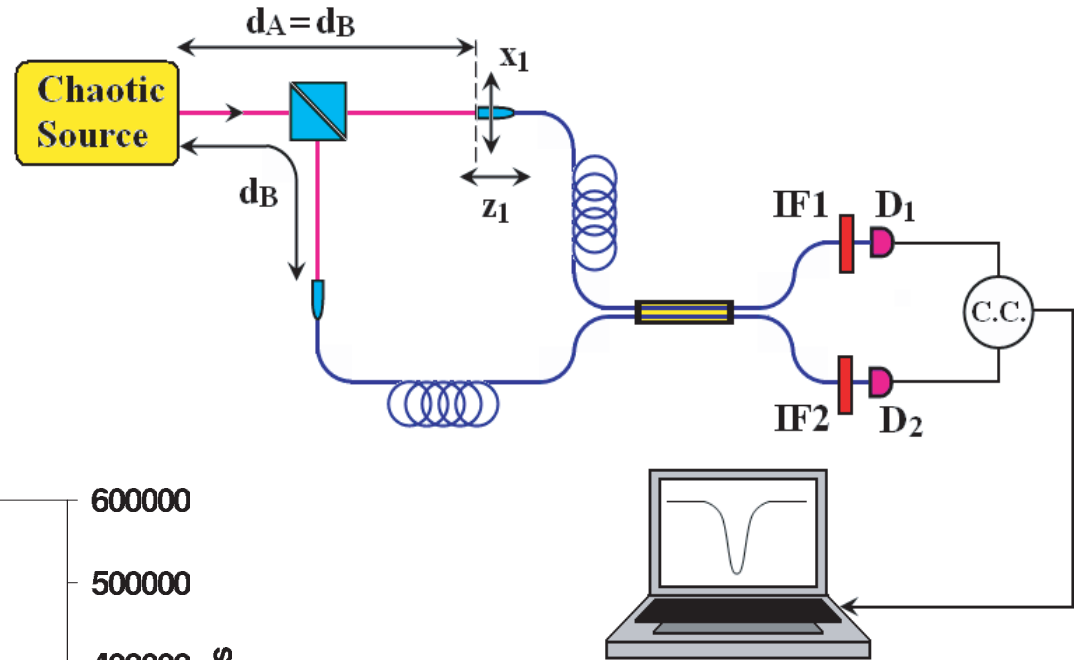
Experiment 2: Robust



UMBC: 10^3 , ARL: 10^4 ...

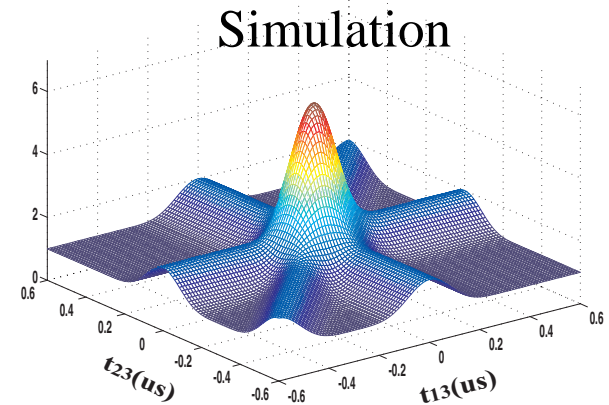
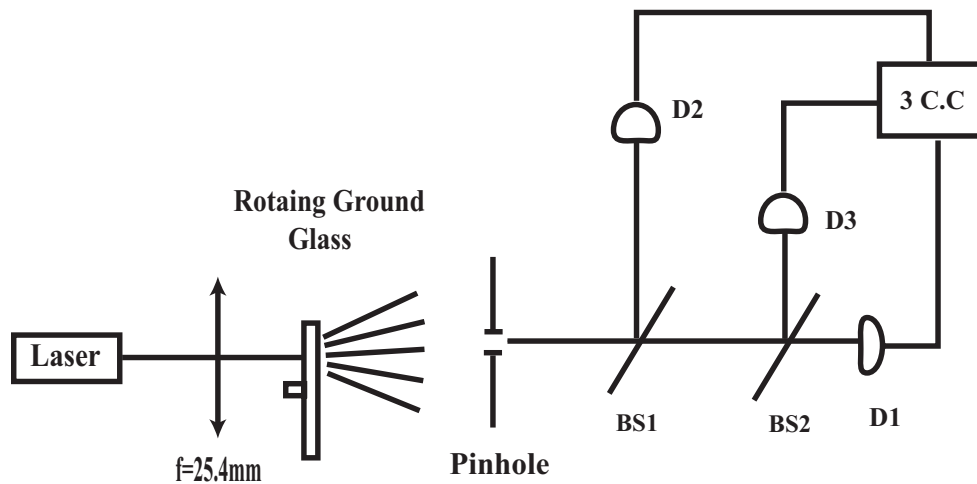
Experiment 3: Two-photon interference – the “Dip”

