Boyd Group Research Program

- Single photon imaging (joint with Howell group)
  - full image encoded on a single photon
- Entanglement propagation through turbulence
- Nature of two-photon interference
  - observation of generalized HOM interference
- Development of photon-number-resolving detectors
  - Bayesian analysis can improve performance of TMD
- Quantum lithography
  - careful dosimetry measurements of recording materials

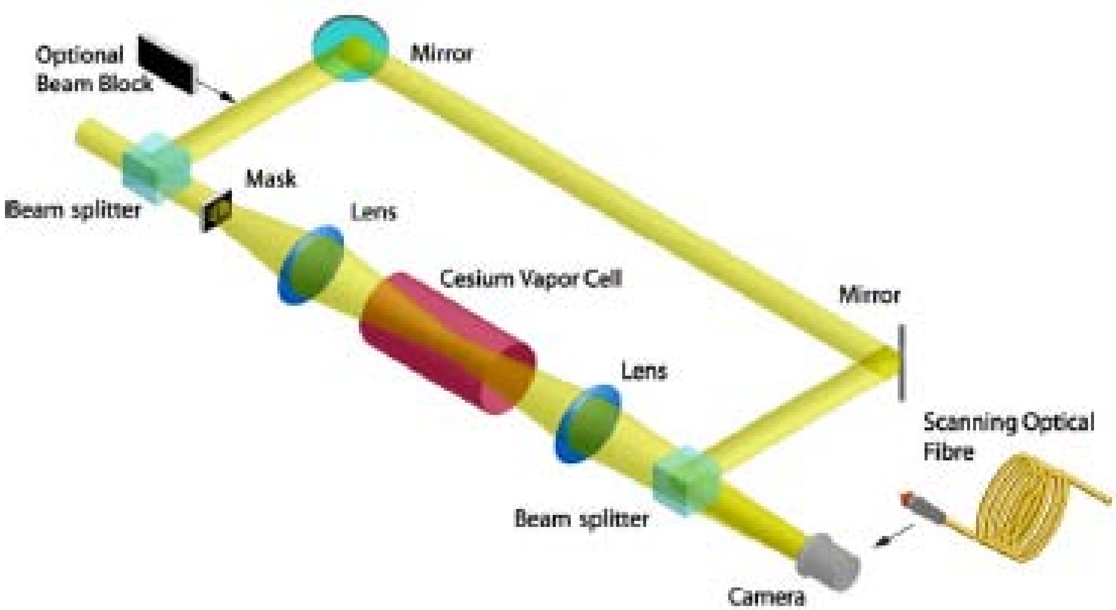
Single-Photon Imaging

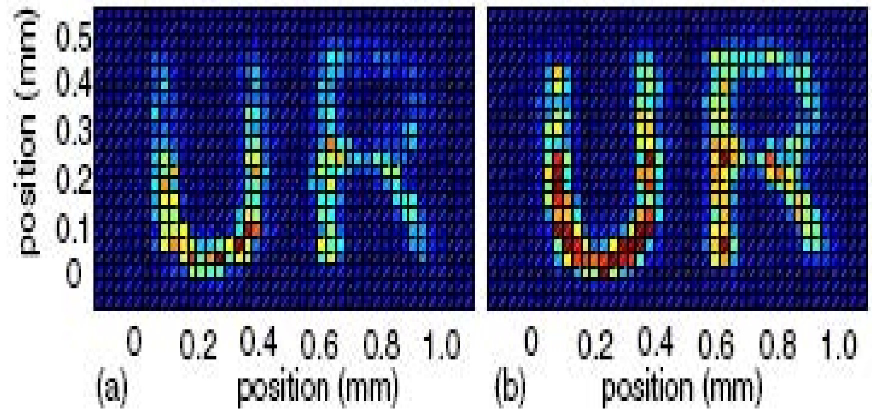
Joint Project: Boyd and Howell Groups

Petros Zerom, Heedeuk Shin, others

- We want to impress an entire image unto a single photo and later recover the image
- Our procedure is to "sort" the photons into classes determined by the image impressed on the photon
- We use holographic matched filtering to do the sorting
- We use heralded single photons created by PDC

