Quantum Information

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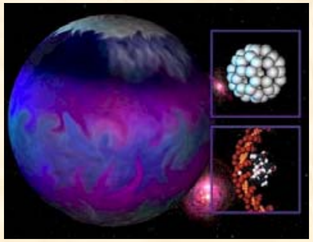
Thanks to

- ARDA
- Army Research Office
- DARPA
- DOD MURI
- University of Rochester
- NSF
- Research Corporation



Overview

- Introduction: Continuously Entangled Biphotons
  - Entanglement
  - Schmidt Decomposition: Information Eigenmodes
- Experiments
  - Pixel Entanglement in Transverse Modes
  - Time-energy

Single Particle Continuous Variable Uncertainty Relations

- Continuous observables position and momentum (or e.g., field quadratures)

$$\left\langle \left(\Delta\hat{x}\right)^2 \right\rangle \left\langle \left(\Delta\hat{p}\right)^2 \right\rangle \geq \frac{\hbar^2}{4}$$

1. Heiserberg's uncertainty relation.

2. Closely related to the space-bandwidth product in imaging .

3. Continuous quantum cryptography

EPR: Continuous Entanglement

Einstein, Podolsky and Rosen questioned the completeness of wavefunction description of Quantum Mechanics in their gedanken experiment [Phys Rev 47, 777 (1935)].

Suppose we have two quantum particles 1 and 2 with their positions governed by

$$\Psi = \iint A(x_1, x_2)\left| x_1, x_2 \right\rangle dx_1 dx_2$$

$$A(x_1, x_2) = \delta(x_1 - x_2)$$

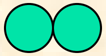
$$\tilde{A}(k_1, k_2) = \frac{1}{2\pi}\int e^{-ik_1 x_1} e^{-ik_2 x_2} A(x_1, x_2) dx_1 dx_2$$

$$\tilde{A}(k_1, k_2) = \delta(k_1 + k_2)$$

EPR entanglement (70 years)

Position $\delta(x_1-x_2)$

Interaction



Particle 2

Particle 1

Momentum $\delta(k_1+k_2)$

EPR: no interaction at distant locations.
particle 2 must be in both a position and momentum eigenstate, which violates Heisenberg's uncertainty principle $\Delta x \Delta k<1/2$.

Separability

$$\rho = \sum_i \rho_{1i} \otimes \rho_{2i}$$

General Statement of Separability

- Continuous systems

$$\left\langle \left(\Delta \hat{x}_{12}\right)^2 \right\rangle \left\langle \left(\Delta \hat{p}_{12}\right)^2 \right\rangle \geq \hbar^2$$

$$\hat{x}_{12} = \hat{x}_1 - \hat{x}_2 \qquad \hat{p}_{12} = \hat{p}_1 + \hat{p}_2$$

Duan et al, PRL 84, 2722 (2000)
Simon, PRL 84, 2726 (2000)
Mancini et al, PRL 88, 120401 (2002)

Entangled statistics

- Uncertainty sum or product vanish for perfect maximal entanglement.

$$\left\langle \left(\Delta\hat{x}_{12}\right)^2 \right\rangle \left\langle \left(\Delta\hat{p}_{12}\right)^2 \right\rangle = 0$$

Howell, Bennink, Bentley and Boyd
Phys. Rev. Lett. **92**, 210403 (2004)

Schmidt Decomposition

- Schmidt Number
  - Number of information eigenmodes
  - Discrete (even for continuous distributions), because of finite trace
  - Bipartite

C. K. Law and J. H. Eberly
Phys. Rev. Lett. **92**, 127903 (2004)

Schmidt Decomposition

- Discrete

$$\left|\Psi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|H_1,V_2\right\rangle - \left|V_1,H_2\right\rangle\right)$$

Schmidt Number:
K=2

- Continuous
  - Ratio of single particle uncertainty over two-particle uncertainty

$$K = \frac{\Delta x_1}{\Delta x_{12}} \geq \frac{1}{2\Delta x_{12}\Delta k_{12}} \approx \frac{1}{EPR}$$

EPR Entanglement: Previous Work

- Squeezed light fields (quadrature squeezed correlations)
  - Reid and Drummond, PRL 60, 2731 (1988)
  - Ou et al, PRL 68, 3663 (1992)
- Collective atomic spin variables (spin observables)
  - Julsgaard, Nature 413, 400 (2001)
- Modern rephrasing of continuous entanglement
  - Duan et al, PRL 84, 2722 (2000)
  - Simon, PRL 84, 2726 (2000)
  - Mancini et al, PRL 88, 120401 (2002)
- Discrete Entanglement (violation of separability bounds)
  - Hofman and Takeuchi PRA 68 032103
  - Ali Khan and Howell Phys. Rev. A **70**, 062320 (2004)

Transverse Momentum-Position Entanglement

- Ghost Imaging and Ghost Diffraction
  - Pittman *et al*, PRA 52, R3429 (1995)
  - D. V. Strekalov *et al,* PRL **74**, 3600-3603 (1995)
- Classical Ghost imaging and Ghost Diffraction
  - Bennink et al, PRL 89, 113601 (2002)
  - Bennink et al PRL **92,** 033601 (2004)]

- Noncommuting observables
  - Gatti et al, PRL 90, 133603 (2003)
  - Howell et al Phys. Rev. Lett. **92**, 210403 (2004)
  - Equivalent to demonstrating Rotational Invariance, but for continuous variables.