Experimental condition:
No first-order interference



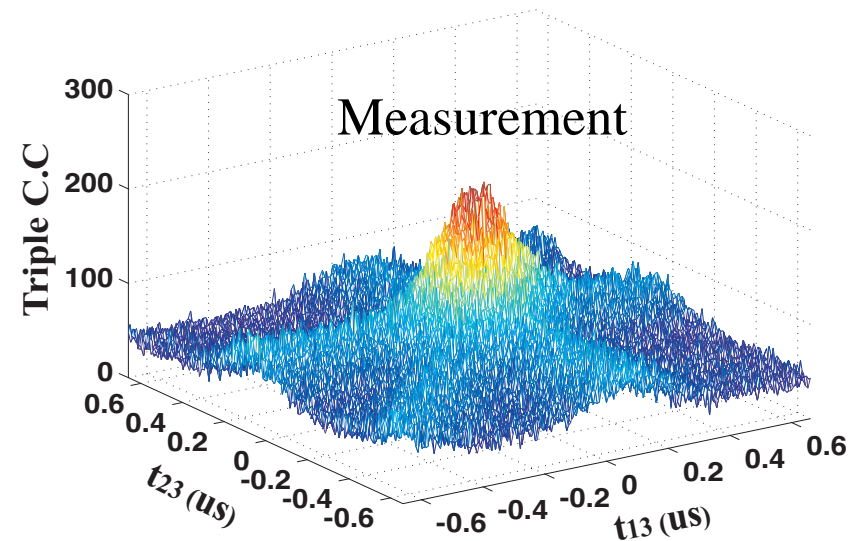
The counting rate of both D_1 and D_2 are constants (no first-order interference). The “dip” is observed in coincidences.

Experiment 4: Thermal light three-photon correlation



$$G^{(3)}(t_1, t_2, t_3)$$

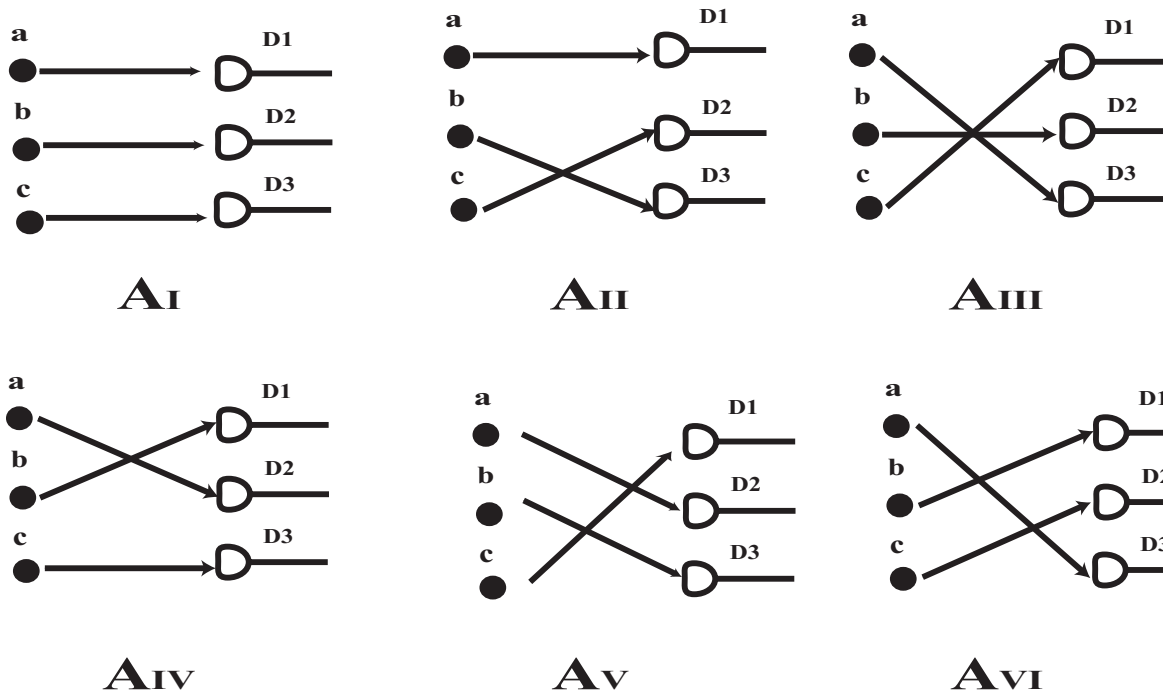
$$= \sum_{\text{Sub-Source}} \left| \frac{1}{\sqrt{6}} (A_I^{(3)} + A_{II}^{(3)} + A_{III}^{(3)} + A_{IV}^{(3)} + A_V^{(3)} + A_{VI}^{(3)}) \right|^2$$



