



# Quantum Imaging Theory

Jonathan P. Dowling

Quantum Science & Technologies Group  
Hearne Institute for Theoretical Physics  
Department of Physics & Astronomy  
Louisiana State University, Baton Rouge

<http://quantum.phys.lsu.edu/>

ARO Quantum Imaging MURI Review, NWU, FRI 13 NOV 09

1. “Quantum Entanglement in Optical Interferometry,” Hwang Lee, Christoph F. Wildfeuer, Sean D. Huver, and Jonathan P. Dowling, invited chapter for: Handbook of Optics, Eric VanStryland, Editor (Optical Society of America, in press).

1. Christoph F. Wildfeuer, Aaron J. Pearlman, Jun Chen, Jingyun Fan, Alan Migdall, Jonathan P. Dowling, Resolution and sensitivity of a Fabry-Perot interferometer with a photon-number-resolving detector, arXiv:0905.1085 (Physical Review A, in press).
2. Uskov, DB; Kaplan, L; Smith, AM; Huver, SD; Dowling, JP. 2009. Maximal success probabilities of linear-optical quantum gates. PHYSICAL REVIEW A 79 (4): art. no.-042326.
3. Huver SD, Wildfeuer CF, Dowling JP, Entangled Fock States for Robust Quantum Optical Sensors, Physical Review A 78, 063828 (2008).
1. Wildfeuer CF, Dowling JP, Strong Violations of Bell-type Inequalities for Werner States, Physical Review A 78, 032113 (2008).

1. Tae-Woo Lee, Sean D. Huver, Hwang Lee, Lev Kaplan, Steven B. McCracken, Changjun Min, Dmitry B. Uskov, Christoph F. Wildfeuer, Georgios Veronis, Jonathan P. Dowling, Optimization of quantum interferometric metrological sensors in the presence of photon loss, arXiv:0908.3008 (preprint, submitted to Physical Review Letters).
2. William N. Plick, Christoph F. Wildfeuer, Petr M. Anisimov, Jonathan P. Dowling, Optimizing the Multi-Photon Absorption Properties of N00N States, arXiv:0908.1370 (preprint, Physical Review A, in press).
3. Yang Gao, Christoph F. Wildfeuer, Petr M. Anisimov, Hwang Lee, Jonathan P. Dowling, Super-Resolution at the Shot-Noise Limit with Coherent States and Photon-Number-Resolving Detectors, arXiv:0907.2382, (preprint, submitted to Journal of Modern Optics).
4. Sulakshana Thanvanthri, Kishore T. Kapale, Jonathan P. Dowling, Ultra-Stable Matter-Wave Gyroscopy with Counter-Rotating Vortex Superpositions in Bose-Einstein Condensates, arXiv:0907.1138 (preprint, submitted to Physical Review Letters).
5. Hugo Cable, Reeta Vyas, Surendra Singh, Jonathan P. Dowling, A non-degenerate optical parametric oscillator as a high-flux source for quantum lithography, arXiv:0903.4268 (preprint, New Journal of Physics, in press).
6. Olson SJ, Dowling JP, Probability, Unitarity, and Realism in Generally Covariant Quantum Information, arXiv:0708.3535 (preprint, submitted to Classical and Quantum Gravity).
7. Petr M. Anisimov and Jonathan P. Dowling, Super resolution with superposition, Physics 2, 52 (2009) (Viewpoint article, invited).
  1. Dowling, JP. 2009. The Age of Entanglement When Quantum Physics Was Reborn. SCIENCE 325 (5938): 269-269 (book review, invited).

## Viewpoint

### Super resolution with superposition

**Petr M. Anisimov and Jonathan P. Dowling**

*Hearne Institute for Theoretical Physics, Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803*

Published June 22, 2009

*A proposal for obtaining optical resolution better than the classical limit by means of spatially entangled quantum states of light opens a new frontier in the fields of quantum optical imaging, metrology, and sensing.*

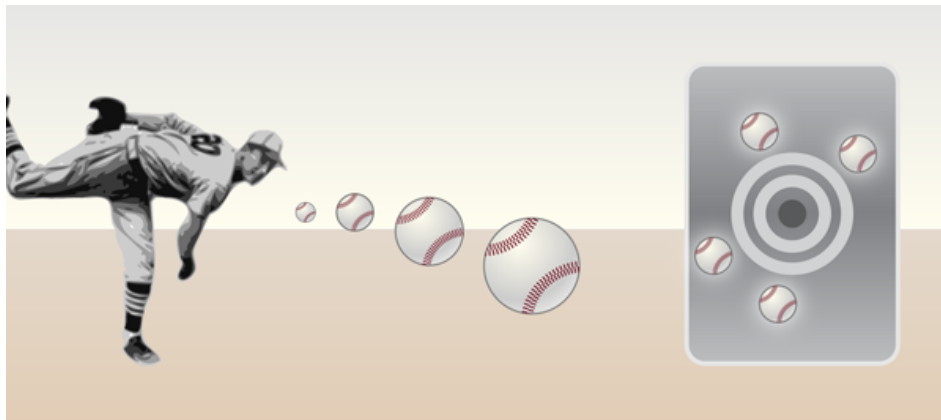
Subject Areas: **Quantum Information, Optics**

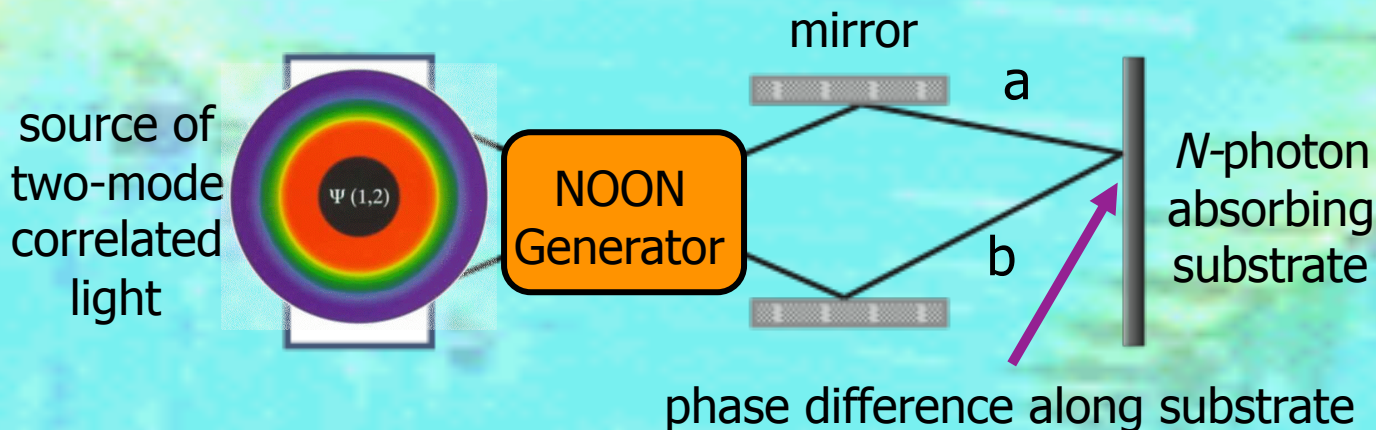
**A Viewpoint on:**

**Quantum Imaging beyond the Diffraction Limit by Optical Centroid Measurements**

Mankei Tsang

*Phys. Rev. Lett.* 102, 253601 (2009) – Published June 22, 2009

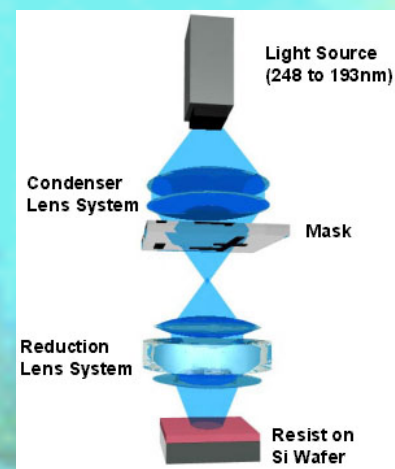




Deposition rate:  $\Delta_N(\varphi) = \left\langle \left( \hat{a}^\dagger + e^{-i\varphi} \hat{b}^\dagger \right)^N \left( \hat{a} + e^{+i\varphi} \hat{b} \right)^N \right\rangle$

Classical input :  $\Delta_N(\varphi) = \cos^{2N}(\varphi / 2)$

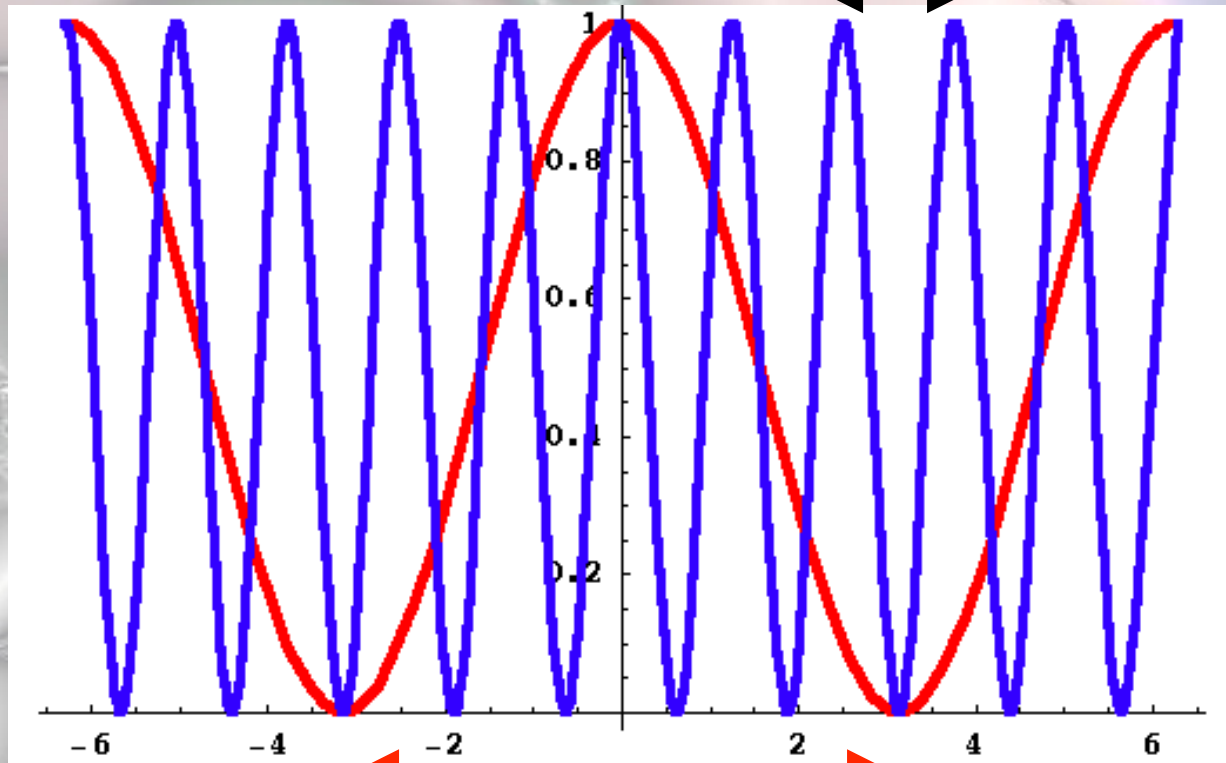
NOON input :  $\Delta_N(\varphi) = \cos^2(N\varphi / 2)$



***Super-resolution, beating the classical diffraction limit.***

# Super-Resolution á la NOON

$$\lambda / N$$



$N=1$  (classical)

$N=5$  (NOON)