Prior Work - Howell Group





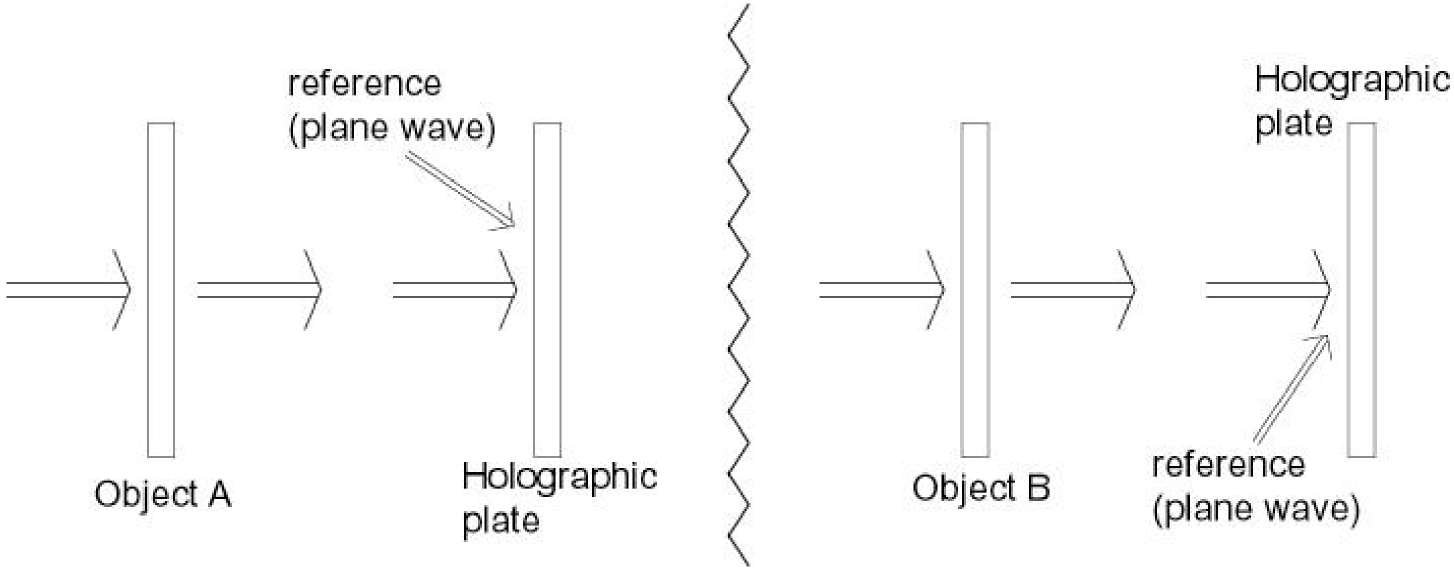
Delayed (9 ns) **Non-delayed**

- Delayed an image (with phase and amplitude characteristics preserved) by many pulse widths
- Delayed image using very weak light pulses (4 ns FWHM, <1 photon/pulse)
- Image reproduced with high fidelity and low noise
- But can read out image only one pixel at a time

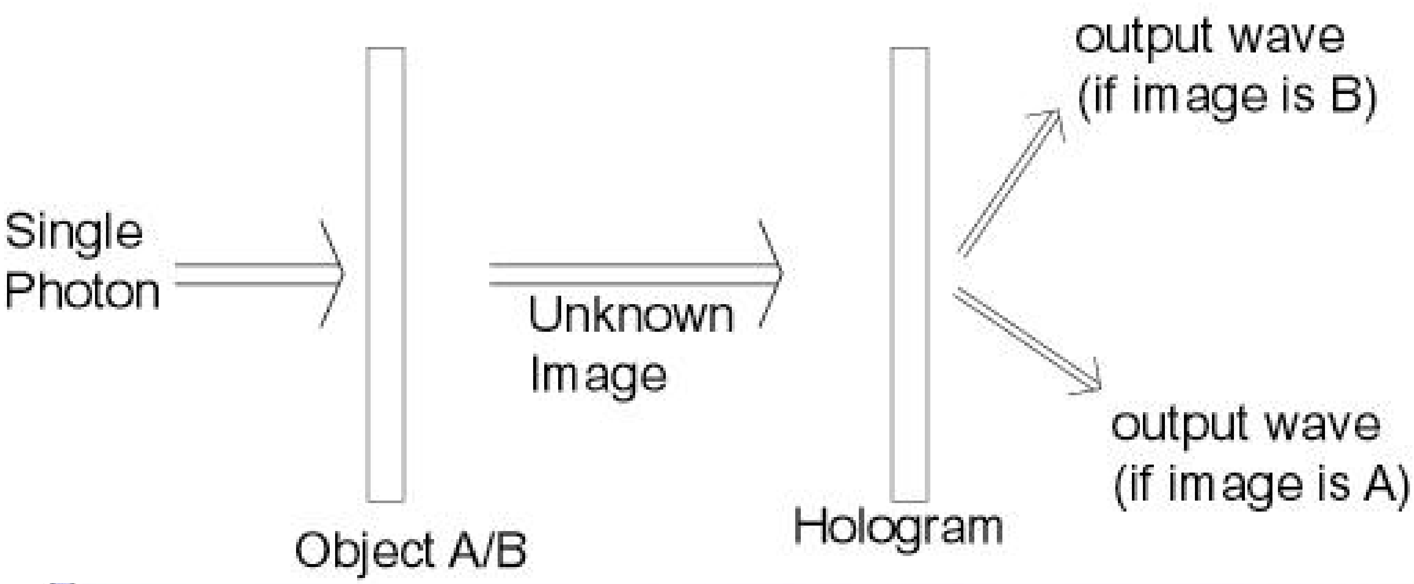
R. M. Camacho, *et al, PRL* ***98****, 043902 (2007)*

Holography, matched filtering, and single-photon Imaging

- Writing the matched filter (a multiple exposure hologram)