Transverse Momentum-Position Entanglement

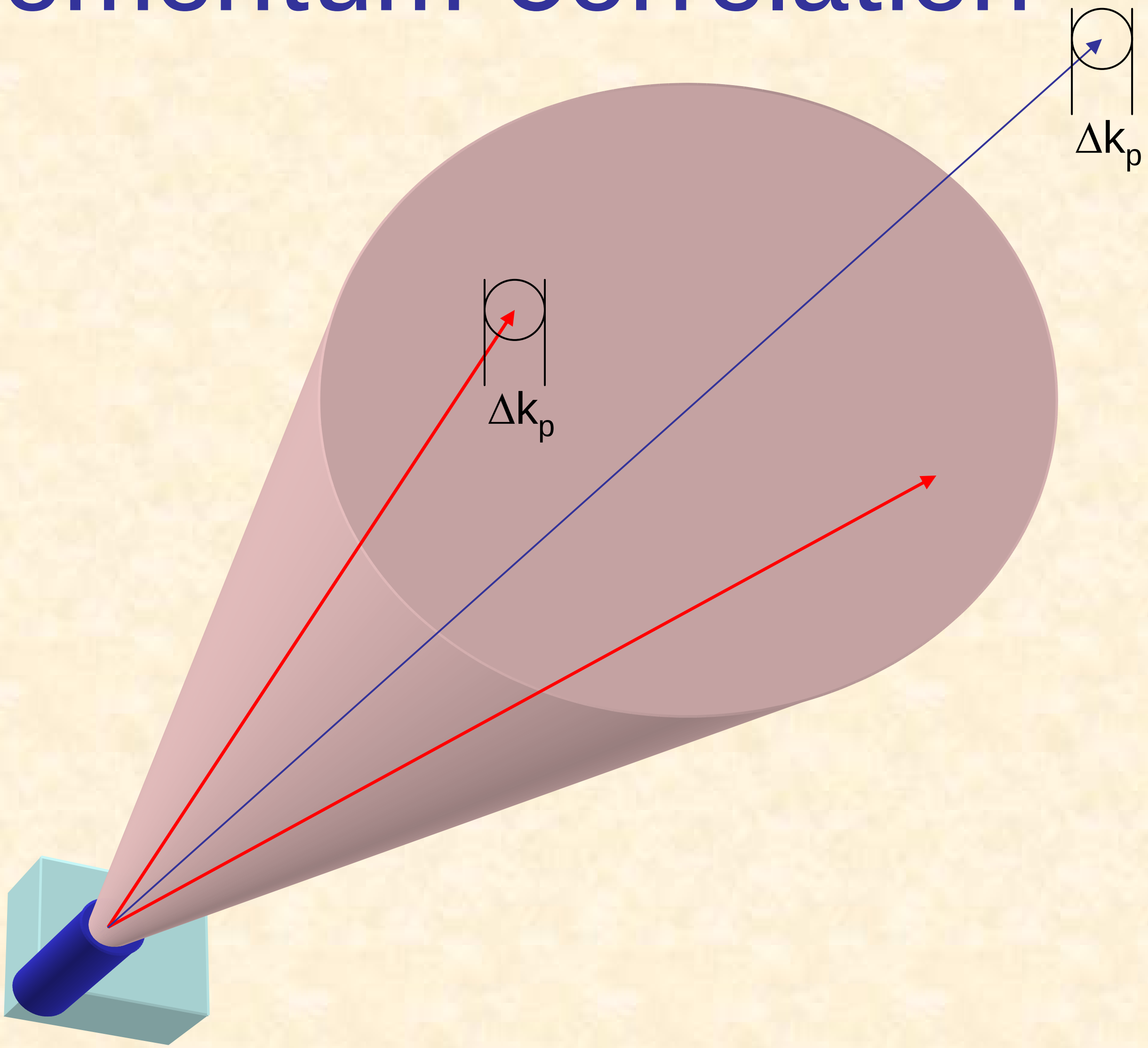
- Created?
  - Used first order (two-photon) spontaneous parametric down conversion.
  - One photon downconverts into two photons.
    - Momentum conserved (momentum correlation)
    - Photons emitted from a small birth place region (position correlation)
- Thin crystal, paraxial and narrow filter approximation

$$\left|\Psi\right\rangle = \left|vac\right\rangle + N \times \int d\vec{k}_s \int d\vec{k}_i E(\vec{k}_i + \vec{k}_s) Sinc\left(\frac{\Delta k_z L}{2}\right) \left|1, k_i\right\rangle \left|1, k_s\right\rangle$$