Can N-photon correlation be considered as photon bunching?

Superposition of three-photon amplitudes

$$G^{(3)}(t_1, t_2, t_3) = \sum_{a,b,c} \left| \frac{1}{\sqrt{6}} (A_I^{(3)} + A_{II}^{(3)} + A_{III}^{(3)} + A_{IV}^{(3)} + A_V^{(3)} + A_{VI}^{(3)}) \right|^2$$



$$G^{(N)}(t_1, \dots, t_N) = \sum_{\text{Sub-Source}} \left| \frac{1}{\sqrt{N!}} (A_I^{(N)} + \dots + A_{N!}^{(N)}) \right|^2$$

Superposition of
N-photon amplitudes

Experiment 5: Thermal light three-photon spatial correlation and imaging

Experiment in progress,
in collaboration with John Howell of Rochester.

Experiment 6: 3-D ghost imaging for medical applications

Experiments in progress,
in collaboration with Harvard Medical School.

Experiment 7: Sun light ghost imaging

Experiments in progress,
in collaboration with ARL and NGC.

Experiment 8: Ghost imaging of the Moon

Experiment in planning,
in collaboration with ARL and NGC.

Part-II

Resolution for a Class of $N+1$ -Photon Entangled States

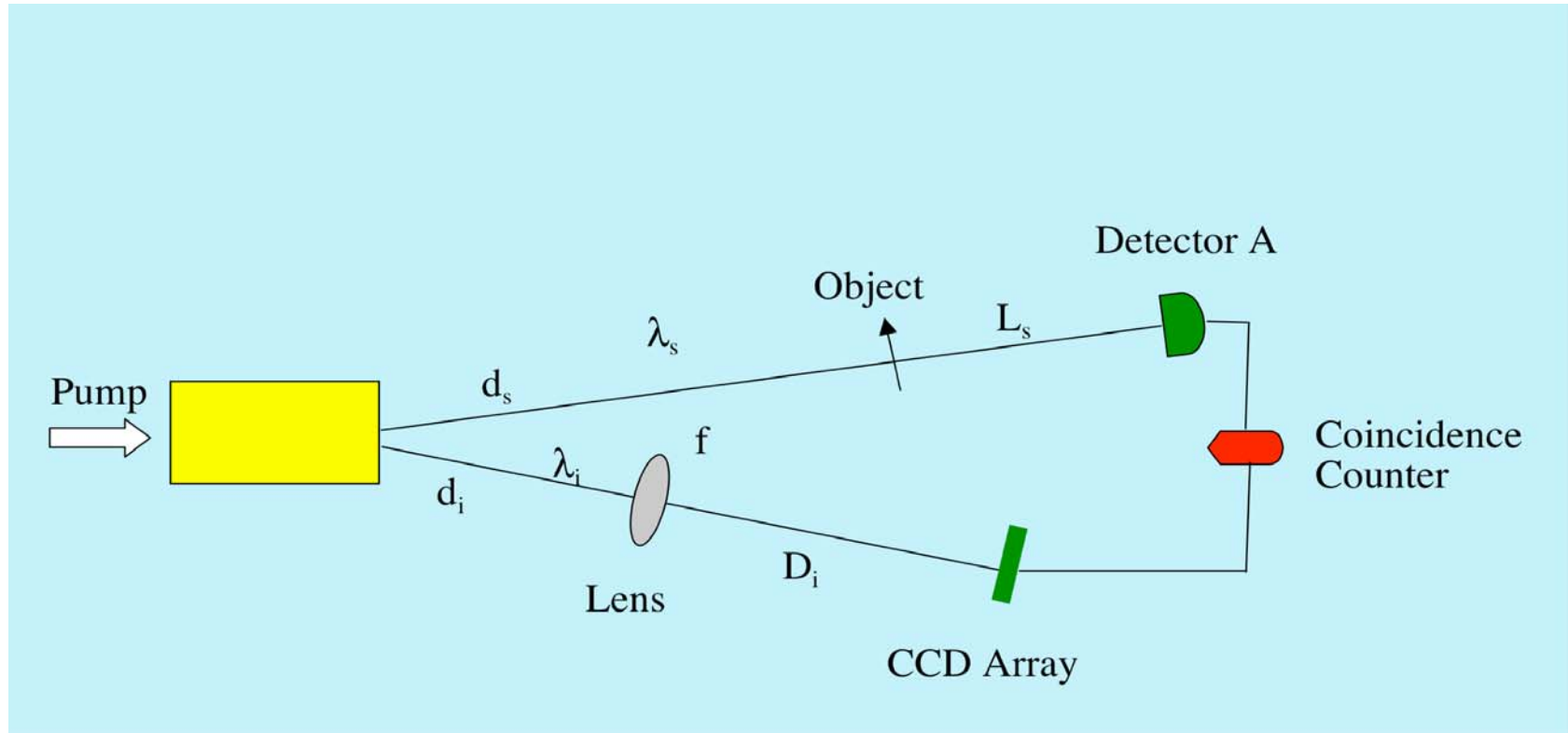
UMBC Quantum Optics Group

Morton H. Rubin

Jianming Wen

Yanhua Shih

Reminder about the two-photon non-degenerate case.