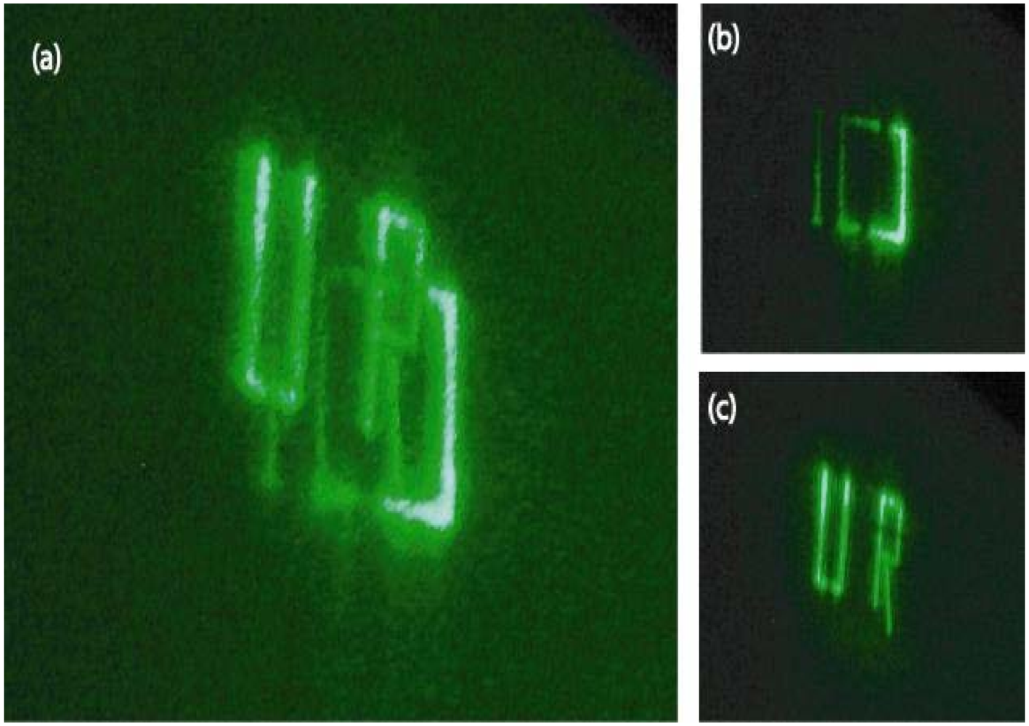


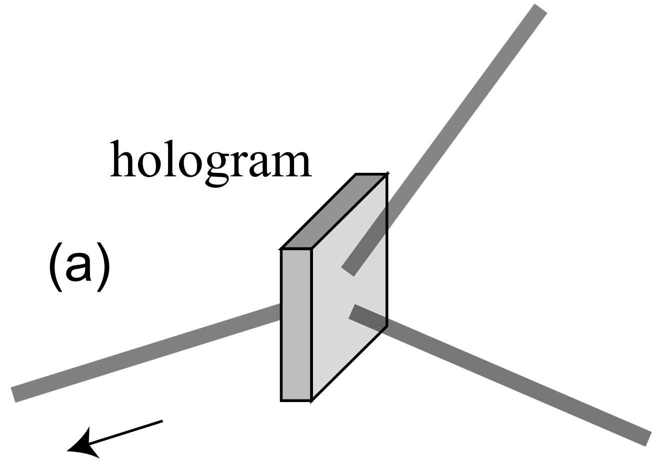
- Reading the hologram (with a single-photon)