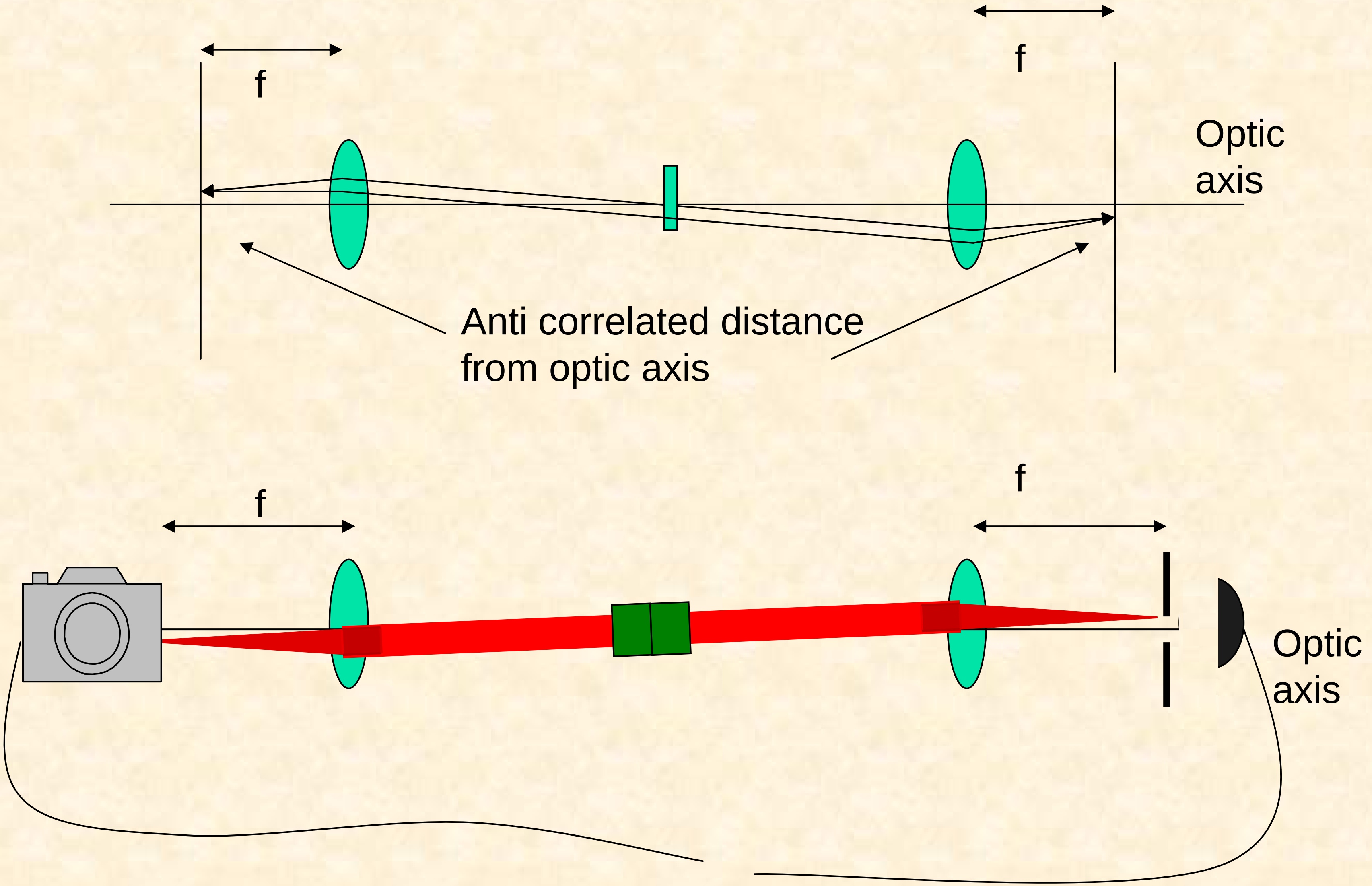
Angular Spectrum of pump

Phase matching condition

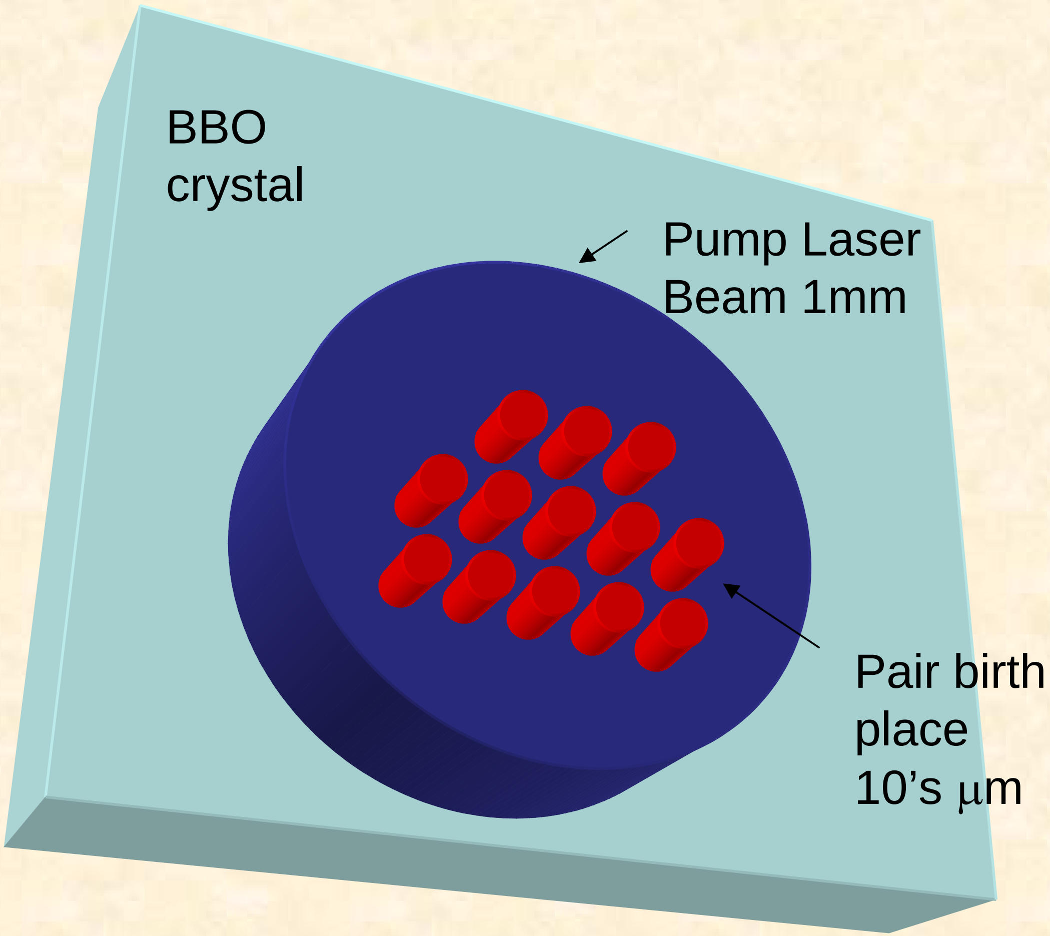
Momentum Correlation



Quantum vs Classical ghost imaging

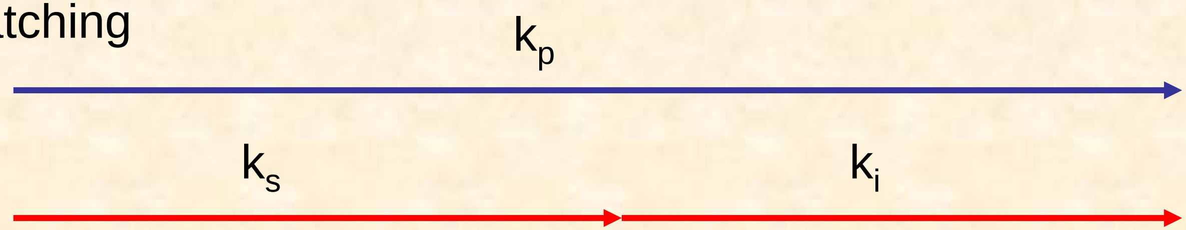


Position Correlation

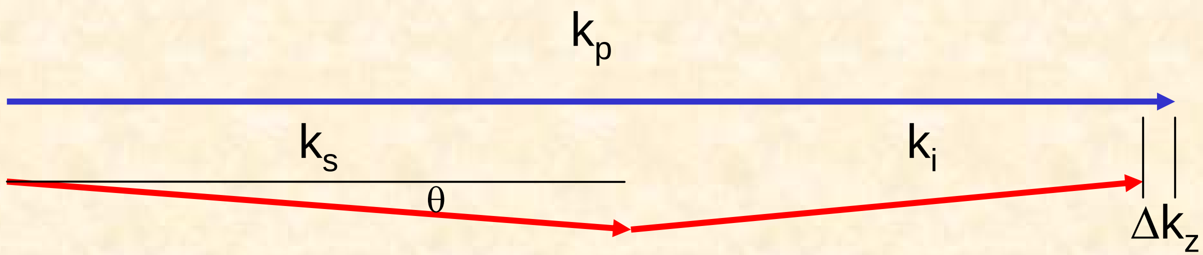


Position Correlation

Collinearly Phase matched type-II in forward direction: Perfect phase matching



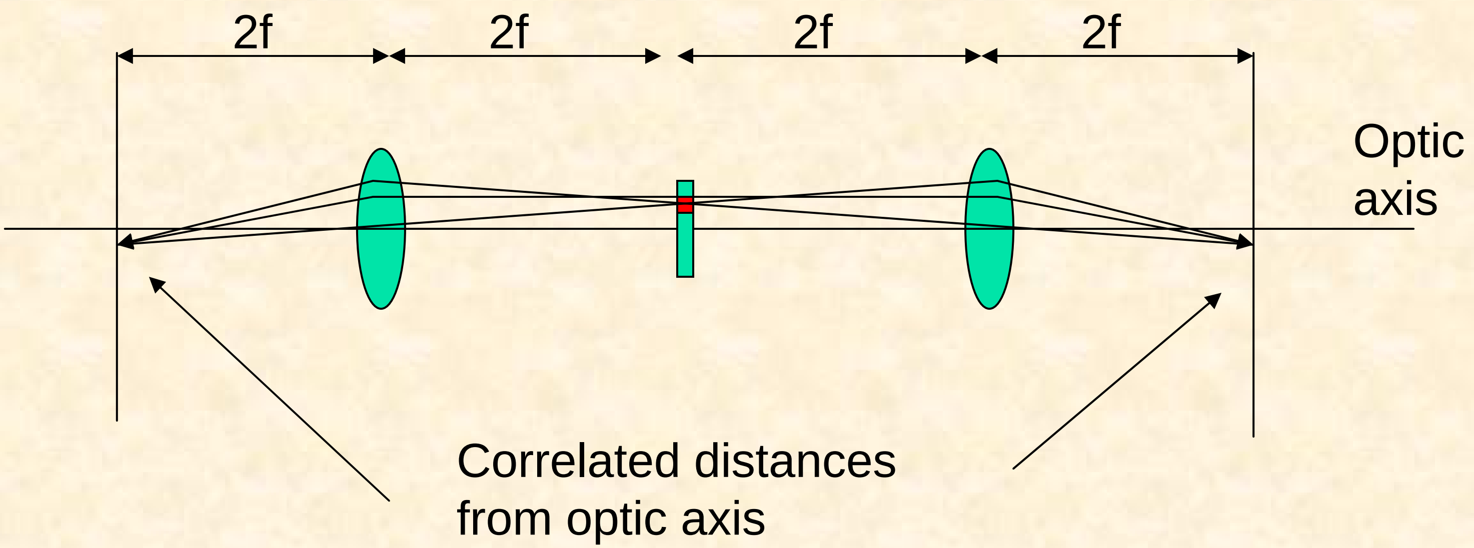
Imperfect Phase matching



Δk_z L=1/2 gives an approximate size to the birth place.