$$\frac{\lambda_i}{\lambda_s} \frac{1}{R_s} + \frac{1}{D_i} = \frac{1}{f}$$

$$R_s = d_s + \frac{\lambda_i}{\lambda_s} d_i$$

Object:

$$t(\vec{\rho}_A) = t_o \delta(\vec{\rho}_A) + t_1 \delta(\vec{\rho}_A - \vec{a})$$

Radius of the Airy disk

$$\alpha_B = x \frac{\lambda_i D_i}{2R_L}$$

R_L = radius of lens

Minimum resolvable
distance (Rayleigh
criterion)

$$\begin{aligned} a_{\min} &= \frac{\lambda_s \alpha_B}{\lambda_i D_i} R_s \\ &= x \frac{\lambda_s d_s + \lambda_i d_i}{2R_L} \end{aligned}$$

No improvement in resolution.

General Formulation for Three Photons

The approach is similar to that used for biphoton imaging:

$$R_{3cc} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt_1 \int_0^T dt_2 \int_0^T dt_3 C(t_1, t_2, t_3) S(t_1, t_2, t_3)$$

Coincidence window

$$C(t_1, t_2, t_3) = \int d^2 \rho_1 \int d^2 \rho_2 \int d^2 \rho_3 \langle \Psi | E_1^{(-)} E_2^{(-)} E_3^{(-)} E_3^{(+)} E_2^{(+)} E_1^{(+)} | \Psi \rangle$$

$$\langle \Psi | E_1^{(-)} E_2^{(-)} E_3^{(-)} E_3^{(+)} E_2^{(+)} E_1^{(+)} | \Psi \rangle = |A|^2 \quad A = \langle 0 | E_3^{(+)} E_2^{(+)} E_1^{(+)} | \Psi \rangle$$