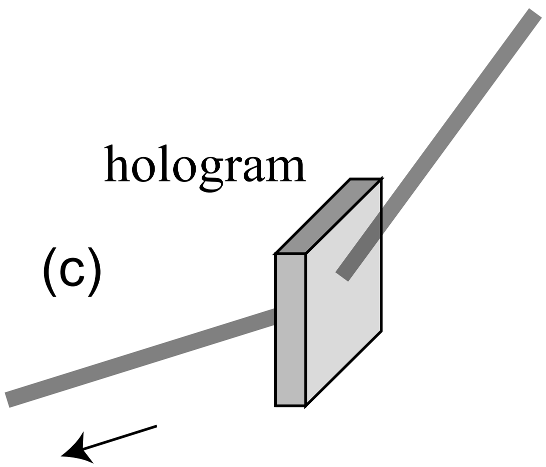


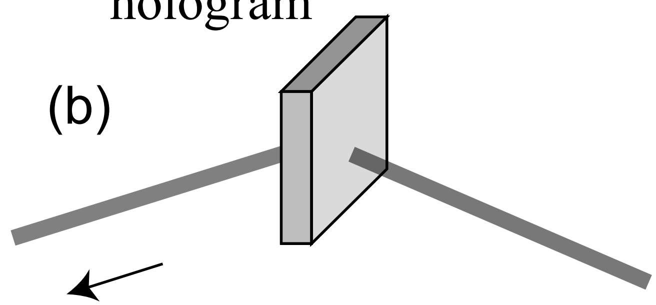
- Generalize to N-exposures

Reconstruction - with plane-wave reference beam

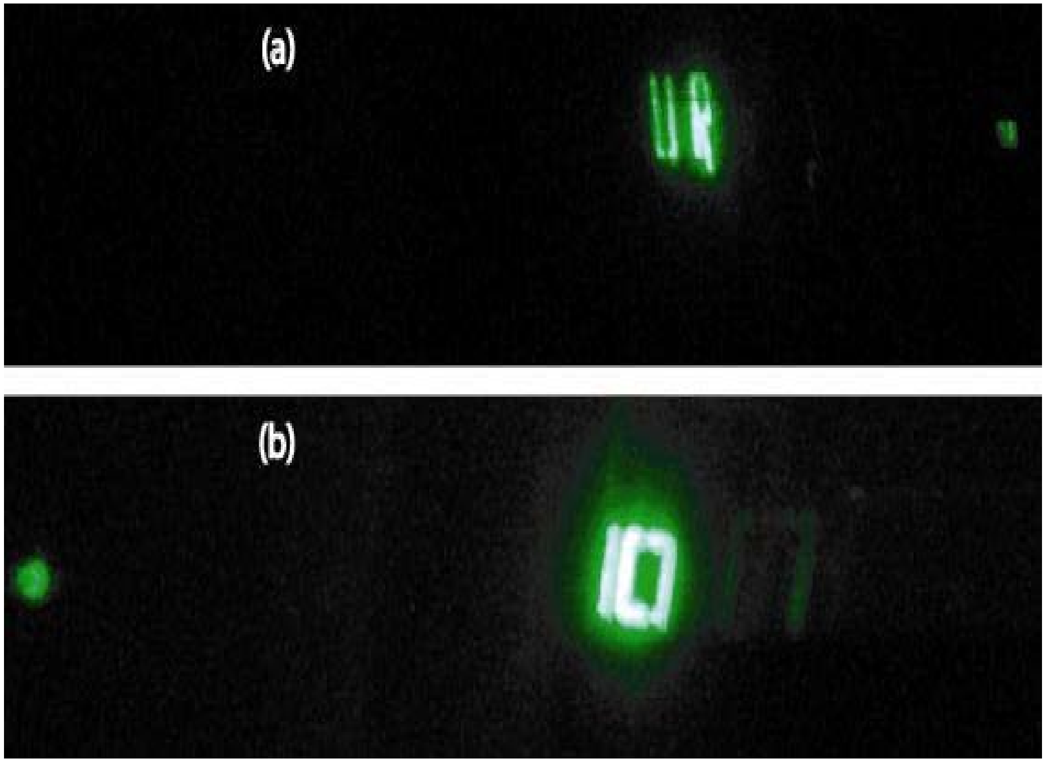


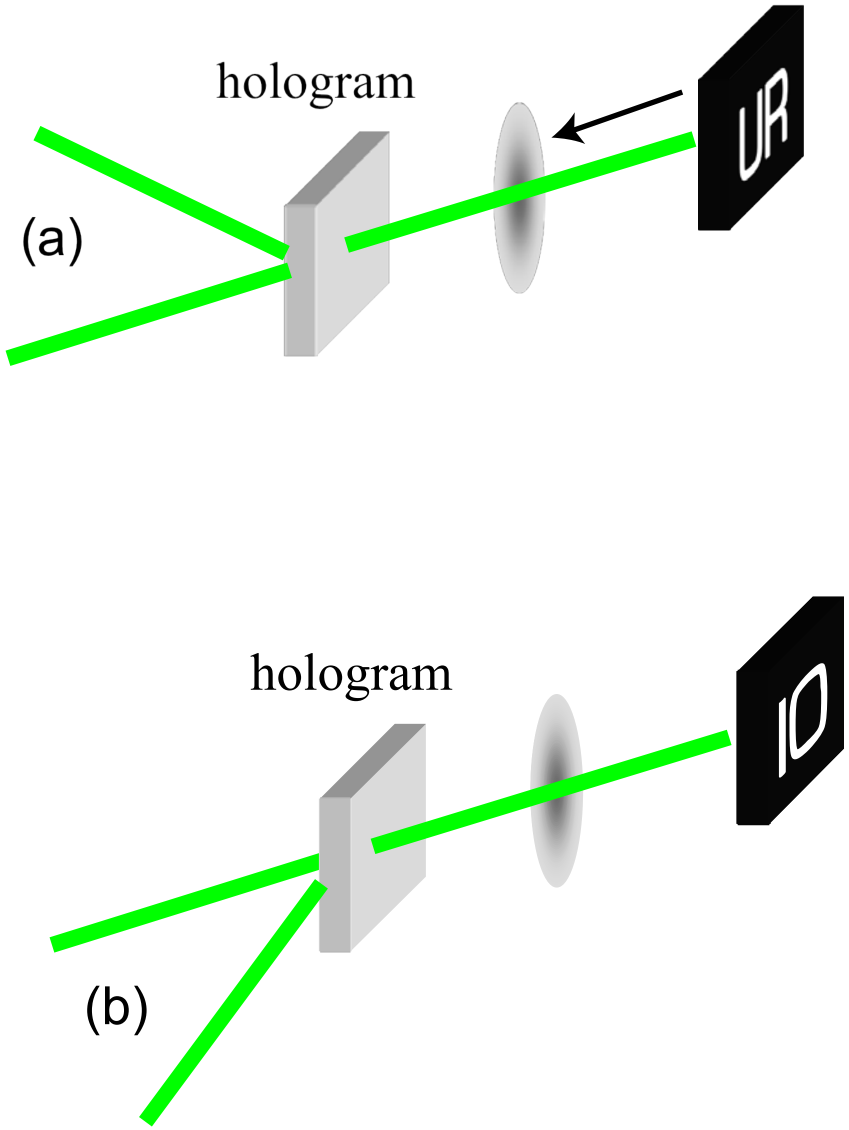






Reconstruction - with structured reference beam





- Very little cross-talk

Single-Photon Imaging - Latest Result

- We have just demonstrated that we can distinguish the "IO" photon from the "UR" photon at the level of an individual single photon
- We use very weak laser light (less than one photon per temporal mode) and place an APD at the location of the diffraction spot