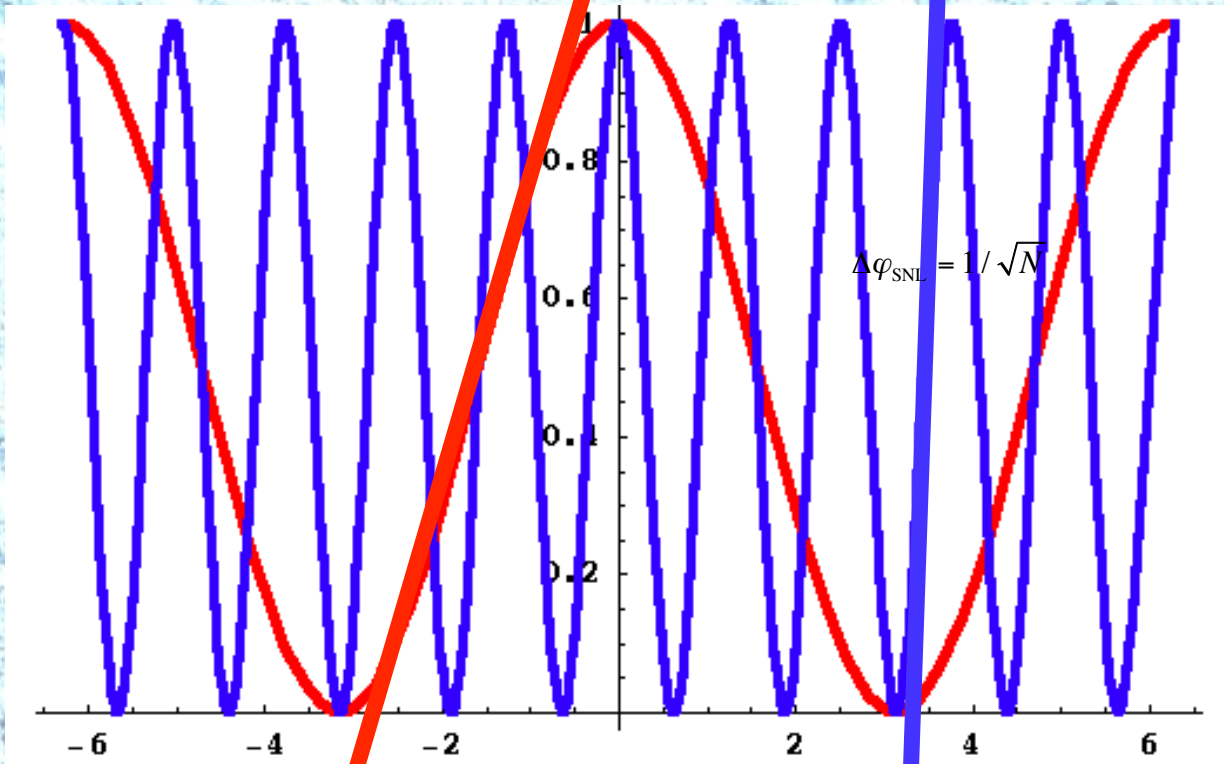
$\lambda$

N=1 (classical)  
N=5 (NOON)

# Super-Sensitivity

$$\Delta\varphi = \frac{\Delta\hat{P}}{\left|d\langle\hat{P}\rangle/d\varphi\right|}$$

$$dP_N/d\varphi$$



Shot-Noise Limit

$$\Delta\varphi_{\text{SNL}} = 1/\sqrt{N}$$

Heisenberg Limit

$$\Delta\varphi_{\text{HL}} = 1/N$$

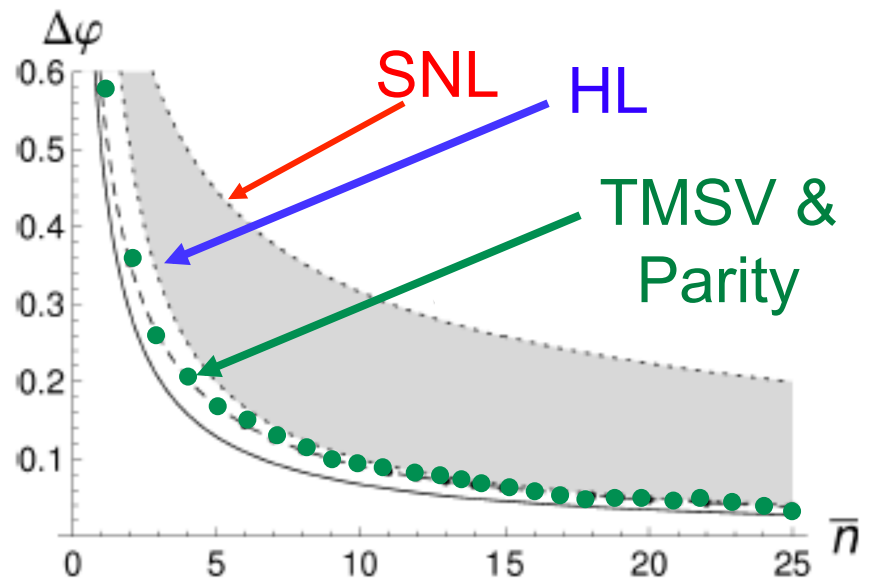
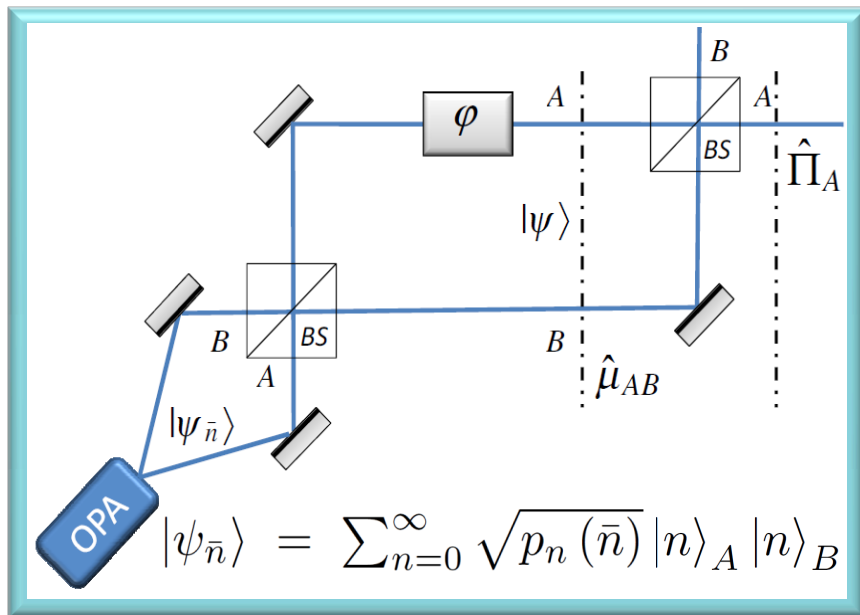
$$dP_1/d\varphi$$

# Quantum Metrology with Two-Mode Squeezed Vacuum: Parity Detection Beats the Heisenberg Limit!

<arXiv:0910.5942v1> (submitted to Phys. Rev. Lett.)

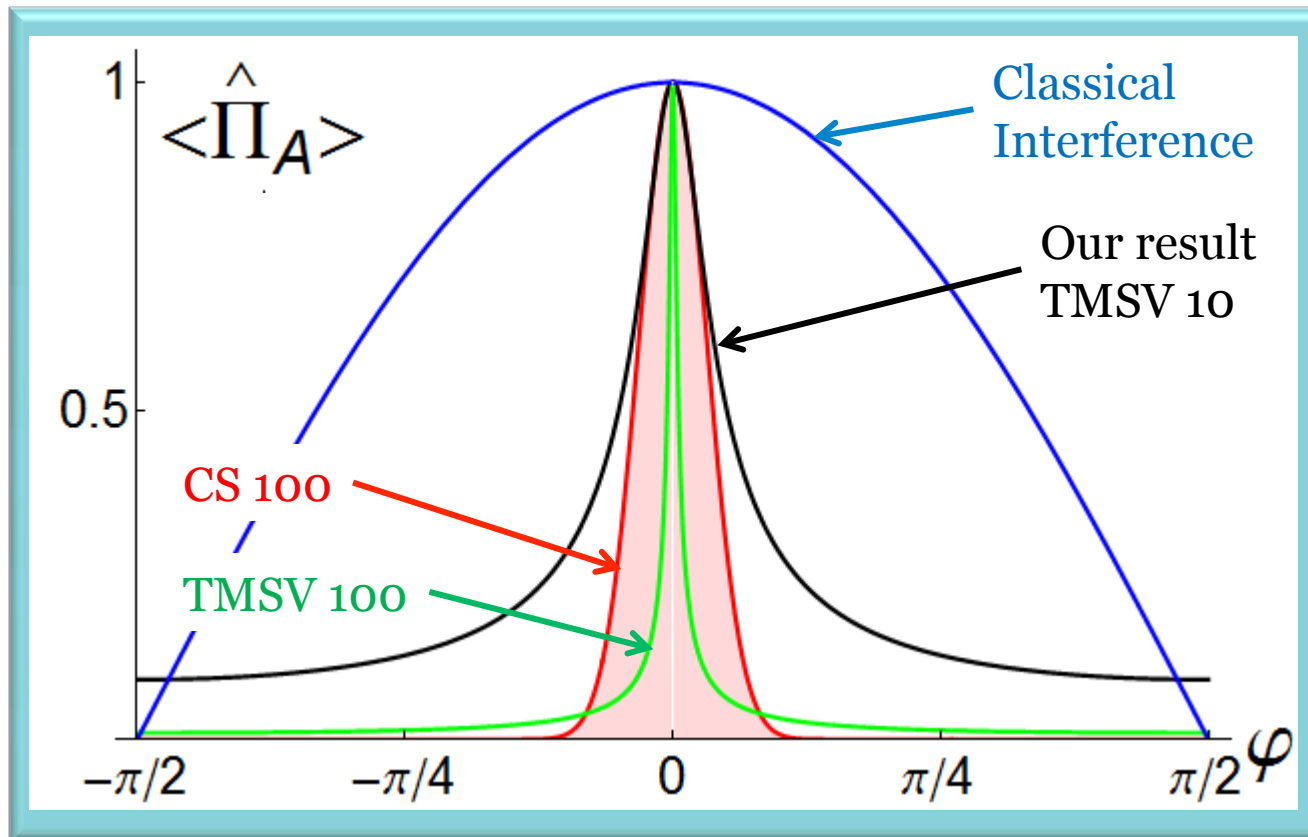
Petr M. Anisimov, Gretchen M. Raterman, Aravind Chiruvelli, William N. Plick,  
Sean D. Huver, Hwang Lee, Jonathan P. Dowling

We study the sensitivity and resolution of phase measurement in a Mach-Zehnder interferometer with two-mode squeezed vacuum ( $\langle n \rangle$  photons on average). **We show that super-resolution and sub-Heisenberg sensitivity is obtained with parity detection.** In particular, in our setup, dependence of the signal on the phase evolves  $\langle n \rangle$  times faster than in traditional schemes, and uncertainty in the phase estimation is better than  $1/\langle n \rangle$ .



# Quantum Metrology with Two-Mode Squeezed Vacuum: Parity Detection Beats the Heisenberg Limit!

<arXiv:0910.5942v1> (submitted to Phys. Rev. Lett.)