Position Correlation

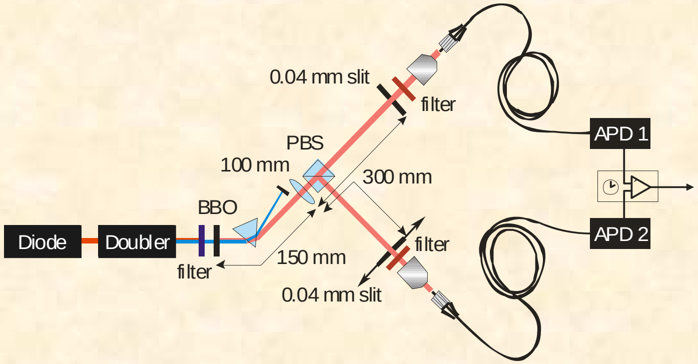
- Both Photons created inside birthplace region.
- Photons measured in near field (image planes) .

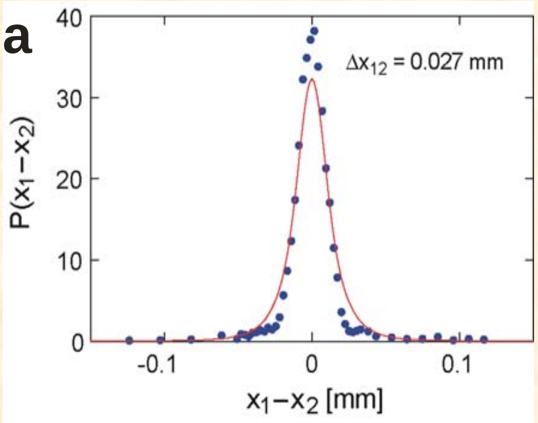


Experiments

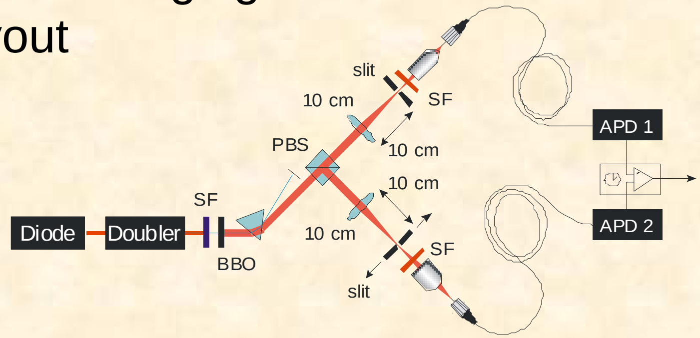
Point Spread Functions

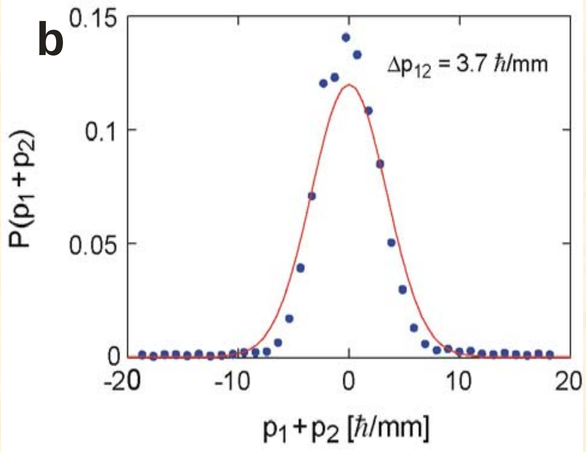
Imaging Layout





Fourier Imaging Layout





EPR Result

- Inferred uncertainty product for particle 2 is approximately

Single-Particle variance product

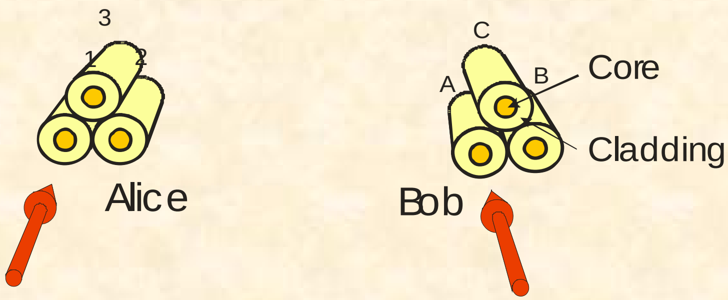
$$(\Delta x)^2 (\Delta p)^2 \geq \hbar^2 / 4$$

Conditional Variance product

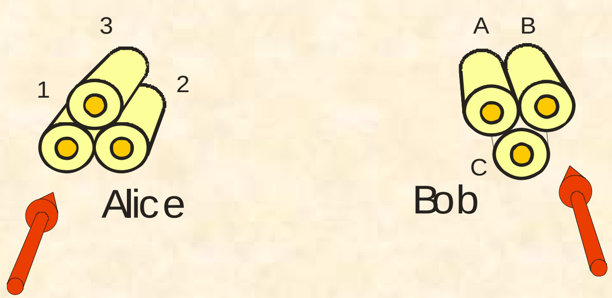
$$(\Delta x_2)^2 |_{x_1} (\Delta p_2)^2 |_{p_1} \approx 0.004\hbar^2$$

Pixel Entanglement: Discretizing continuous entanglement

Position Correlation



Momentum Correlation



Same Basis: correlated or anticorrelated measurements. (3 possible coincidence measurements)

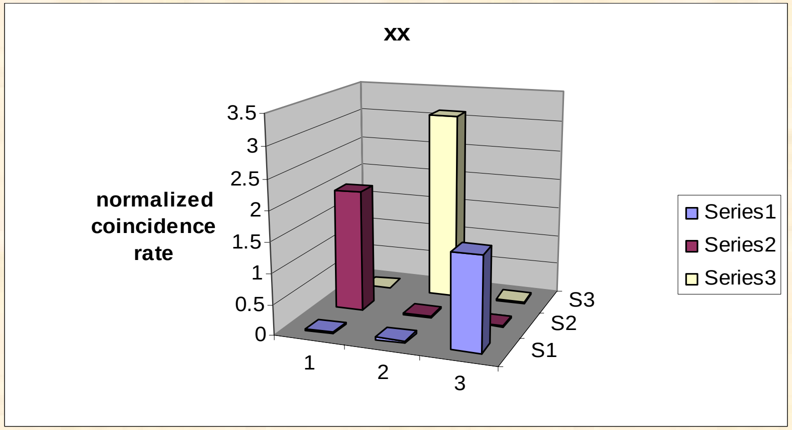
Different basis: uncorrelated measurements (9 possible coincidence measurements).