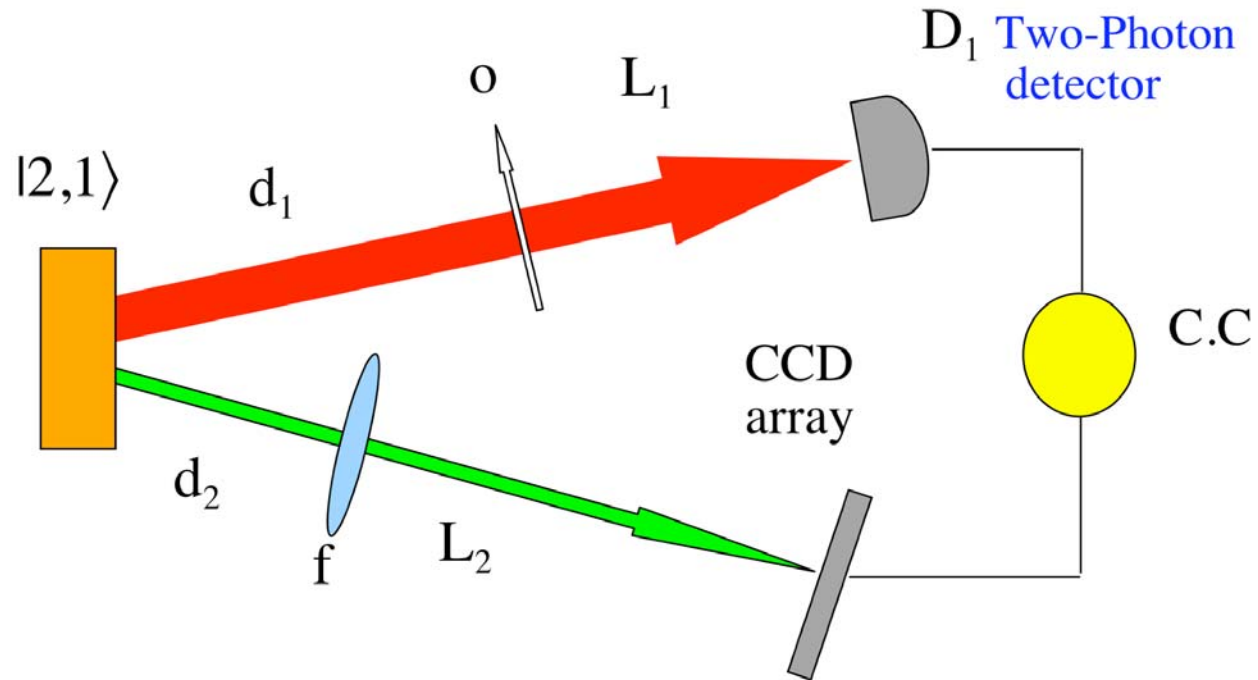
$$A = \sum_{\alpha\beta\gamma} G_{123, \alpha\beta\gamma} A_{\alpha\beta\gamma}$$

$$A_{\alpha\beta\gamma} = \langle 0 | a_\alpha a_\beta a_\gamma | \Psi \rangle \quad \text{Source}$$

$$G_{123, \alpha\beta\gamma} = g_{1\alpha} g_{2\beta} g_{3\gamma} \quad \text{Classical Propagation}$$

Triphoton amplitude

2+1 state



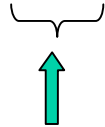
Two *identical photons* are sent to the object and detected with a two-photon detector. These photons are entangled with the third photon that is detected on a CCD array.

Idealized case in which we assume that the source emits a pair of degenerate photons that are entangled with a non-degenerate third photon.

$$A = \sum_{\alpha\beta\gamma} G_{112,\alpha\alpha\beta} A_{\alpha\alpha\beta}$$

$$A_{\alpha\alpha\beta} = \langle 0 | a_{\alpha}^2 a_{\beta} | \Psi \rangle \quad \text{Source}$$

$$G_{112,\alpha\alpha\beta} = g_{1\alpha} g_{1\alpha} g_{2\beta} \quad \text{Classical Propagation}$$



Pair is detected by a single two-photon point detector

Assumptions:

The three photons are entangled:

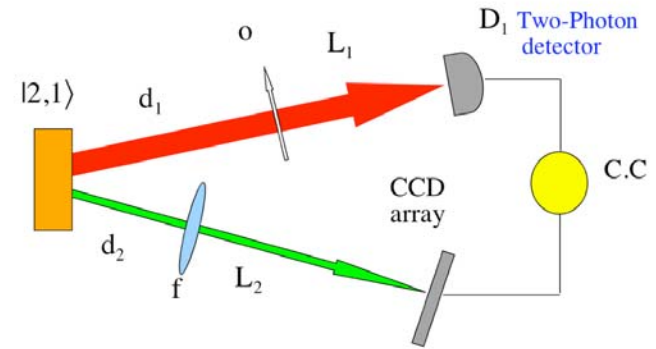
$$A_{\alpha\alpha\beta} \propto \delta(2\omega_\alpha + \omega_\beta - \omega_p) \delta(2\vec{k}_\alpha + \vec{k}_\beta)$$

The state contains two-degenerate photons which *illuminate a single point* on the object and are detected by a two-photon point detector;

the third photon goes to an CCD array after passing through the imaging lens.