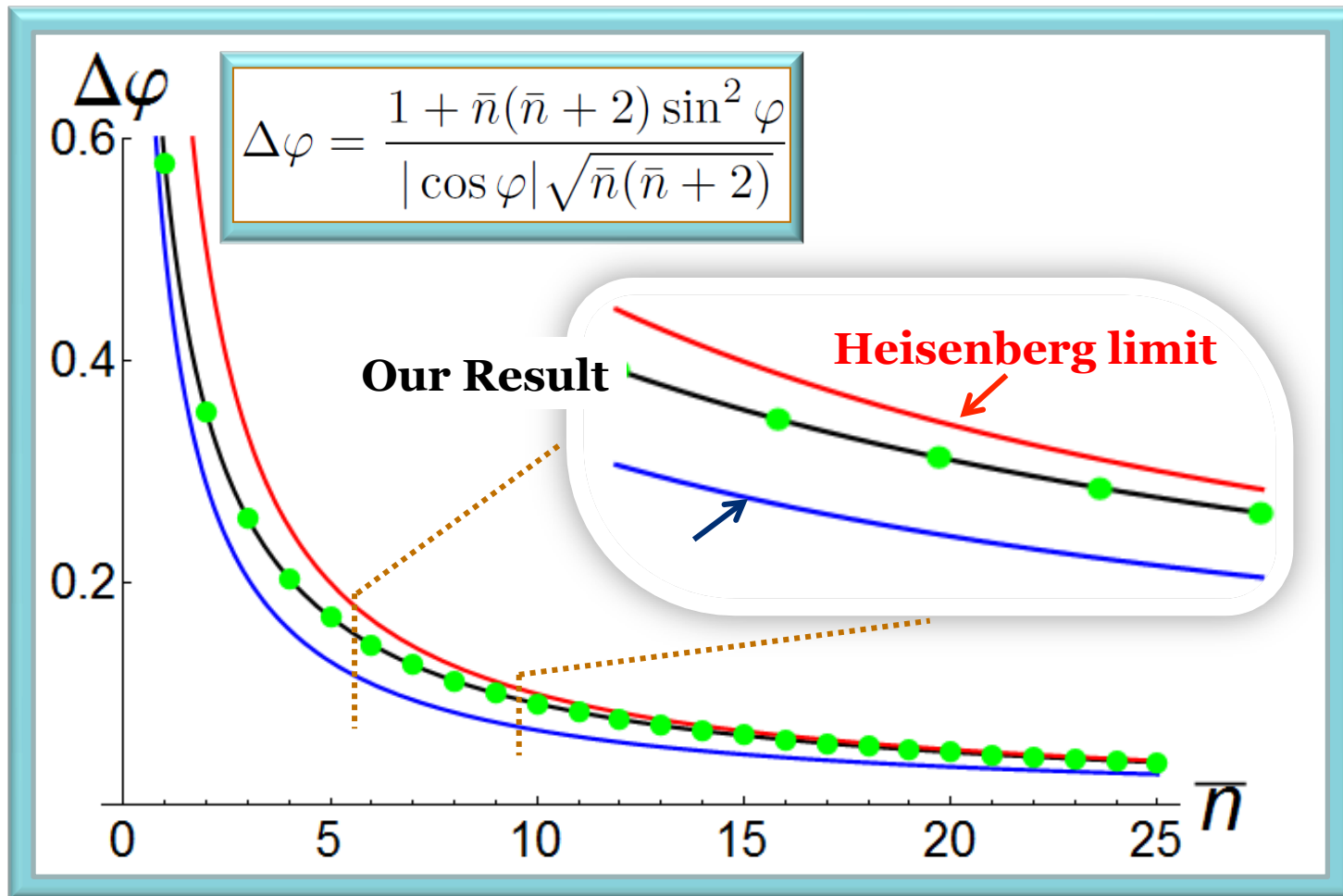


## Super-Duper Phase Resolution



# Quantum Metrology with Two-Mode Squeezed Vacuum: Parity Detection Beats the Heisenberg Limit!

<arXiv:0910.5942v1> (submitted to Phys. Rev. Lett.)



## Super-Duper Phase Sensitivity

# Optimization of Quantum Interferometric Metrological Sensors In the Presence of Photon Loss

<arXiv:0908.3008v1> (Phys. Rev. A, in press)

Tae-Woo Lee, Sean D. Huver, Hwang Lee, Lev Kaplan, Steven B. McCracken, Changjun Min, Dmitry B. Uskov, Christoph F. Wildfeuer, Georgios Veronis, Jonathan P. Dowling

We optimize two-mode, entangled, number states of light in the presence of loss in order to maximize the extraction of the available phase information in an interferometer. Our approach optimizes over the entire available input Hilbert space with no constraints, other than fixed total initial photon number.

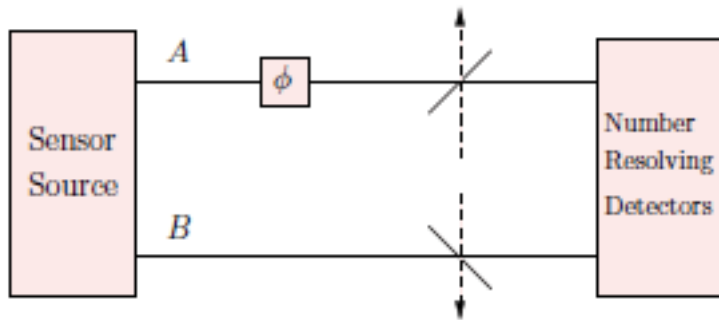
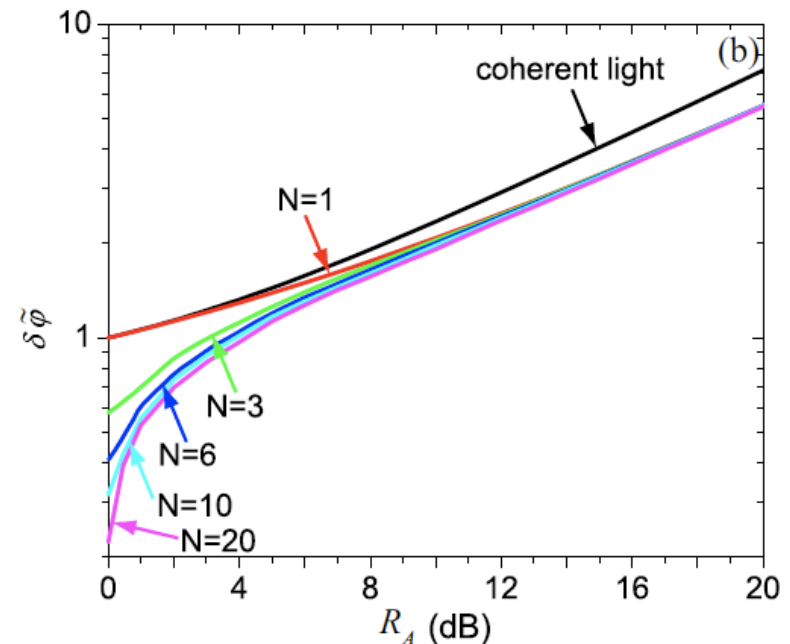
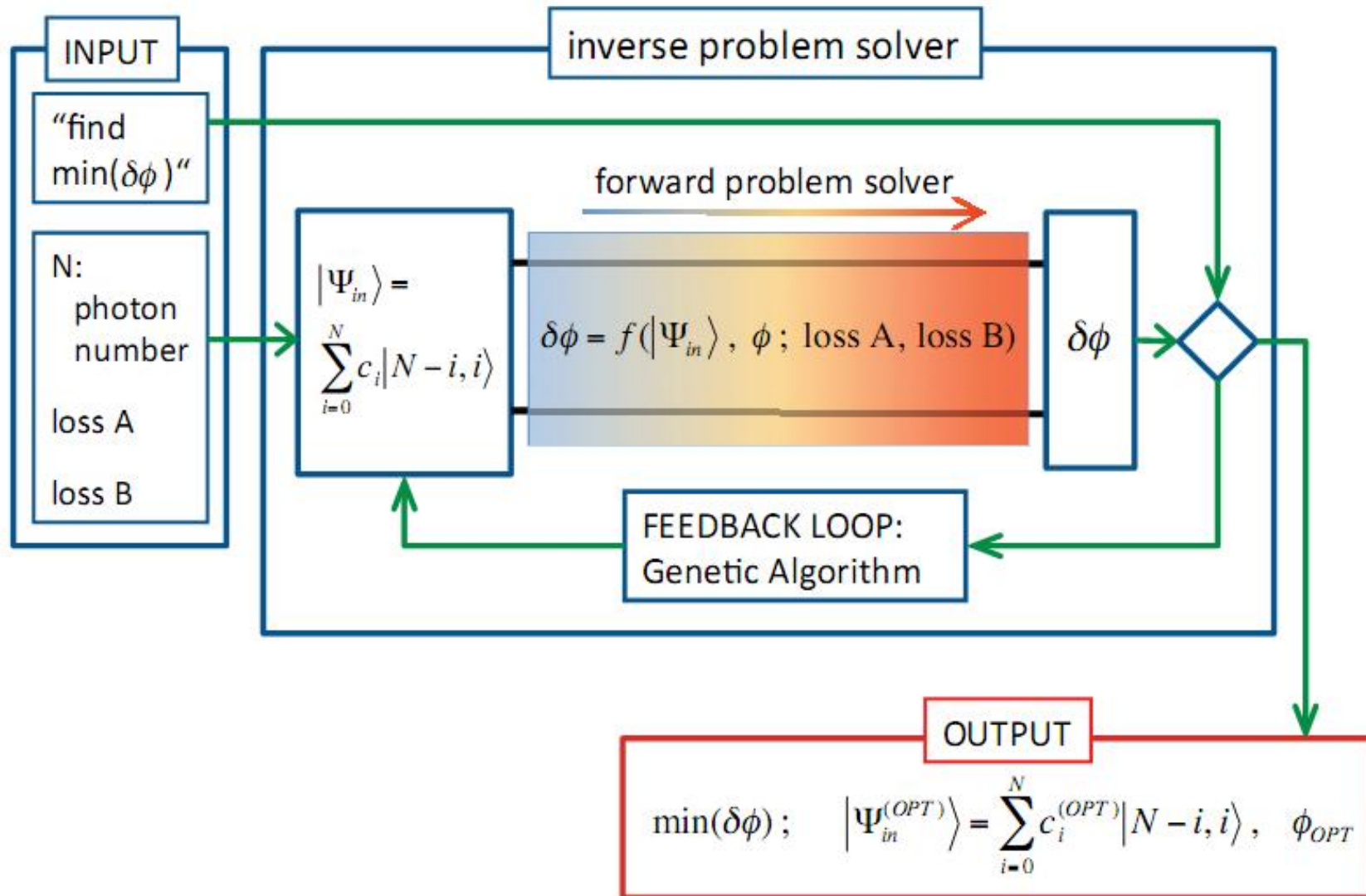


FIG. 1: (Color online) Abstract interferometer condensing the input state plus the first beam splitter into the first box, followed by two propagating modes with loss modeled by additional beam splitters. The box on the right includes a beam splitter and the photon-number resolving detectors.