Generalization of Ekert cryptographic protocol to qudits of arbitrary dimension d (d=3)

Ray Beausoleil

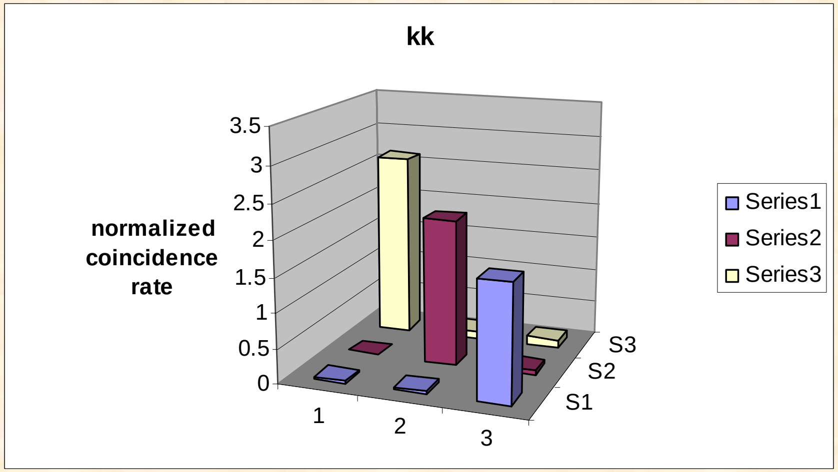
Pixel Entanglement Results

Position-Position

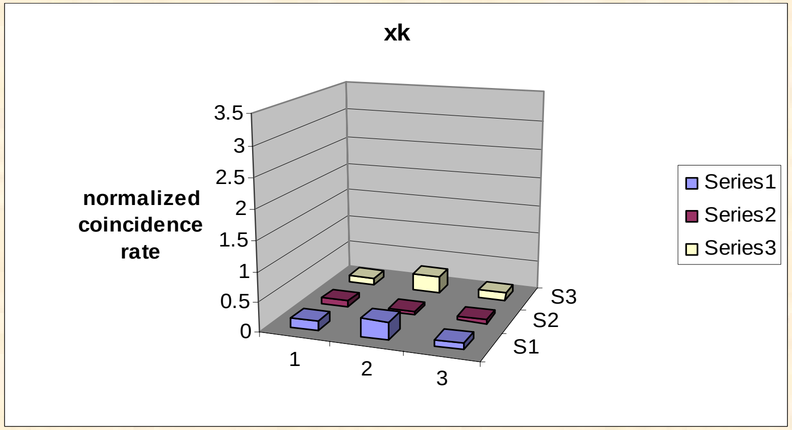


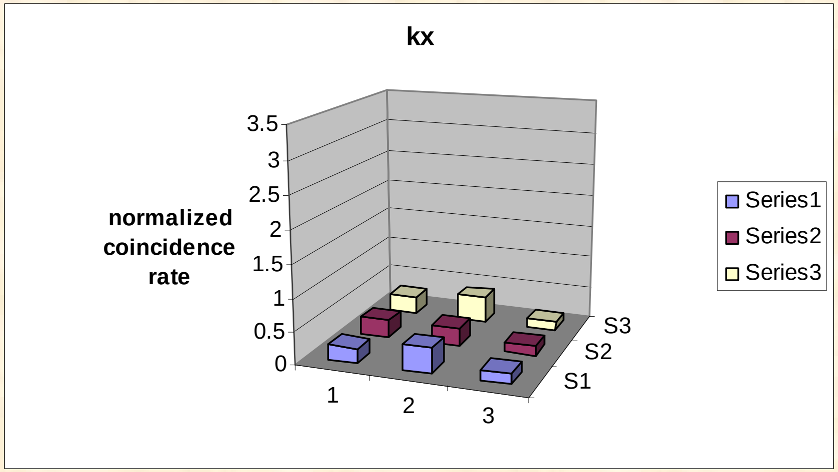
O'sullivan Hale, Ali Khan, Boyd and Howell
PRL (in press)