High light level

Low light level

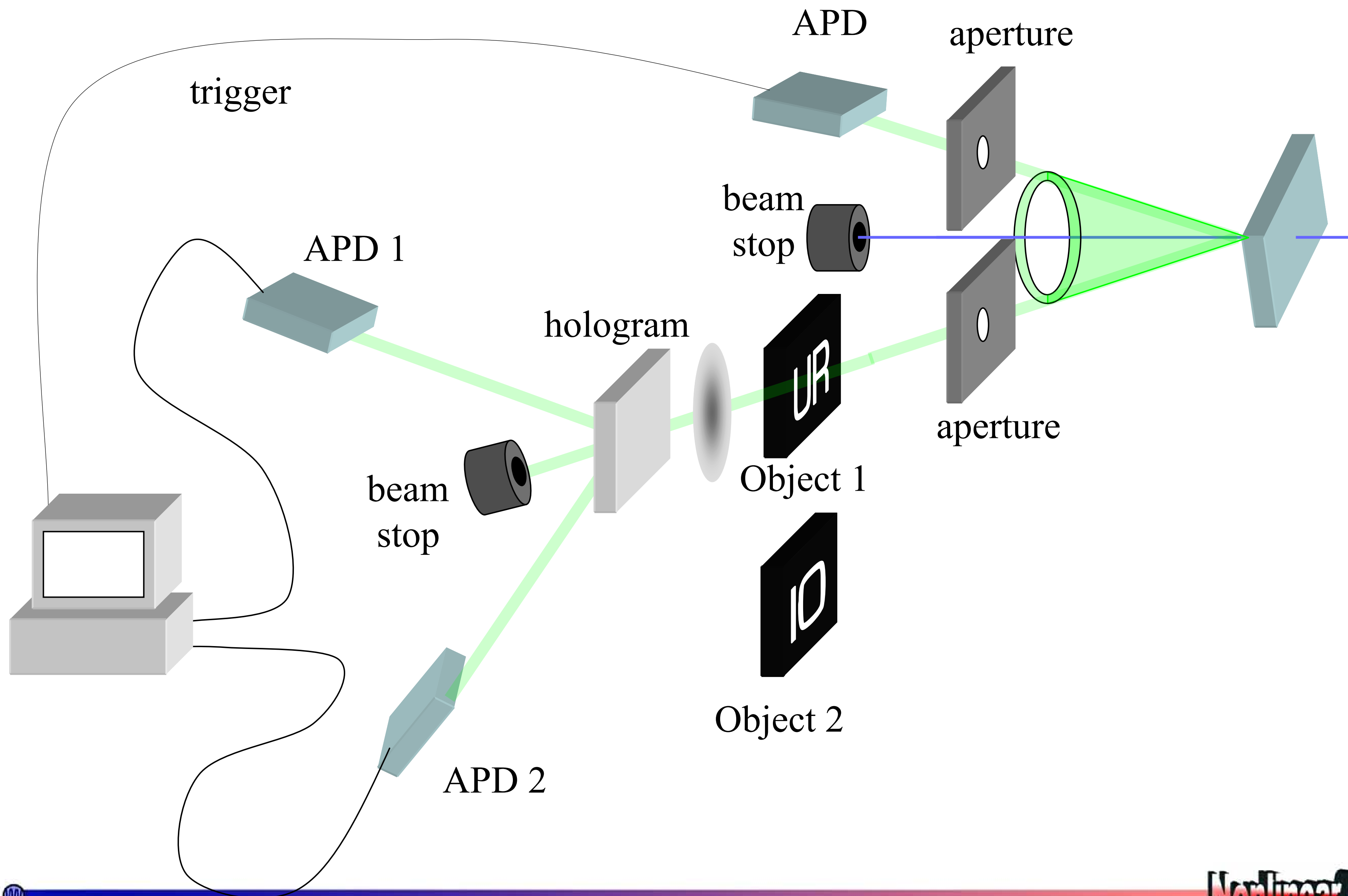
Count rate (1/s) **146** **24506**

High light level

Low light level

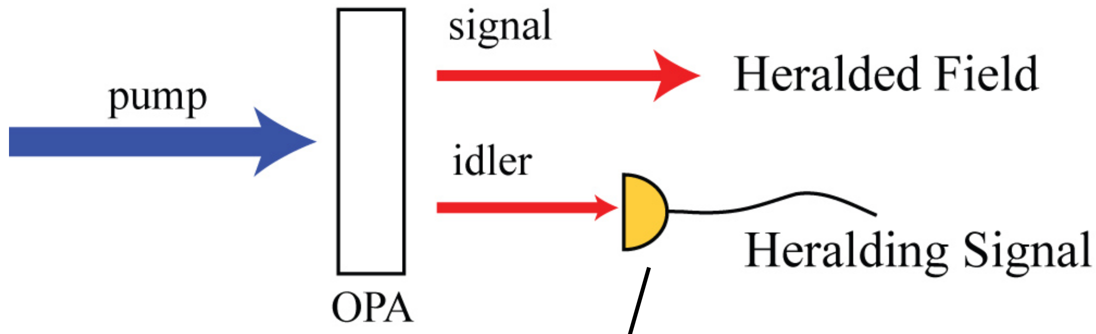
Count rate (1/s) **41387** **444**

Next step: use heralded single photons

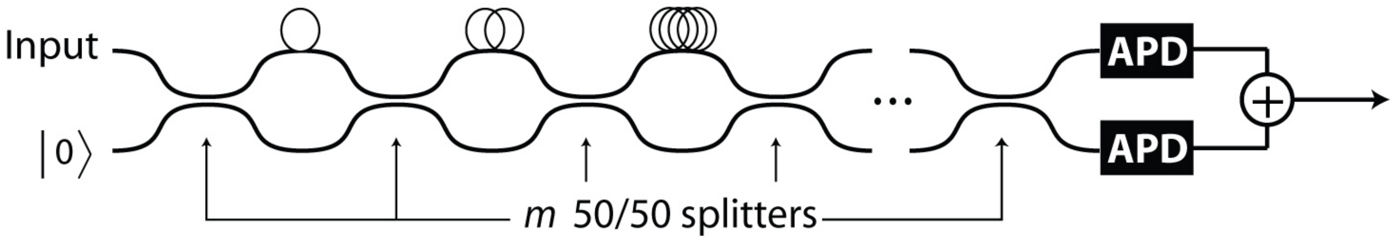


Heralded Photon-Number States

Scheme for producing heralded photon-number states



Time-Multiplexed Photon-Number Resolving Detection



TMDs do not provide perfect photon-number-resolving capabilities (but are easy to implement in the lab) because of loss etc.