# Numerical Implementation



# Optimization of Quantum Interferometric Metrological Sensors In the Presence of Photon Loss

<arXiv:0908.3008v1> (Phys. Rev. A, in press)

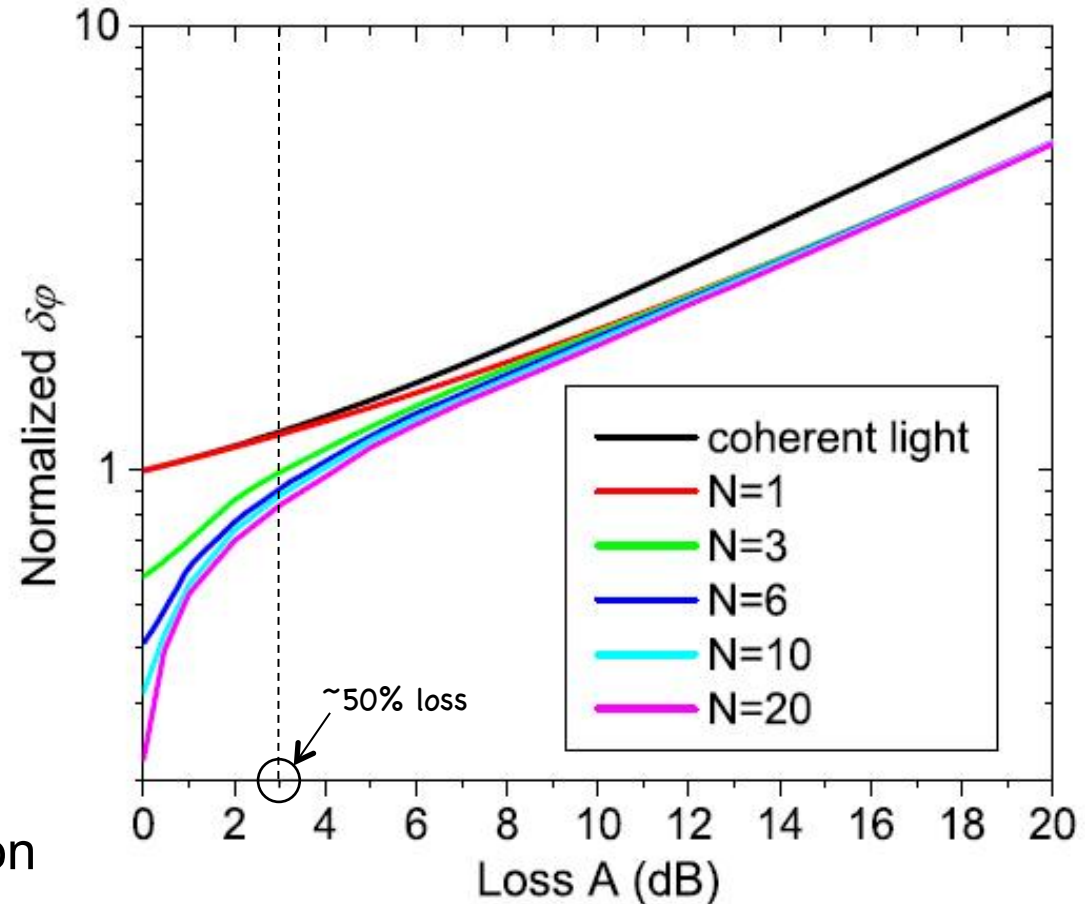
- Low loss regime benefits from quantum state input and improves with increased photon number
- Coherent baseline for 50:50 initial beam splitter

"Normalized"  $\delta\phi = \delta\tilde{\phi}$

$$\tilde{F} = \frac{F}{N}$$

$$\delta\tilde{\phi} = \frac{1}{\sqrt{\tilde{F}}} = \delta\phi\sqrt{N}$$

Quantum Fisher Information



# Optimization of Quantum Interferometric Metrological Sensors In the Presence of Photon Loss

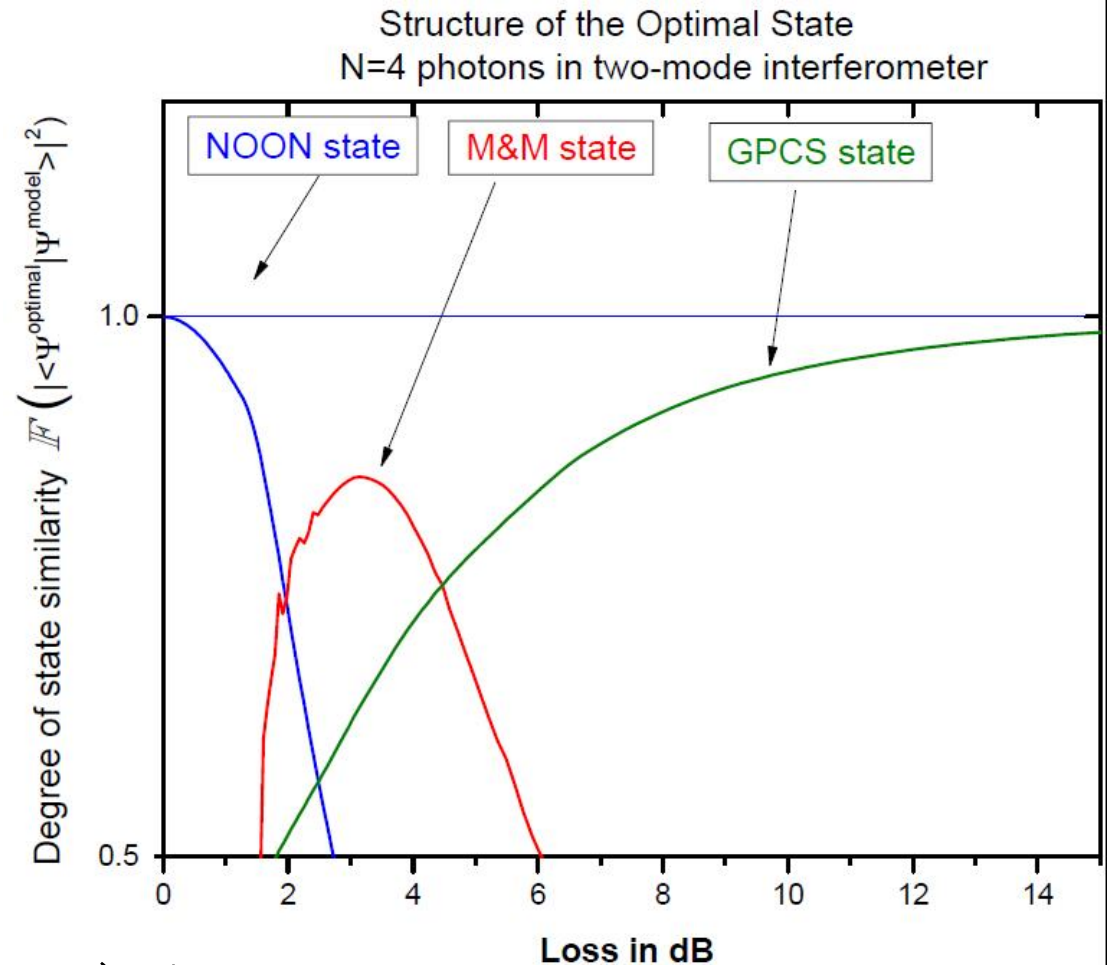
<arXiv:0908.3008v1> (Phys. Rev. A, in press)

- Measure of “fitness” of known state classifications

$$N00N = |N,0\rangle + |0,N\rangle$$

$$m \& m' = |m,m'\rangle + |m',m\rangle$$

$$PGCS = \left( \hat{a}_1^+ e^{i\beta} \cos \alpha - \hat{a}_2^+ e^{-i\beta} \sin \alpha \right)^N / \sqrt{N!} |0\rangle$$



# Optimizing the Multi-Photon Absorption Properties of NOON States

<arXiv:0908.1370v1> (submitted to Phys. Rev. A)

William N. Plick, Christoph F. Wildfeuer, Petr M. Anisimov, Jonathan P. Dowling

In this paper we examine the N-photon absorption properties of "NOON" states, a subclass of path entangled number states. We consider two cases. The first involves the N-photon absorption properties of the ideal NOON state, one that does not include spectral information. **We study how the N-photon absorption probability of this state scales with N.** We compare this to the absorption probability of various other states. The second case is that of two-photon absorption for an  $N = 2$  NOON state generated from a type II spontaneous down conversion event. In this situation we find that the absorption probability is both better than analogous coherent light (due to frequency entanglement) and highly dependent on the optical setup. **We show that the poor production rates of quantum states of light may be partially mitigated by adjusting the spectral parameters to improve their two-photon absorption rates.**

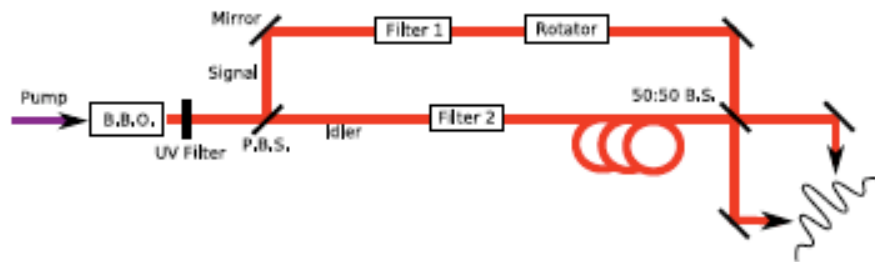


FIG. 2: The basic setup. A nonlinear crystal (BBO in this case) creates a degenerate pair of photons. Each photon is subjected to a filter. A polarization rotator ensures that the two photons are indistinguishable. A beam splitter creates a  $|2 :: 0\rangle$  state that results in an interference pattern.

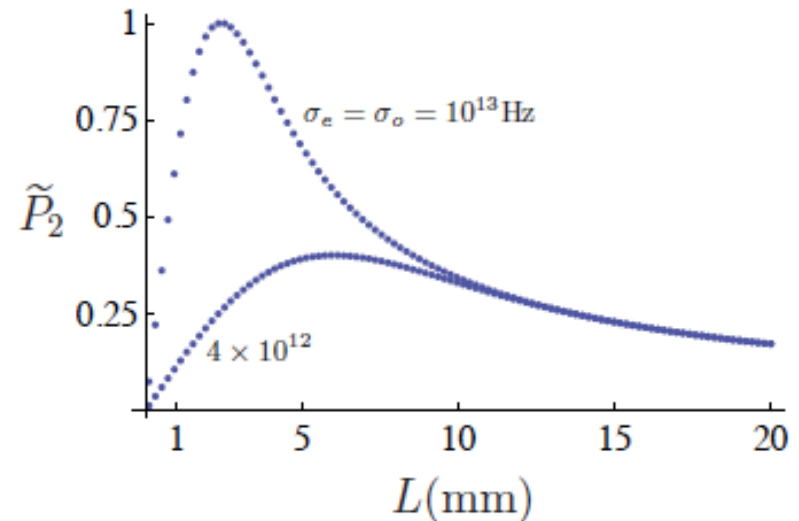


FIG. 6: A plot of the scaled two photon absorption probability for the realistic  $|2 :: 0\rangle$  state as a function of the length of the crystal for two different settings of the filters (in Hz).  $\sigma_p = 10^{12}$  Hz and  $\kappa_f = 10^{14}$  Hz

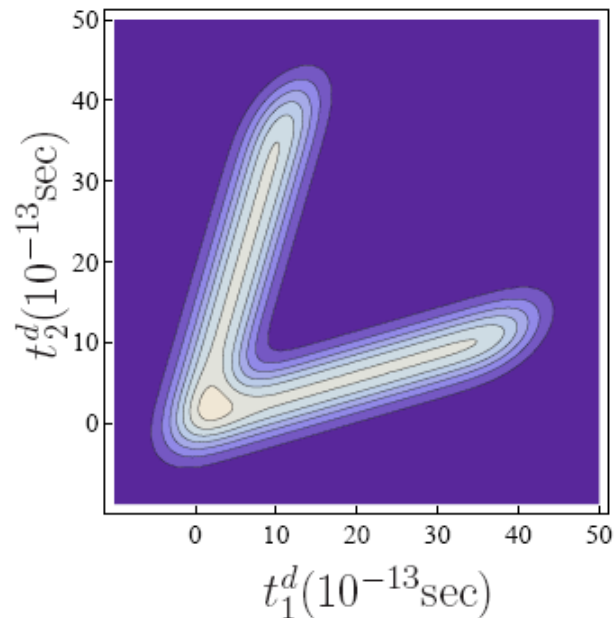
# Optimizing the Multi-Photon Absorption Properties of NOON States

<arXiv:0908.1370v1> (submitted to Phys. Rev. A)

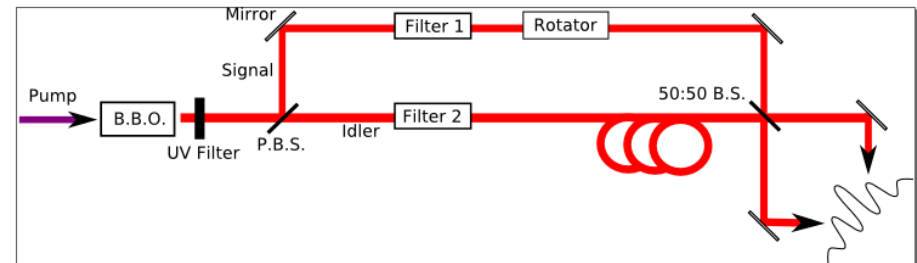
**Problem:** Quantum lithography depends on the N-photon absorption of very weak sources of NOON States.