Momentum-Momentum

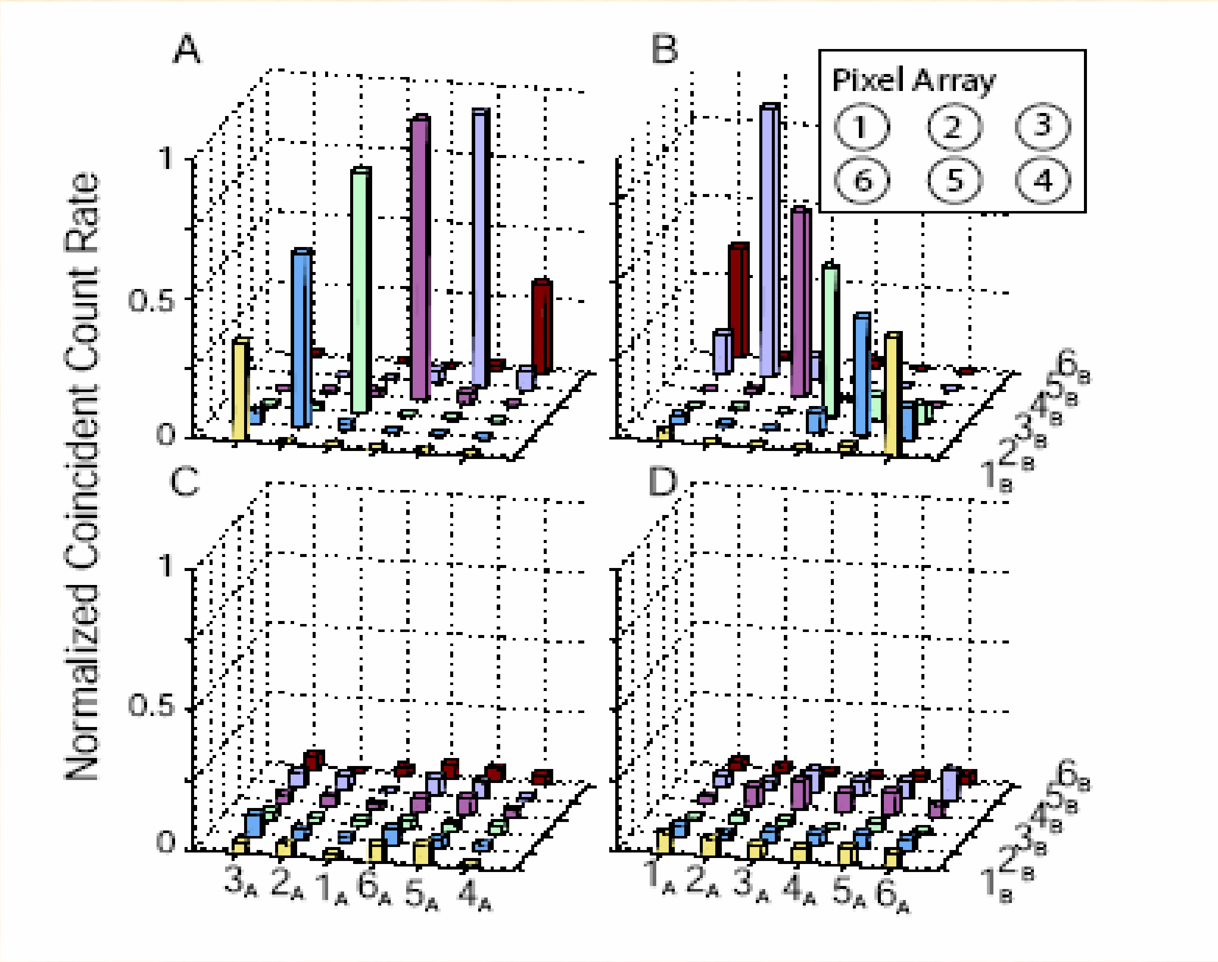


Pixel Entanglement



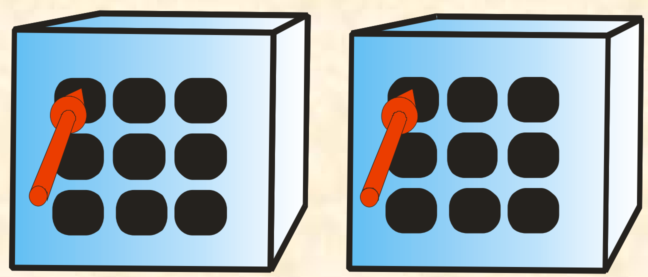


6 pixel array



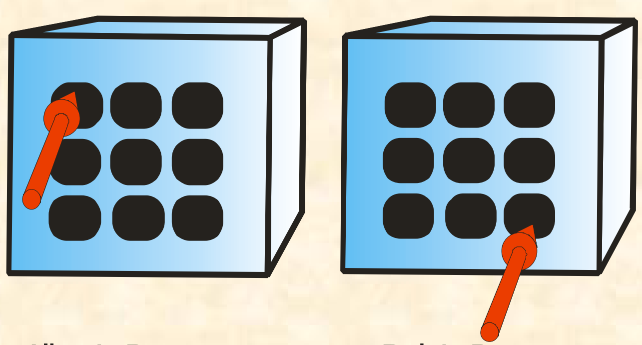
Generalization to large state spaces

Position Correlation



Alice's Detector Bob's Detector

Momentum Correlation



Alice's Detector Bob's Detector

Current limit to dimensionality is due to detectors.

Generalization to arbitrarily large APD arrays.

Reminder: APD arrays inside single photon emission cones.

Time-Energy: Why?

- Quantum Communication
  - Transverse entanglement requires wavefront preservation: multimode
  - Time-Energy: Single Mode (fiber transportable)
  - Very High Bandwidth (qubits vs. large d qudits)

Time-Energy Correlations

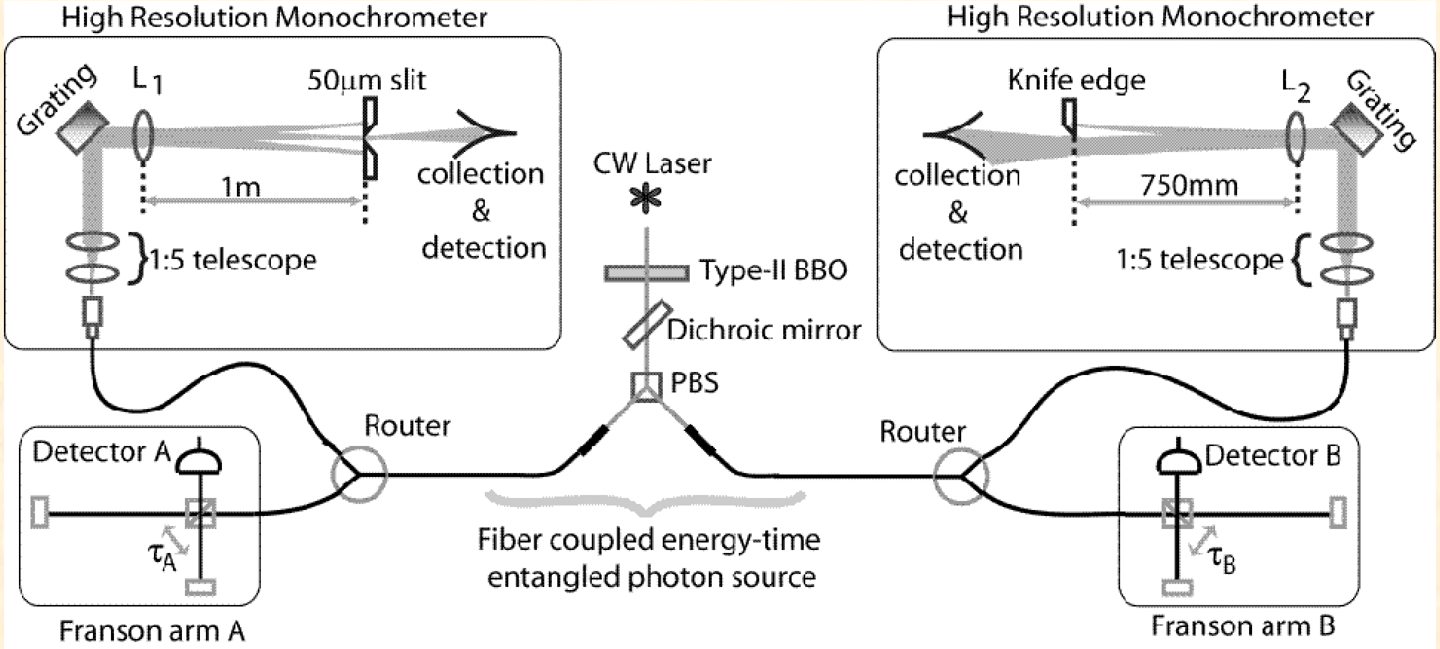
- Time Correlations (100's of fs)
  - Need ultra fast detectors
  - HOM dip is local measurement
  - Use Franson Interferometer to measure fourth order correlations: space-like separated detection $x^2 > (ct)^2$
- Energy Correlations (MHz set by pump)
  - Grating spectral decomposition
- Large Potential Information Content
  - Bandwidth of Down Conversion divided by the Bandwidth of the Pump Laser

$$K = \frac{\Delta\omega_{PDC}}{\Delta\omega_{pump}} \approx \frac{10THz}{1MHz} = 10^7$$