The three photon coincidence gives rise to an image provided the GTLE is satisfied:

$$\frac{1}{f} = \frac{1}{L_2} + \frac{1}{d_2 + (\lambda_1 / 2\lambda_2)d_1}$$



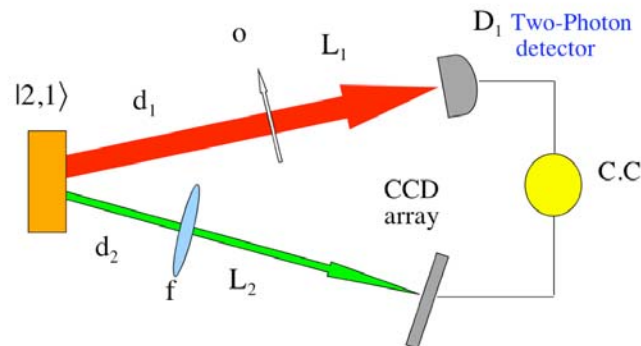
If we assume that the aperture of the system is determined by the lens and is given by R , then, using the Rayleigh criterion

Airy disk

$$1.22 \frac{\lambda_2 L_2}{2R}$$

Minimum resolution

$$1.22 \frac{\lambda_2}{2R} \left(d_2 + \frac{\lambda_1}{2\lambda_2} d_1 \right)$$



Under the assumptions given, we see that the degenerate pair acts like a single photon of wavelength $\lambda_1/2$.

The results above are the same as those found for ghost imaging with two non-degenerate photons.

Morton H. Rubin and Yanhua Shih, *Resolution of Ghost Imaging for Non-degenerate Spontaneous Parametric Down Conversion*, Phys. Rev. A 78, 033836 (2008).

We can replace the 2-photon point detector with a 2-photon bucket detector ($\rho_1 \neq \rho_2$). The effect of this is to change the image from a coherent image to an incoherent image.

Issues:

Projecting the degenerate pair on a single point of the source will not be easy.

(A single point means a small area such that the object reflectance does not vary. We have estimated that the spot size is of order

$\sqrt{\lambda_1 L_1}$ for the case of a two-photon point detector.)

Unlike the original ghost imaging with two photons, the *object must be scanned*.

For the “GHZ” type of state used the loss of one photon destroys any entanglement, this has two effects:

There is no three photon coincidence so the resolution is not affected but rather the exposure time increases.

Generalization to N+1 photon imaging

Assumptions:

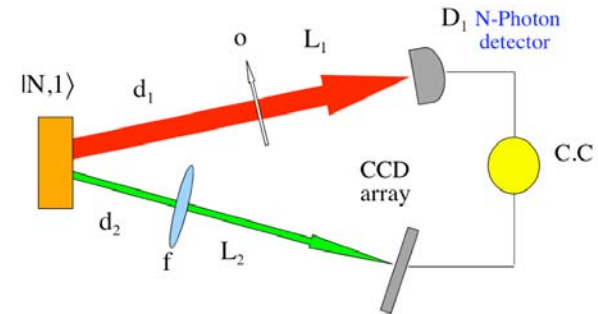
The N+1 photons are entangled;

The state contains N-degenerate photons which illuminate a single point on the object and are detected by an N-photon point detector;

The non-degenerate photon goes to an CCD array after passing through the imaging lens.

The N+1 photon coincidence gives rise to an image provided the GTLE is satisfied:

$$\frac{1}{f} = \frac{1}{L_2} + \frac{1}{d_2 + (\lambda_1 / N\lambda_2)d_1}$$



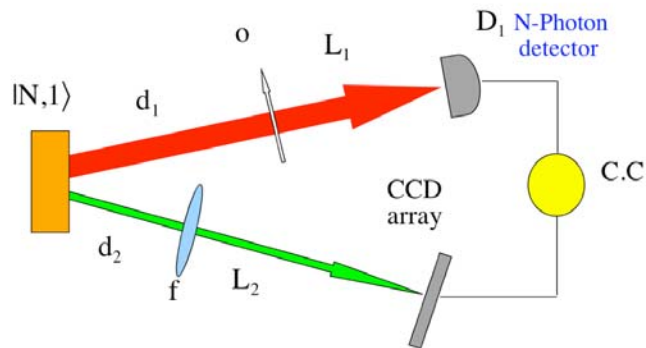
If we assume that the aperture of the system is determined by the lens and is given by R , then, using the Rayleigh criterion

Airy disk

$$1.22 \frac{\lambda_2 L_2}{2R}$$

Minimum resolution

$$1.22 \frac{\lambda_2}{2R} \left(d_2 + \frac{\lambda_1}{N \lambda_2} d_1 \right)$$



Under the assumptions given, we see that the degenerate N photons act like a single photon of wavelength λ_1/N .