**Question:** By manipulating the spectral properties of NOON states can their absorption properties be significantly improved?

Below is an example of the biphoton amplitude for our simple setup:



Take as a proof of principle experiment a two photon NOON state generated and used as in the setup below:

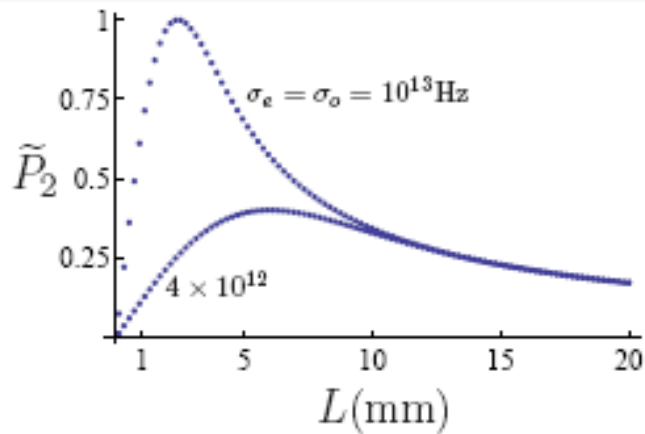


The setup is simple but it provides several parameters to adjust: crystal length, pump pulse shape, and the filters in the arms.

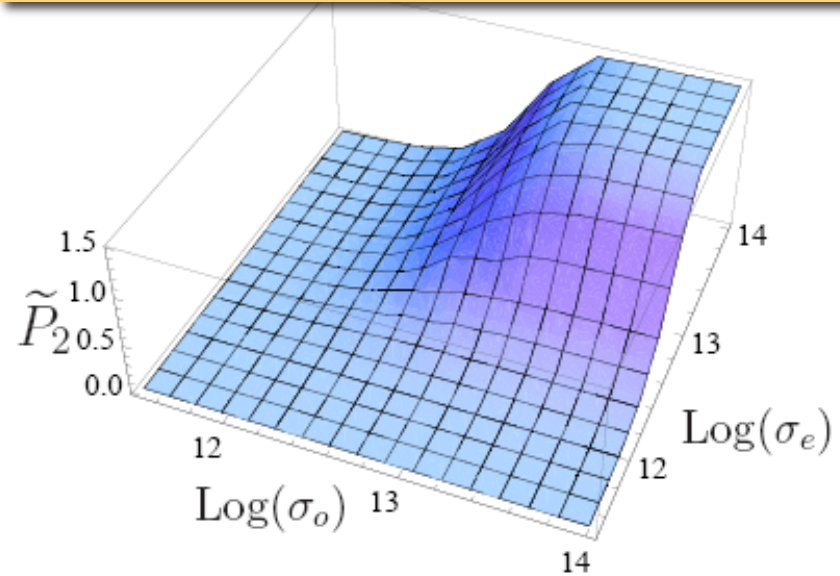
# Optimizing the Multi-Photon Absorption Properties of NOON States

<arXiv:0908.1370v1> (submitted to Phys. Rev. A)

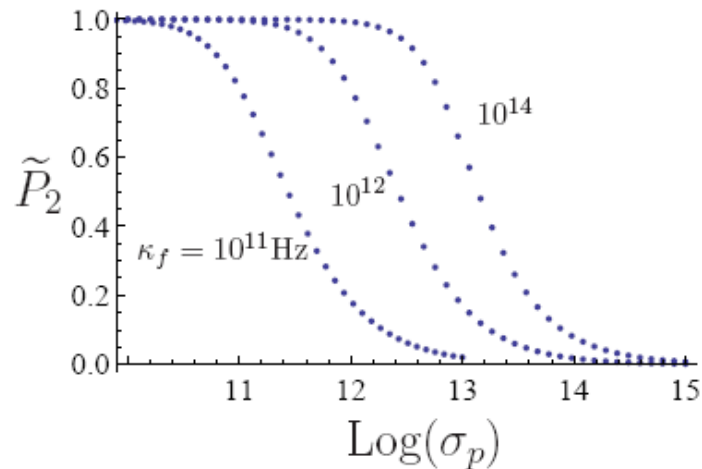
Two-photon absorption vs. crystal length (for two different settings of filter bandwidth):



Another example, with absorption a function of the filters in the arms:



Two-photon absorption vs pump pulse width for three atomic line widths:



**Result:** Even in this simple example the absorption probability may be improved by as much as 5 orders of magnitude by adjusting the parameters.

Furthermore, it can be shown that entanglement provides an improvement in absorption by many orders of magnitude over coherent light with the same spectral properties and intensity.

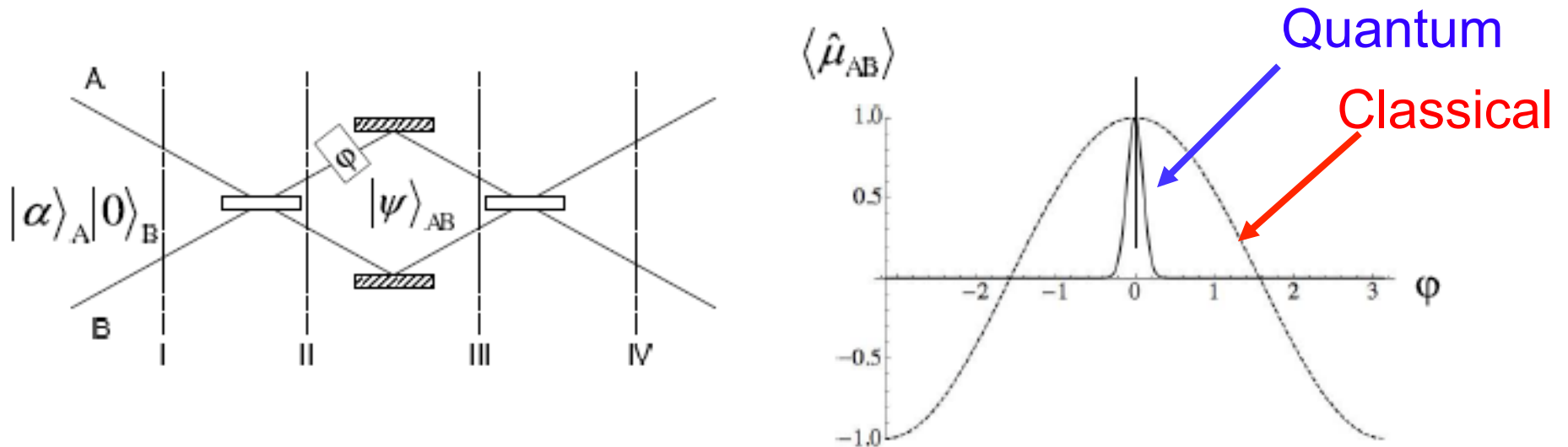


# Super-Resolution at the Shot-Noise Limit with Coherent States and Photon-Number-Resolving Detectors

<arXiv:0907.2382v2> (submitted to New J. Phys.)

Yang Gao, Christoph F. Wildfeuer, Petr M. Anisimov, Hwang Lee, Jonathan P. Dowling

There has been much recent interest in quantum optical interferometry for applications to metrology, **sub-wavelength imaging**, and remote sensing, such as in quantum laser radar (LADAR). For quantum LADAR, atmospheric absorption rapidly degrades any quantum state of light, so that for high-photon loss the optimal strategy is to transmit coherent states of light, which suffer no worse loss than the Beer law for classical optical attenuation, and which provides sensitivity at the shot-noise limit. **This approach leaves open the question -- what is the optimal detection scheme for such states in order to provide the best possible resolution?** **We show that coherent light coupled with photon number resolving detectors can provide a super-resolution much below the Rayleigh diffraction limit, with sensitivity no worse than shot-noise in terms of the detected photon power.**

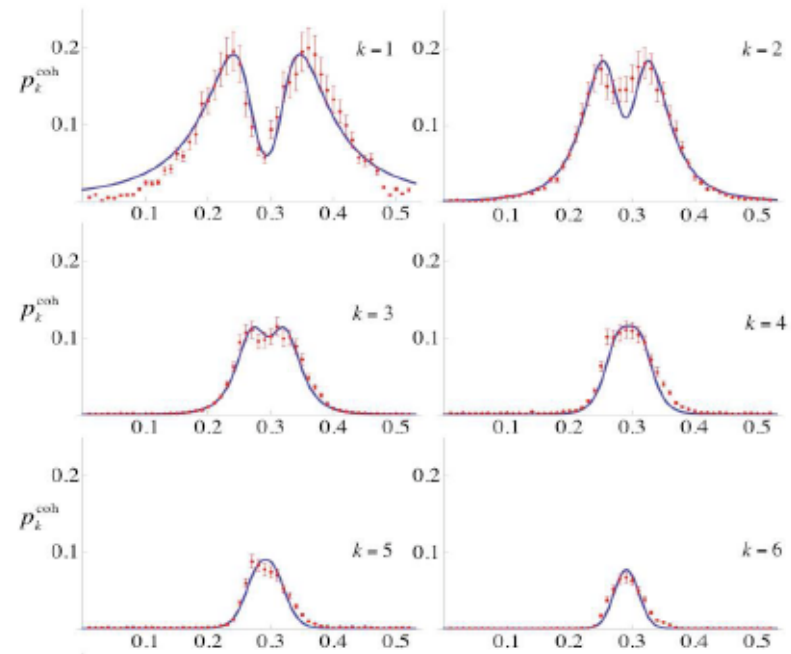
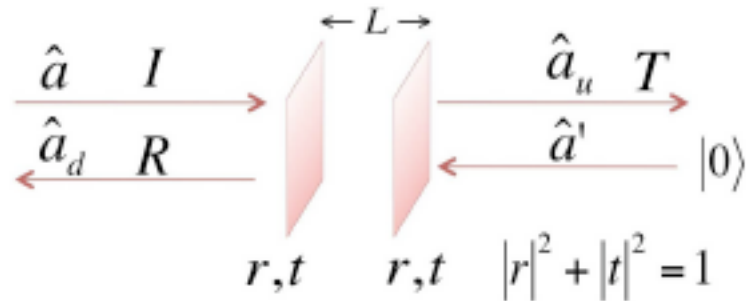


# Resolution and Sensitivity of a Fabry-Perot Interferometer With a Photon-number-resolving Detector

<arXiv:0905.1085v2> Phys. Rev. A (in press)

Christoph F. Wildfeuer, Aaron J. Pearlman, Jun Chen, Jingyun Fan,  
Alan Migdall, Jonathan P. Dowling

With photon-number resolving detectors, we show compression of interference fringes with increasing photon numbers for a Fabry-Perot interferometer. This feature provides a higher precision in determining the position of the interference maxima compared to a classical detection strategy. We also theoretically show supersensitivity if N-photon states are sent into the interferometer and a photon-number resolving measurement is performed.