Information Eigenmmodes

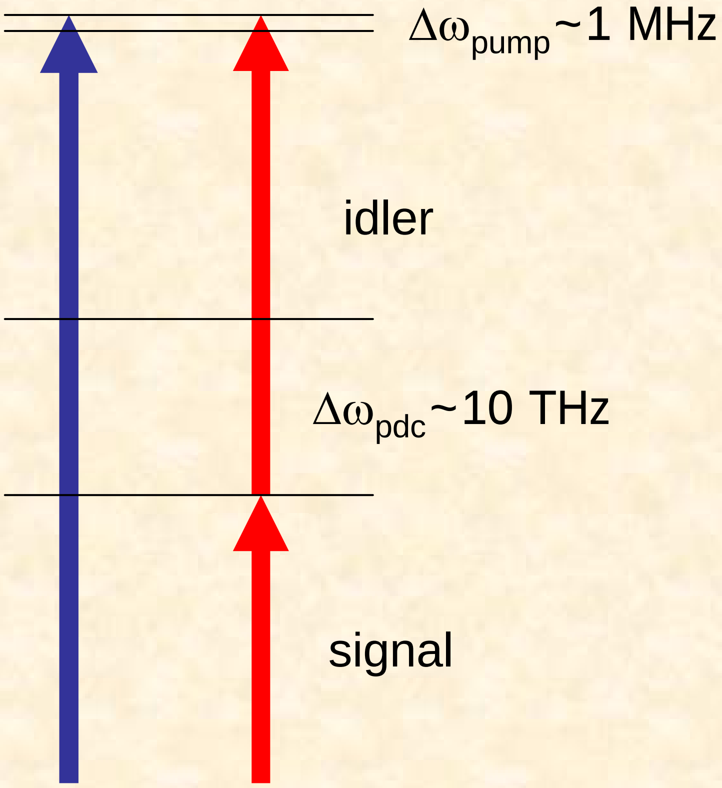
C. K. Law and J. H. Eberly
Phys. Rev. Lett. **92**, 127903 (2004)

Time Energy Entanglement



Energy-Energy Correlation

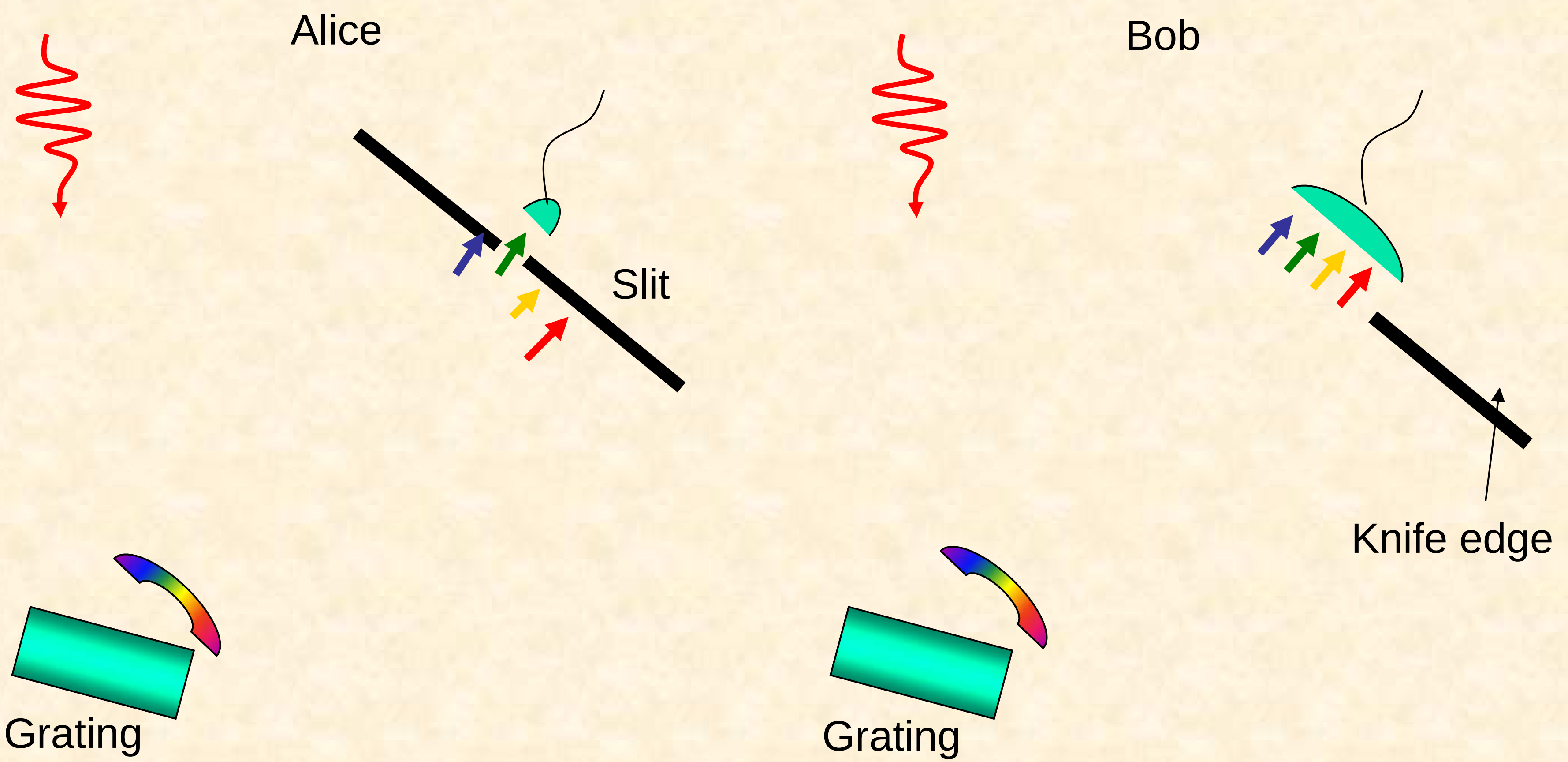
$$\omega_{pump} = \omega_i + \omega_s$$



- Energy Energy correlations set by phase matching conditions
- Energy conservation yields high energy correlations for CW pump