Under what conditions will this method work?

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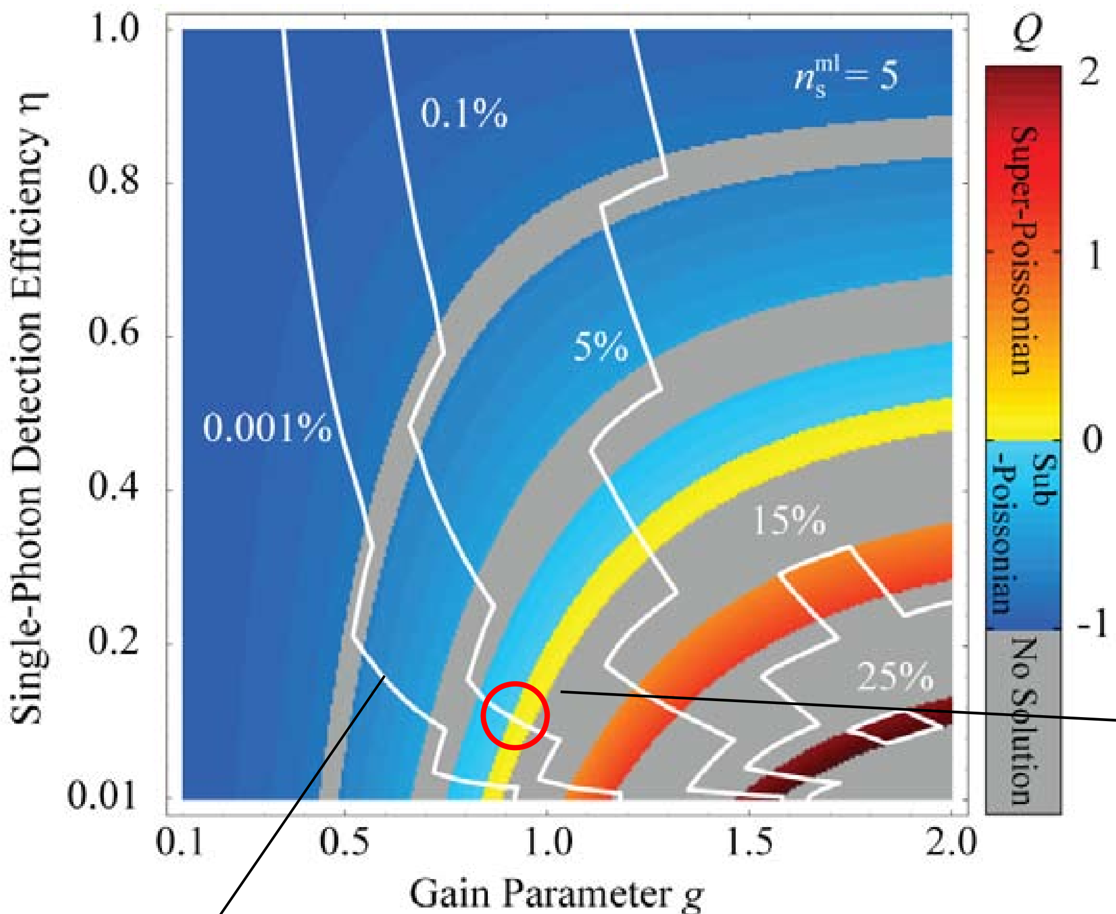
Results

Using Bayes' theorem and *a priori* knowledge of the statistics of the OPA (characterized by gain *g*), we calculate the Mandel's *Q*-parameter to characterize the resulting heralded state for various detector parameters.



white lines – efficiency of creating the heralded state

<u>Example</u>

Create a 5-photon heralded state

Use a TMD with 5 beam splitters and detection efficiency η.