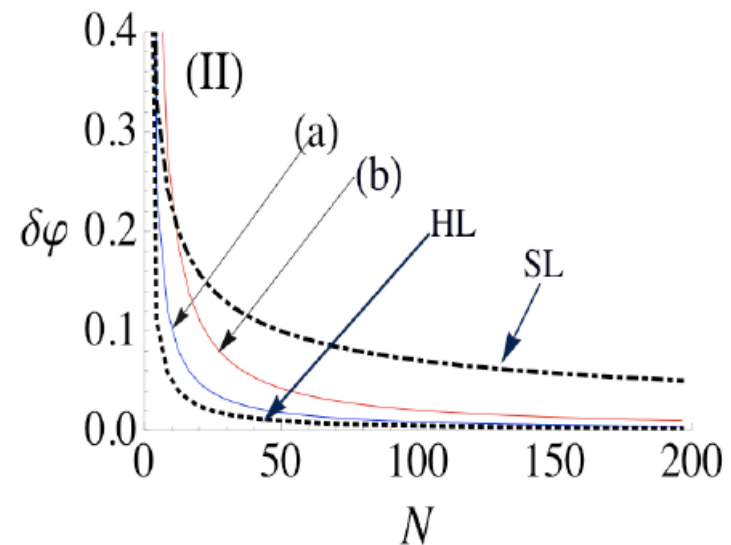
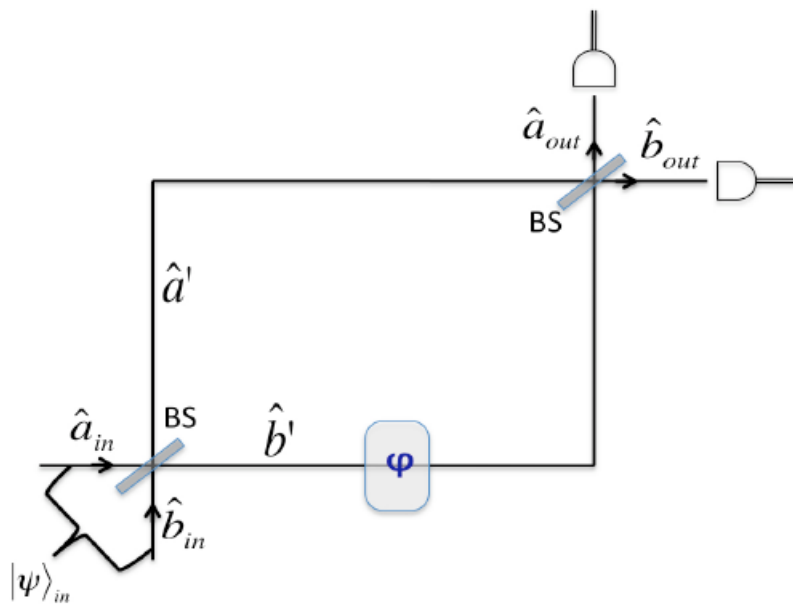


# Parity Measurements in Quantum Optical Metrology

<arXiv:0901.4395v2> (submitted to Phys. Rev. A)

Aravind Chiruvelli, Hwang Lee

We investigate the utility of parity detection to achieve Heisenberg-limited phase estimation for optical interferometry. We consider the parity detection with several input states that have been shown to exhibit sub shot-noise interferometry with their respective detection schemes. **We show that with parity detection, all these states achieve the sub-shot noise limited phase estimate. Thus making the parity detection a unified detection strategy for quantum optical metrology.** We also consider quantum states that are a combination of a NOON states and a dual-Fock state, which gives a great deal of freedom in the preparation of the input state, and is found to surpass the shot-noise limit.

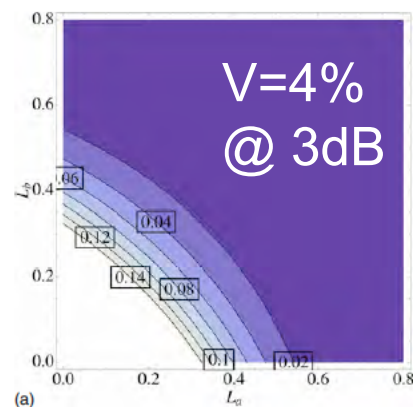
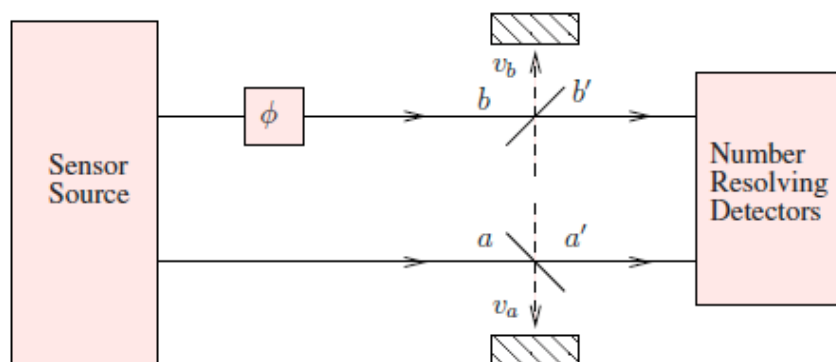


# Entangled Fock states for Robust Quantum Optical Metrology, Imaging, and Sensing

(Phys. Rev. A 78 Art. No. 063828 DEC 2008)

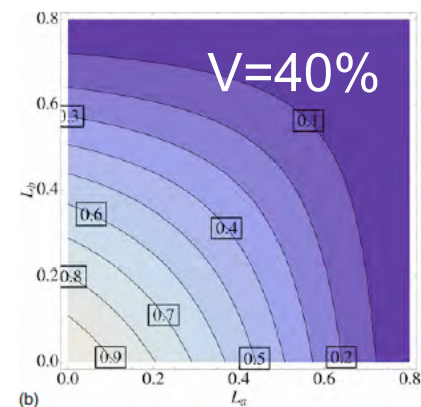
Sean D. Huver, Christoph F. Wildfeuer, Jonathan P. Dowling

We propose a class of path-entangled photon Fock states for robust quantum optical metrology, imaging, and sensing in the presence of loss. We model propagation loss with beam-splitters and derive a reduced density matrix formalism from which we examine how photon loss affects coherence. It is shown that particular entangled number states, which contain a special superposition of photons in both arms of a Mach-Zehnder interferometer, are resilient to environmental decoherence. **We demonstrate an order of magnitude greater visibility with loss, than possible with NOON states. We also show that the effectiveness of a detection scheme is related to super-resolution visibility.**



NOON

$$|10 :: 0\rangle$$



M&M

$$|20 :: 10\rangle$$

## Sub-Shot-Noise Quantum Optical Interferometry: A Comparison of Entangled State Performance within a Unified Measurement Scheme

Journal of Modern Optics Vol. 55, Nos. 19–20, 10–20 November 2008, 3319–3327

Yang Gao, Hwang Lee

Phase measurement using a lossless Mach-Zehnder interferometer with certain entangled  $N$ -photon states can lead to a phase sensitivity of the order of  $1/N$ , the Heisenberg limit. However, previously considered output measurement schemes are different for different input states to achieve this limit. We show that it is possible to achieve this limit just by the parity measurement for all the commonly proposed entangled states. Based on the parity measurement scheme, the reductions of the phase sensitivity in the presence of photon loss are examined for the various input states.

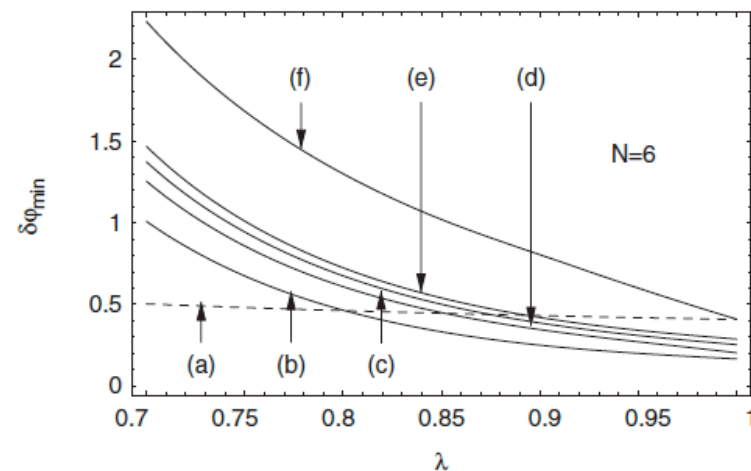
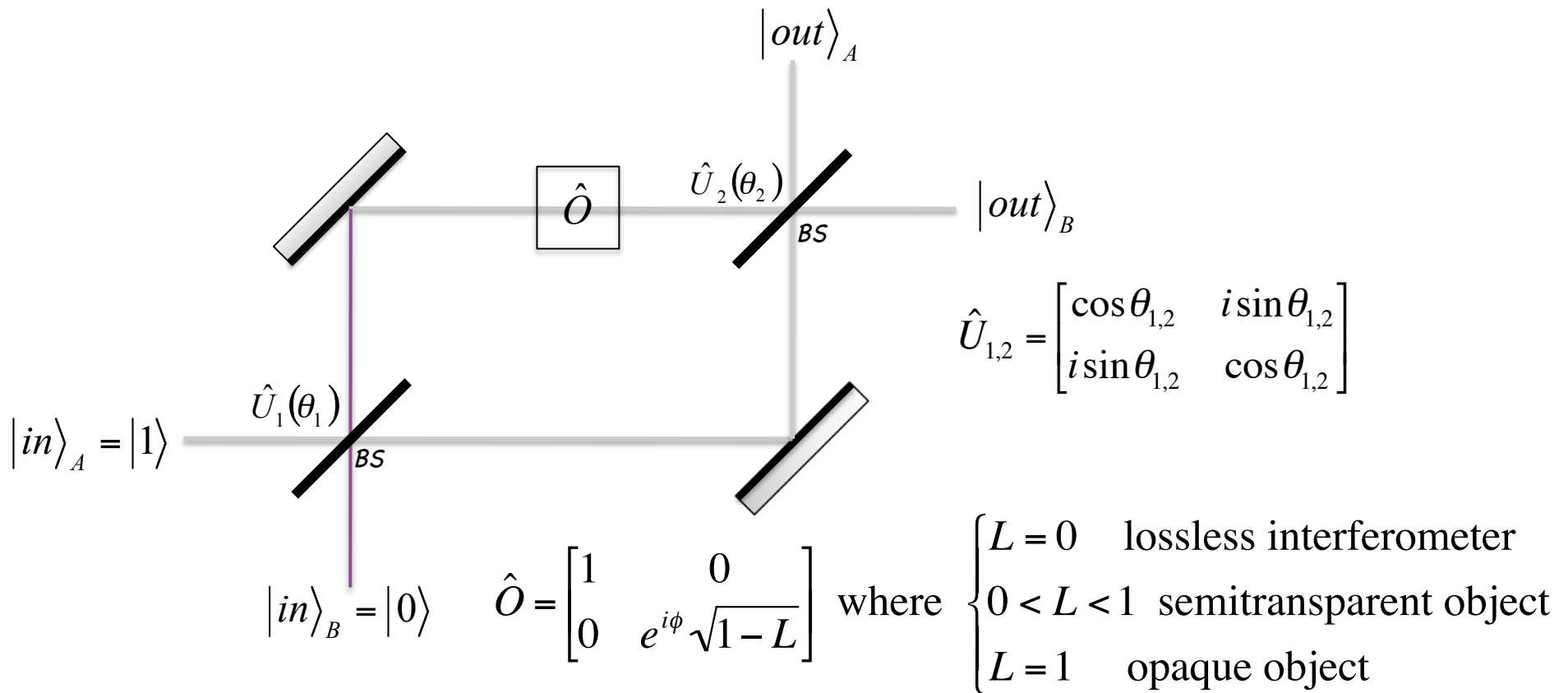


Figure 2. The minimum phase sensitivity,  $\delta\varphi_{\min}$ , for the various entangled states as a function of  $\lambda$ , the transmission coefficient. The upper and lower figures are for  $N=4$  and  $N=6$ , respectively. The dotted line (a) represents that of the uncorrelated input state [37]. The solid lines represent (b) the NOON state, (c) the dual Fock state, (d) the intelligent ( $\eta=10$ ) state, (e) the Yurke, and (f) the intelligent ( $\eta=1$ ) state, respectively.