The results above are the same as those found for ghost imaging with two non-degenerate photons.

We can replace the N -photon point detector with an N -photon bucket detector. The effect of this is to change the image from a coherent image to an incoherent image.

The **issues** here are the same as those for three photons only the difficulties increase with increasing N .

Different types of entangled three photon states

1. GHZ type state

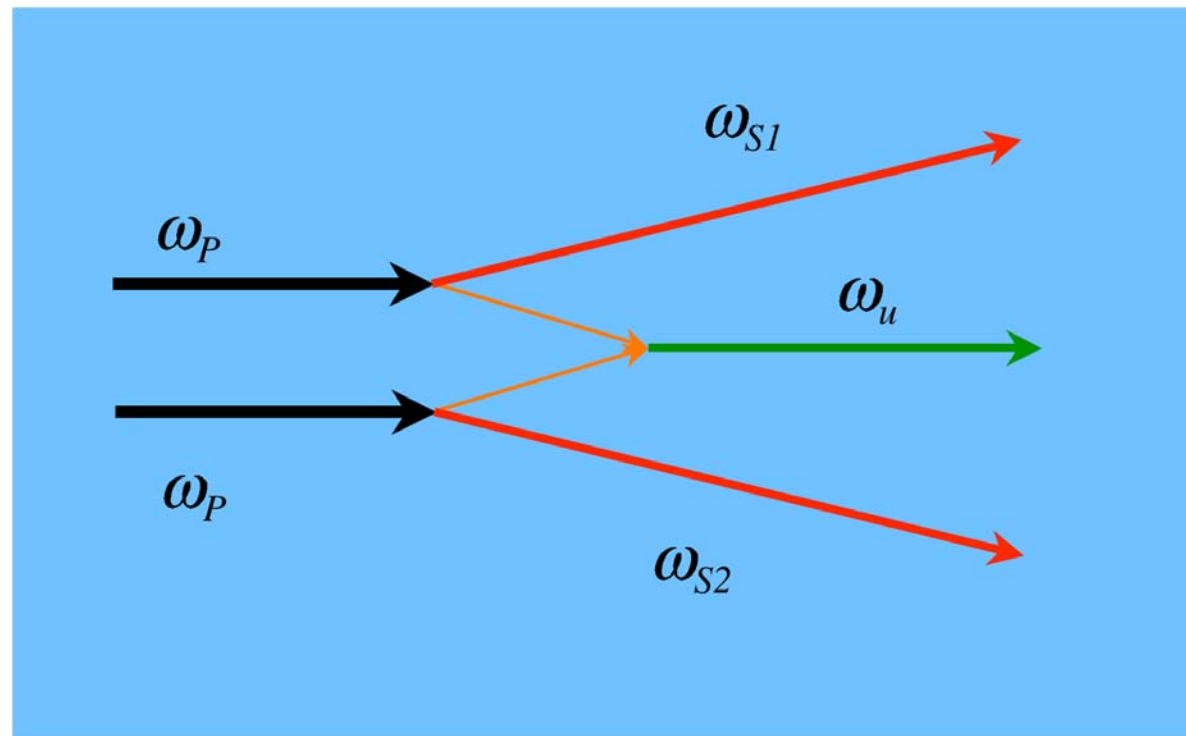
If one photon is lost the remaining pair is not entangled.

2. W type state

If one photon is lost the remaining pair is entangled, in general, it is not maximally entangled.