Energy-Energy Correlations

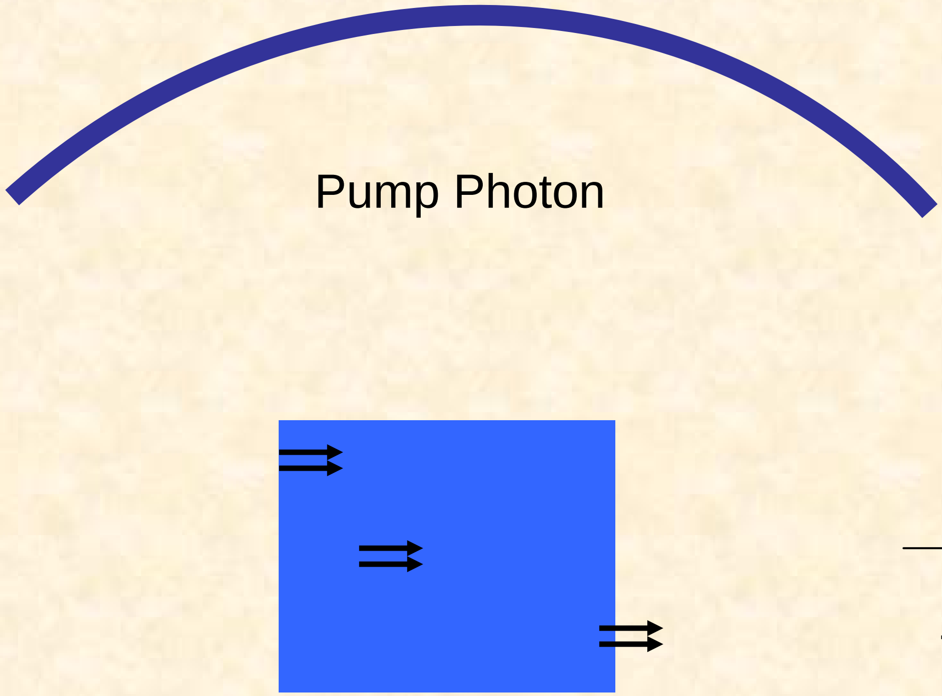
$$\Phi(\omega_s,\omega_i) \propto Exp\left(\frac{-(\omega_s+\omega_i)^2}{2(\Delta\omega_p)^2}\right)$$

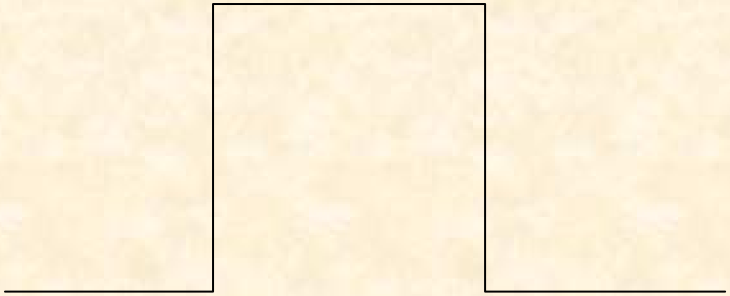


Type II time-time correlations

1. Horizontal, Vertical different velocity (birefringence)
2. Spontaneous emission equally likely at any point in crystal

$$\Omega(t_1, t_2) = R(t_1 - t_2)$$





Temporal Correlation Function

Temporal Correlation of Franson Interferometer

Output ports of Michelson with postselection of short-short and long long

$$\left|\Psi(t_1,t_2)\right\rangle = \int dt_1 \int dt_2 A(t_1,t_2)[a^\dagger(t_1)a^\dagger(t_2) + e^{i(\varphi_{12})}a^\dagger(t_1-\tau_1)a^\dagger(t_2-\tau_2)]\left|0\right\rangle$$

$$g_{1,2} = \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \left\langle\Psi\left|a_1^\dagger(t_1)a_2^\dagger(t_2)a_1(t_1)a_2(t_2)\right|\Psi\right\rangle$$

Repeated use of equal time boson commutator relation and normal ordering

Franson Envelope

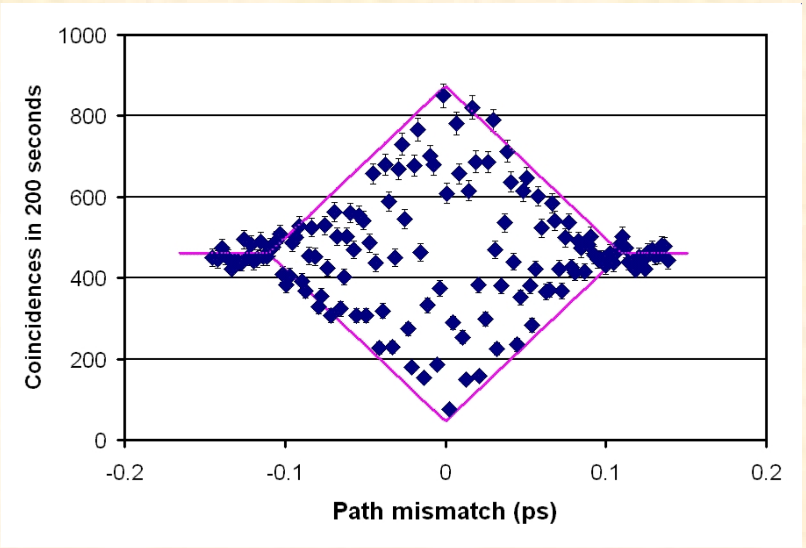
$$g_{1,2} = 1 - \cos(\varphi_{12})\Lambda(\tau_1 - \tau_2)$$

Hong-Ou-Mandel dip

$$g_{1,2} = 1 - \Lambda(\tau_1 - \tau_2)$$

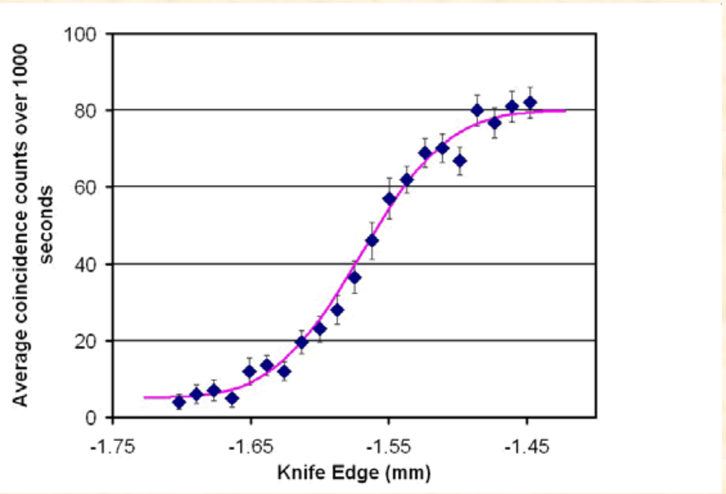
Time-Energy Results

100 fs RMS



Time-Time Correlations

0.048 nm RMS



Energy-Energy Correlations
Knife Edge Sweep

Experimental Apparatus

Curtis Broadbent

Irfan Ali Khan

Time-Energy Results

$$\left\langle \left(\Delta E_{12}\right)^2 \right\rangle \left\langle \left(\Delta t_{12}\right)^2 \right\rangle \approx 0.00022\hbar^2$$

- Measured Time-Energy Variance Product
- Single Mode (fiber transportable)
- Limitations
  - Low flux per spectral window
  - Limited spectral resolving power: Could violate variance product by many more orders of magnitude

Conclusion

- Showed discrete and continuous entanglement
- Violated EPR bound (security measure) by two orders of magnitude
- Demonstrated Pixel Entanglement (correlated pixels in nonorthogonal bases).
  - Quantum information with large Hilbert spaces
- Fiber transportable giant entanglement
  - Long distance capabilities
  - Up to 10 million pixels (10 million entangled states)
  - Working on a fiber based large qudit cryptosystem