# Interaction-Free Measurement (IFM) With Loss

Daniel Lum, Blane McCracken, Petr Anisimov, Jonathan P. Dowling  
(in preparation)

- Mach-Zehnder interferometer (MZI) with the possible presence of an object (in mode  $A$ ) denoted by operator  $\hat{O}$



# Interaction-Free Measurement: Review of Lossless Case with One Pass

- Dark port configuration:

$$\theta_1 + \theta_2 = \pi/2$$

$$\phi = 0$$

- Outputs assuming dark port configuration:

- $0 \leq L \leq 1$ : 
$$\hat{U}_2 \hat{O} \hat{U}_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \theta_1 \cos \theta_2 - \sqrt{1-L} \sin \theta_1 \sin \theta_2 \\ i(\cos \theta_1 \sin \theta_2 + \sqrt{1-L} \sin \theta_1 \cos \theta_2) \end{bmatrix} = \begin{bmatrix} \frac{1 - \sqrt{1-L}}{2} \\ \frac{i + i\sqrt{1-L}}{2} \end{bmatrix}$$

- Maximum efficiency of 50%

$$\eta = \frac{P_{ifm}}{P_{ifm} + P_{abs}} = \frac{R_1 R_2}{R_1 R_2 + T_1} = \frac{1 - R_1}{2 - R_1} \rightarrow \frac{1}{2}$$

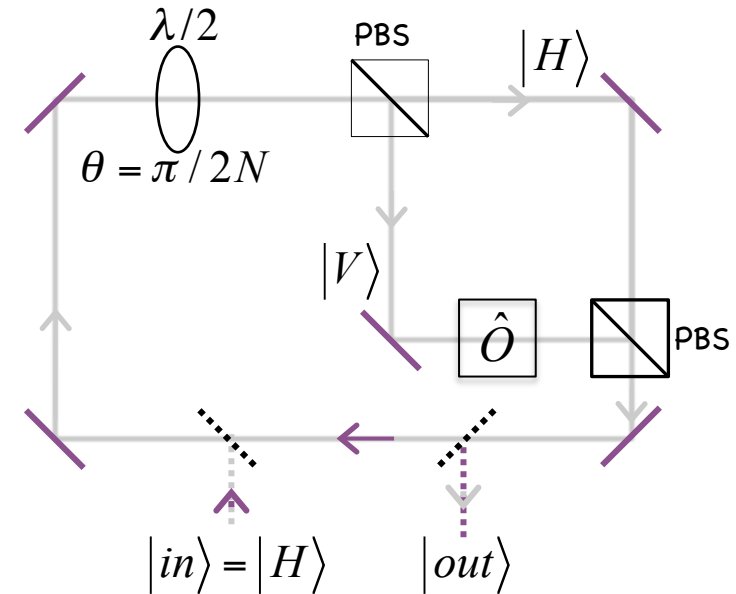
if

$$R_1 = 0$$

# IFM High Efficiency Lossless Detection: Multi-Pass Protocol

- The quantum Zeno effect
- The balancing condition requires the following dark port arrangement:

$$\sum_{n=1}^N \theta_n = \frac{\pi}{2} \Rightarrow \theta_n = \frac{\pi}{2N}$$



P.G. Kwiat *et al.*, Phys. Rev. Lett. **83**, 4725 (1999)

- In the limit of large **N→Infinity**, efficiency approaches **100%**.

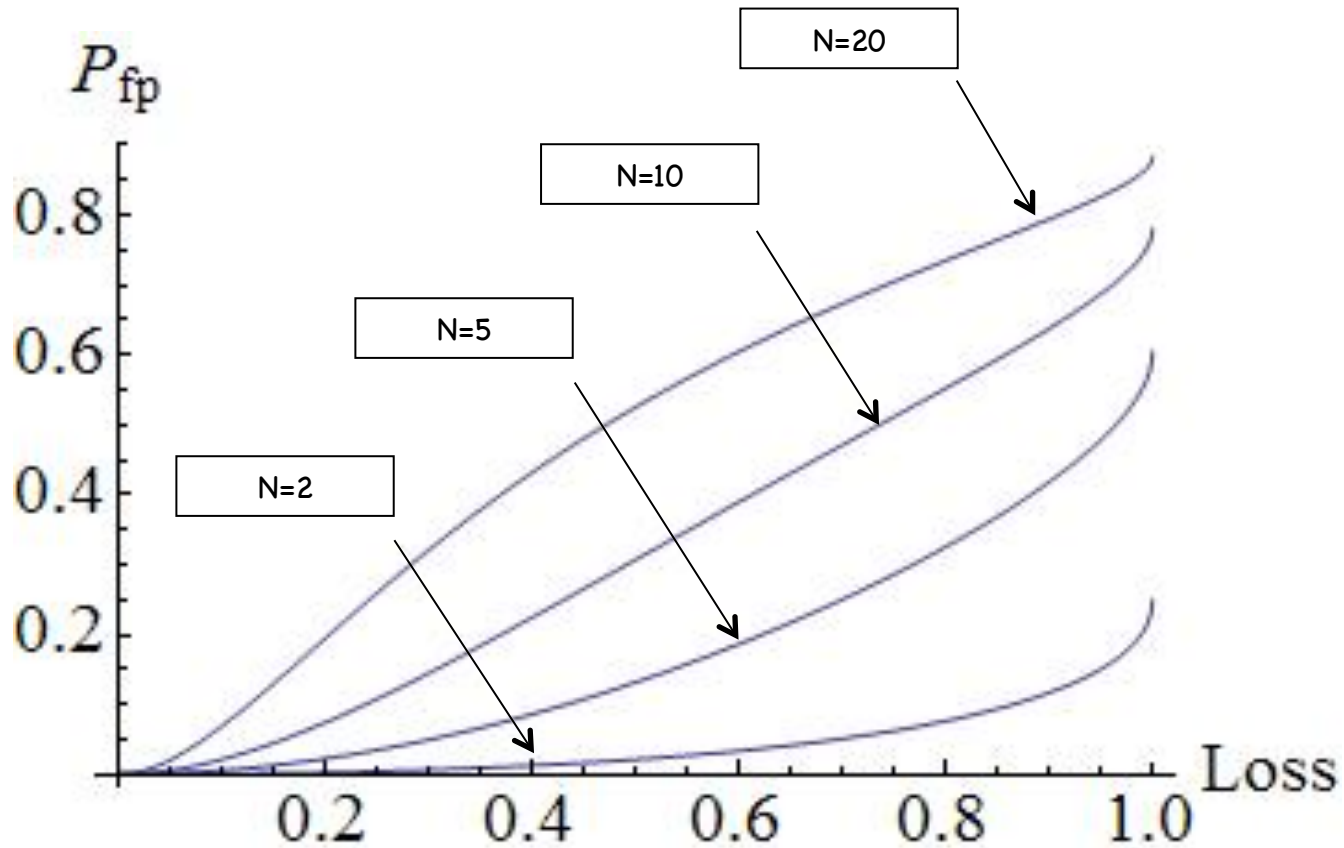
$$\lim_{N \rightarrow \infty} P_{ifm} = \lim_{N \rightarrow \infty} (\cos^{2N} \theta_n) \approx \lim_{N \rightarrow \infty} \left( 1 - \frac{\pi^2}{4N} \right) = 1$$

$$\lim_{N \rightarrow \infty} \eta = \lim_{N \rightarrow \infty} \frac{P_{ifm}}{P_{ifm} + P_{abs}} = 1$$



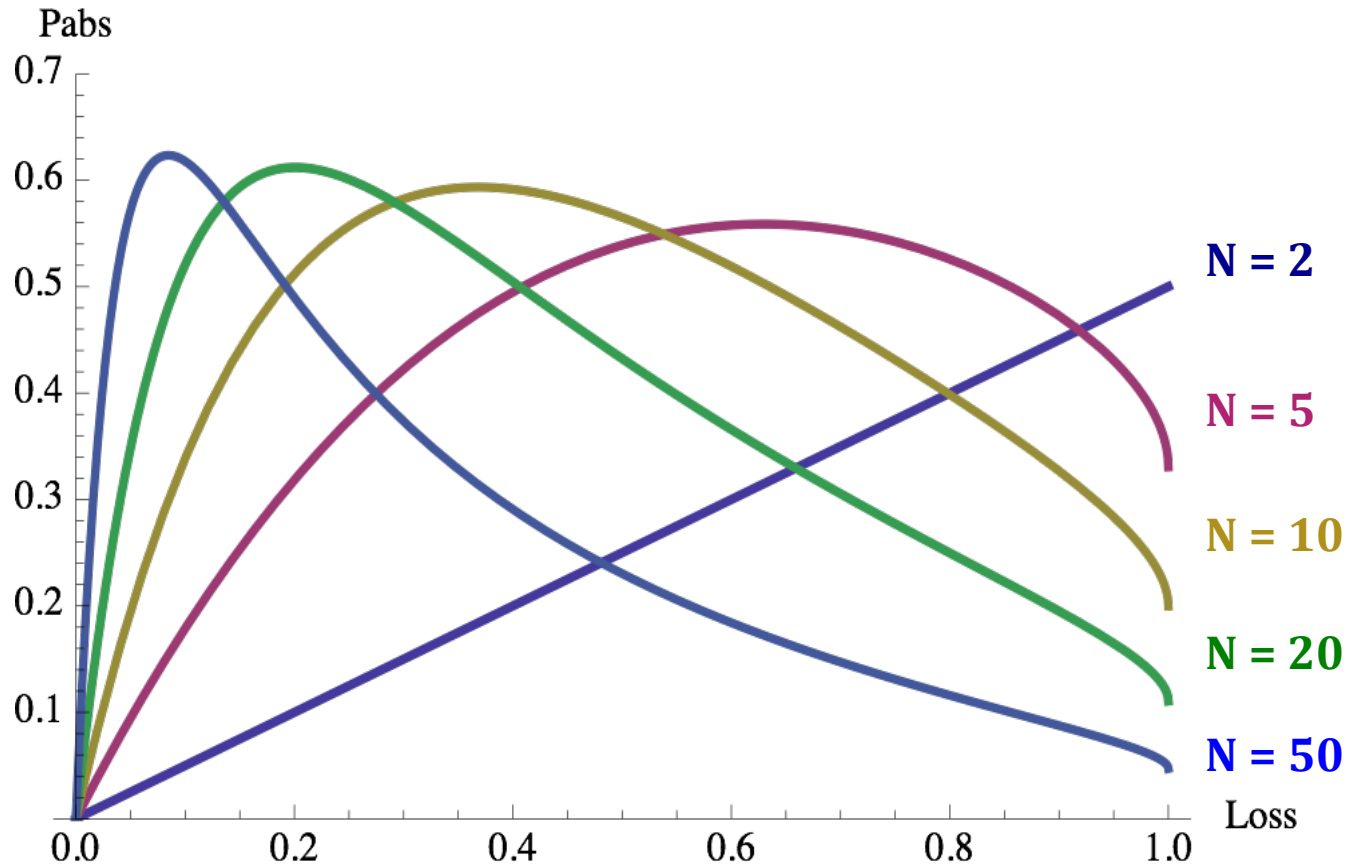
# Interaction-Free Measurement (IFM) With Loss