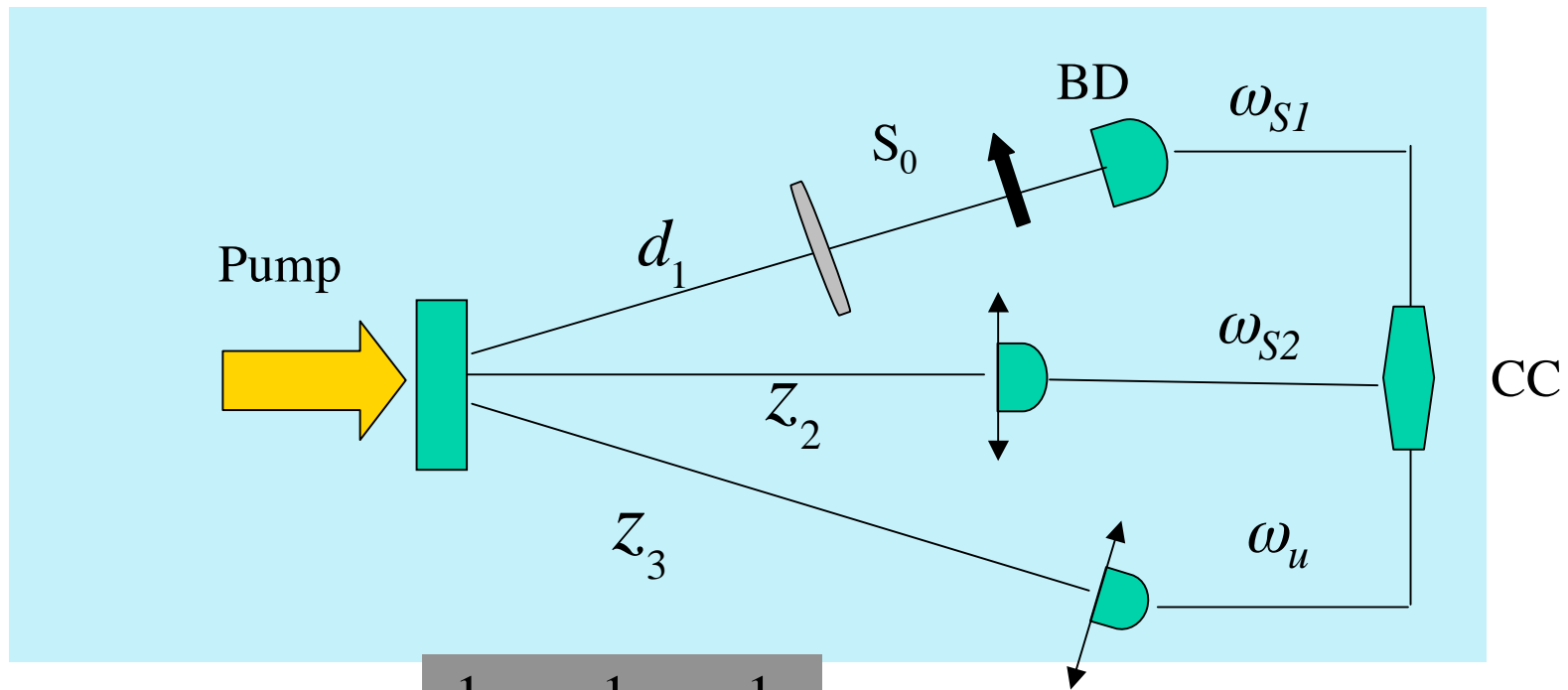
Brief review of results using “W” type states

Triphoton Entanglement Generation:
Two down conversions and one up conversion



Keller, Rubin, Shih and Wu, PRA **57**, 2076, (1998)

Example



Gaussian Equation:

$$\frac{1}{S_0} + \frac{1}{d_1 + \zeta} = \frac{1}{f}$$

$$\frac{1}{\zeta} = \frac{\lambda_{s1}}{\lambda_{s2} z_2} + \frac{\lambda_{s1}}{\lambda_u z_3}$$



Parallel Combination

We have shown that:

A ghost image occurs in the three photon coincident counting

A Gaussian thin lens equation holds.

If one photon is not detected there is still an image but the GTLE changes and the image is blurred.

Jianming Wen, Thesis, UMBC, *Transverse Correlation in Entangled Photons and Light-Matter Interactions* (Jan. 20007)

Jianming Wen, P. Xu, Morton H. Rubin, and Yanhua Shih, *Phys. Rev. A* **76**, 023828 (2007)

Jianming Wen, Morton H. Rubin, and Yanhua Shih, *Phys. Rev. A* **76**, 045802 (2007)

Current Work

Use of entangled and thermal beams for imaging.

Three photon imaging with beams.

Analysis of non-Gaussian entangled states at the photon level and for beams.