Trade-offs between purer Fock states and

- heralding efficiency (determined by *g*)
- TMD detection efficiency η

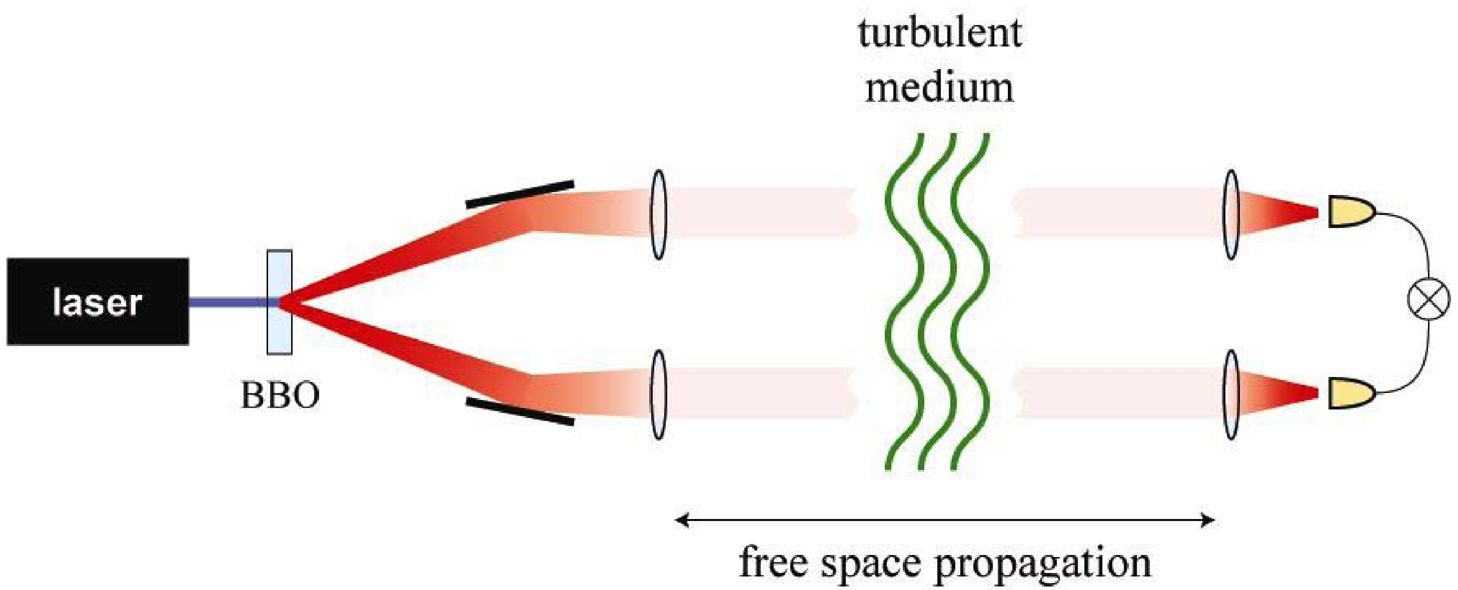
But, for detector efficiency as low as η ~ 10%, sub-Poissionian states can be created with reasonable efficiency.

Entanglement Propagation

Goal

To understand and develop the tools to study how the transverse spatial correlations between photons produced in SPDC change as the photons propagate:

- through free-space (develop formalism, merit functions, experimental techniques)
- through distorting and turbulent media



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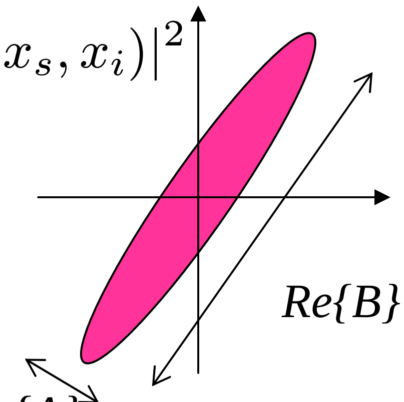
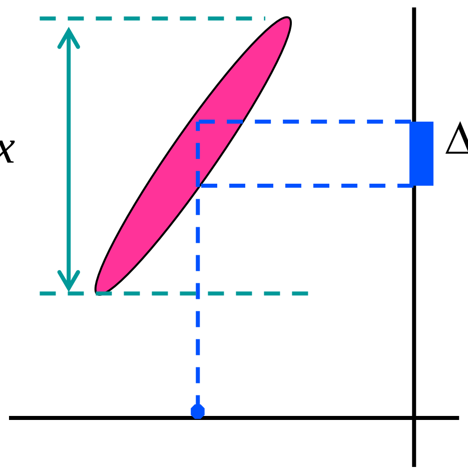
Measures of Entanglement





To quantify amount of entanglement, we use the Schmidt Number:

$$K \equiv \left(\sum_n \lambda_n^2\right)^{-1}$$

$$\Psi(x_1, x_2) = \sum_n \sqrt{\lambda_n}\, \phi_n(x_1)\, \psi_n(x_2)$$

More conveniently, we can use the Fedorov Ratio [2,3]:

$$R_x \equiv \frac{\Delta x}{\Delta x^{\text{cond}}}$$

[1] D'Angelo et al, PRL. **92**, 233601 (2004).
[2] Fedorov et al., PRA **69**, 052117 (2004).
[3] Chan and Eberly, quant-ph/0404093.