- What if loss enters this system?
  - Increasing N number of beam-splitters and loss contributes to false positives



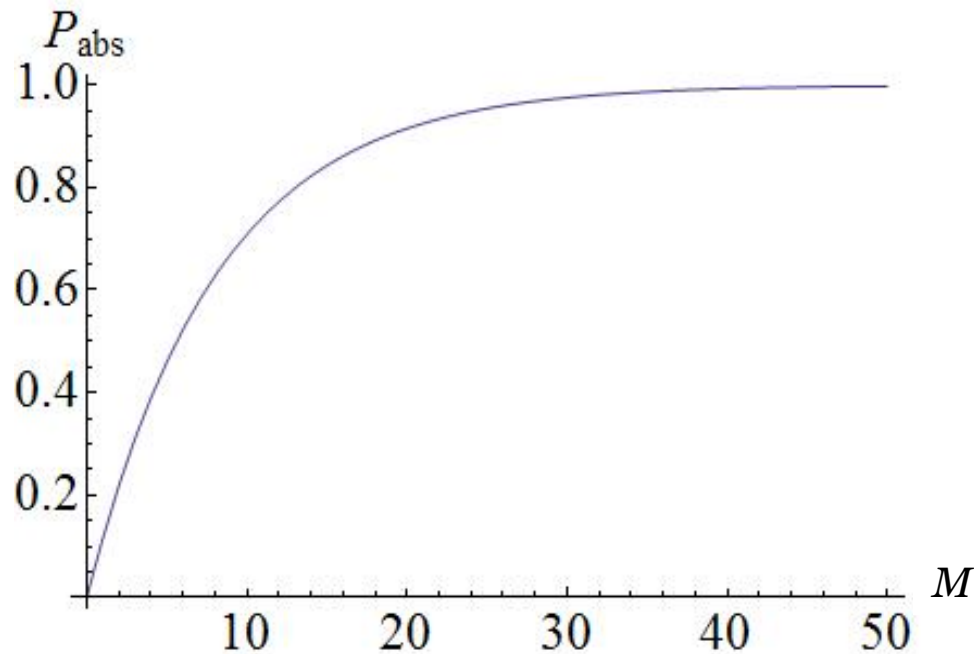
# Interaction-Free Measurement (IFM) With Loss

- For  $N > 2$ , total absorption peaks at  $0 < L_0 < 1$
- Loss = 1 makes a hard measurement on the lossy path, effectively implementing the quantum Zeno effect.



# Interaction-Free Measurement (IFM) With Loss

- System is increasingly sensitive to low loss with increased number of beam-splitters ( $N$ )
- Experimental measure of probabilities requires infinite number of trials ( $M$ )
- Increasing  $M$  guarantees a photon-object interaction



# Interaction-Free Measurement (IFM) With Loss: Hypotheses Testing

- The Classical Chernoff bound
  - an upper limit on the probability of an erroneous conclusion (the object is present or it is not).

$$P_e \leq \frac{1}{2} \min_{s \in [0,1]} \left( \sum_b p_0^s(b) p_1^{1-s}(b) \right)^M$$

- Subscripts 0,1 denote the two hypotheses:
  - 0, no object present
  - 1, object present
- Parameter  $b$  represents all possible outcomes of the experiment:

# Error and Transmission Probabilities

- **Error probability:**  $P_{er} = e^{-MC_I}$   
where  $C_I$  is the "Chernoff distance:"

$$C_I(p, q) = \xi \log \frac{\xi}{p} + \bar{\xi} \log \frac{\bar{\xi}}{\bar{p}}$$
$$\xi = \frac{\log(\bar{q}/\bar{p})}{\log(p/\bar{p}) + \log(\bar{q}/q)}$$

Binary Notation:

$$p_0(0) = p$$

$$p_1(0) = q$$

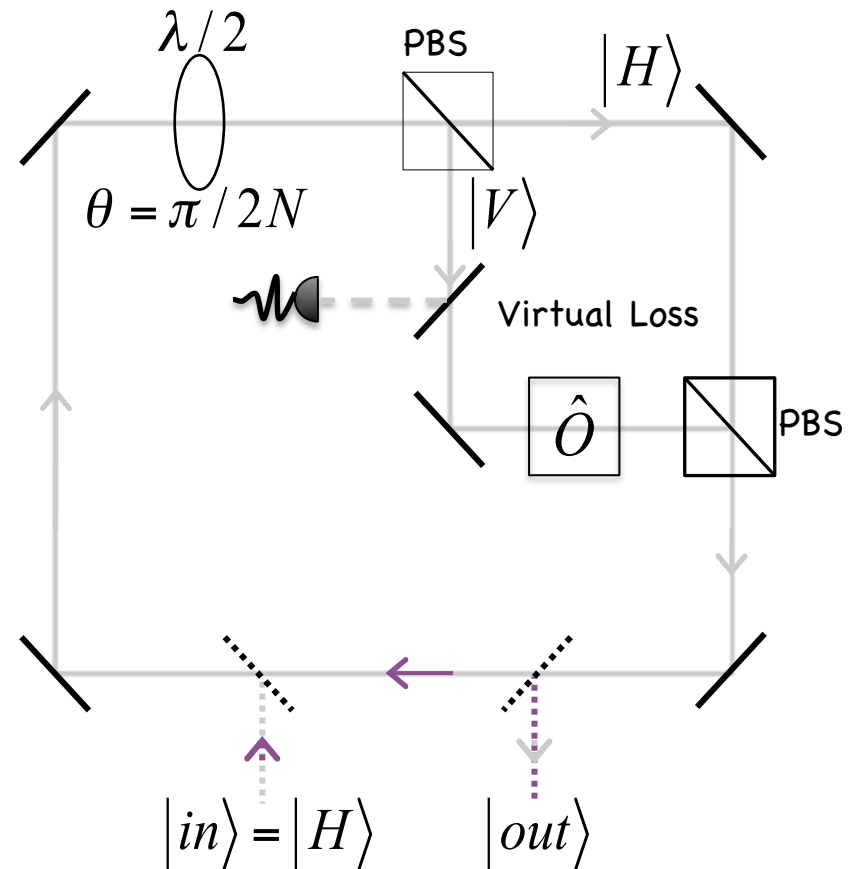
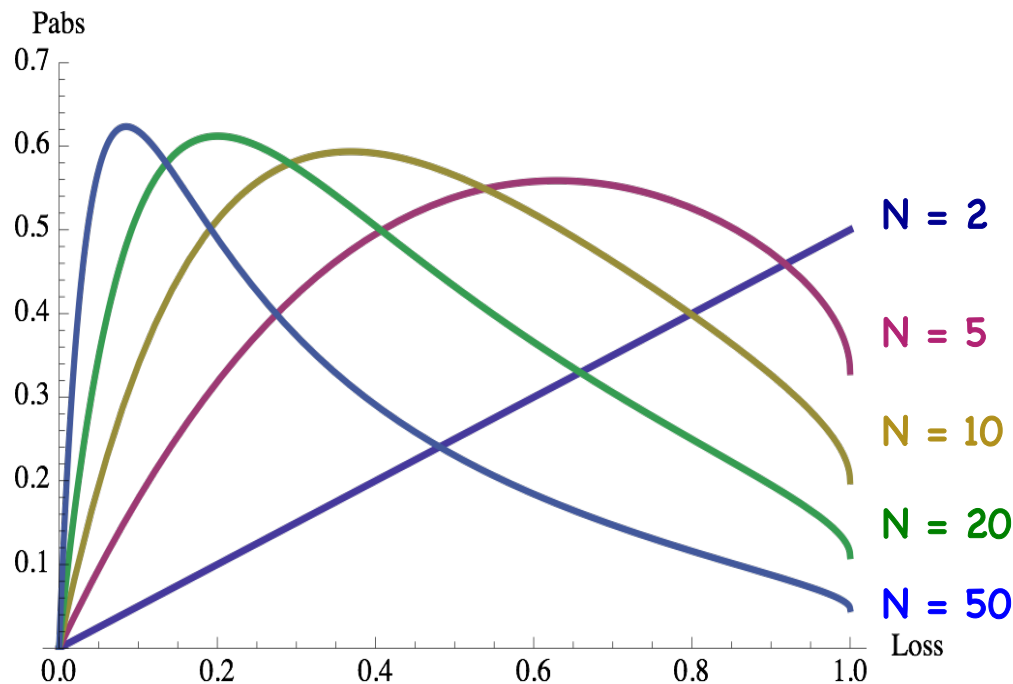
$$\bar{x} = 1 - x$$

- **Transmission probability:**  $P_{tr} = e^{-MA_I}$   
where  $A_I$  is the "absorption distance:"

$$A_I = \log \frac{1}{1 - P_{abs}}$$

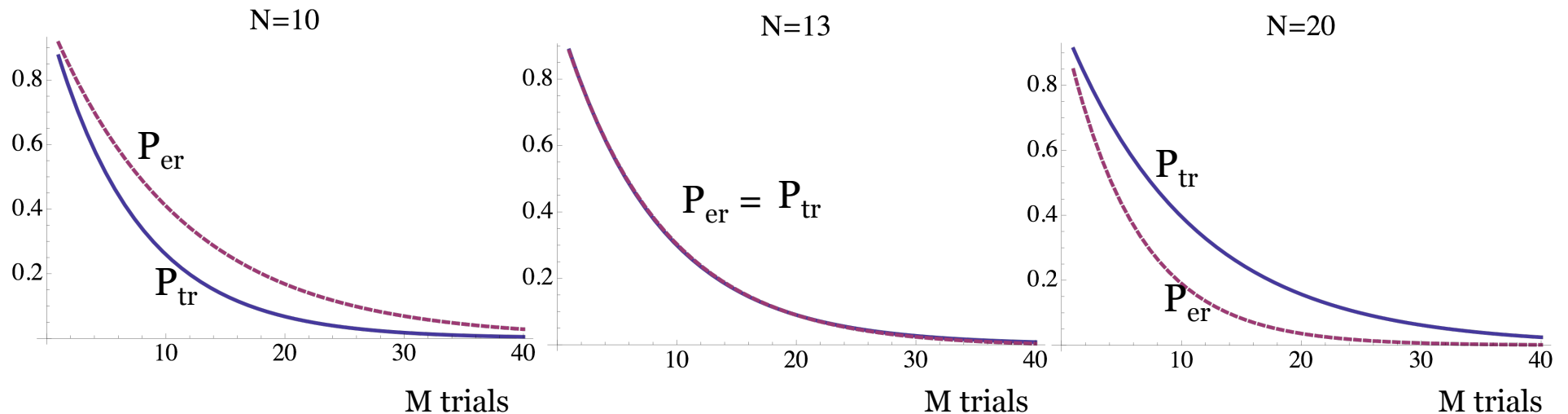
# IFM with Loss: Invisible Quantum Tripwire

- Detect at output without modal discrimination.
- Note that for  $N > 2$  and  $L = 0$ ,  $P_{\text{abs}}$  becomes quite unstable
  - Solution: implement a virtual loss



# Confidence vs. Absorption

- Increased trials ( $M \uparrow$ ) improves confidence ( $P_{er} \downarrow$ ), but also increases cumulative absorption probability ( $P_{abs} \uparrow$ )
- Practically, the error rate ( $P_{er}$ ) must decrease more rapidly than the transmission probability rate ( $P_{tr}$ ).



## Interaction-Free Measurement (IFM) With Loss

- Successful implementation of the invisible quantum tripwire requires:
  - More than 13 passes through beam-splitters
  - Adding loss to maximize absorption
- When utilized properly, the trip-wire is invisible to detection.