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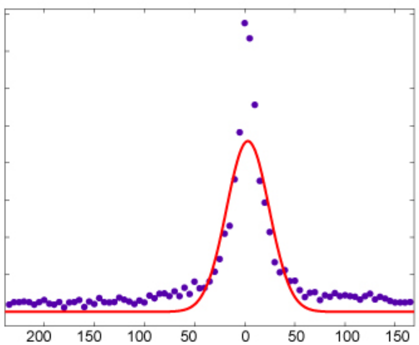
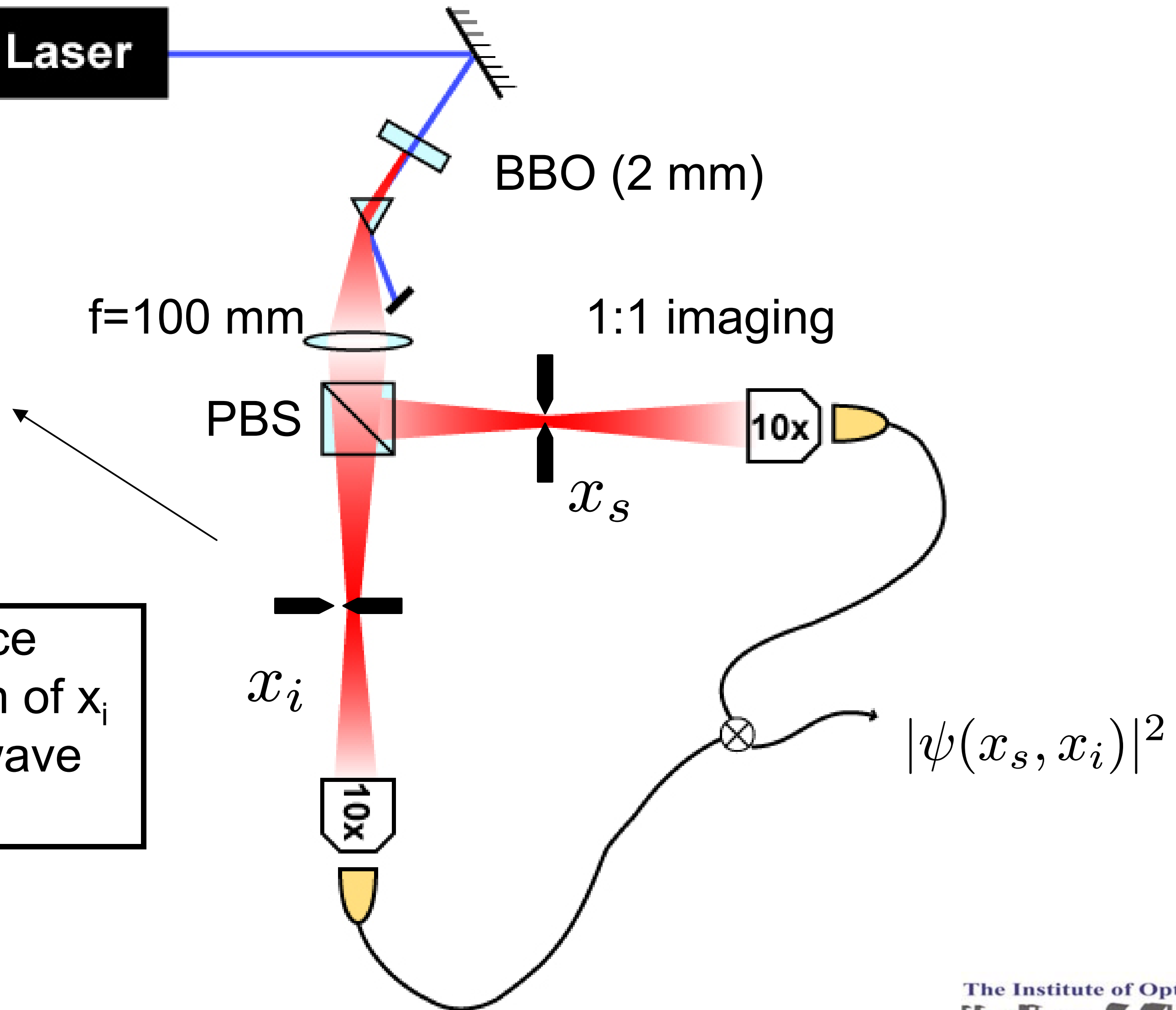
Experimental Set-up

$\lambda = 363.8$ nm
$w_0 = 850\ \mu$m





Measure coincidence events as a function of x_i and x_s to map out wave function.

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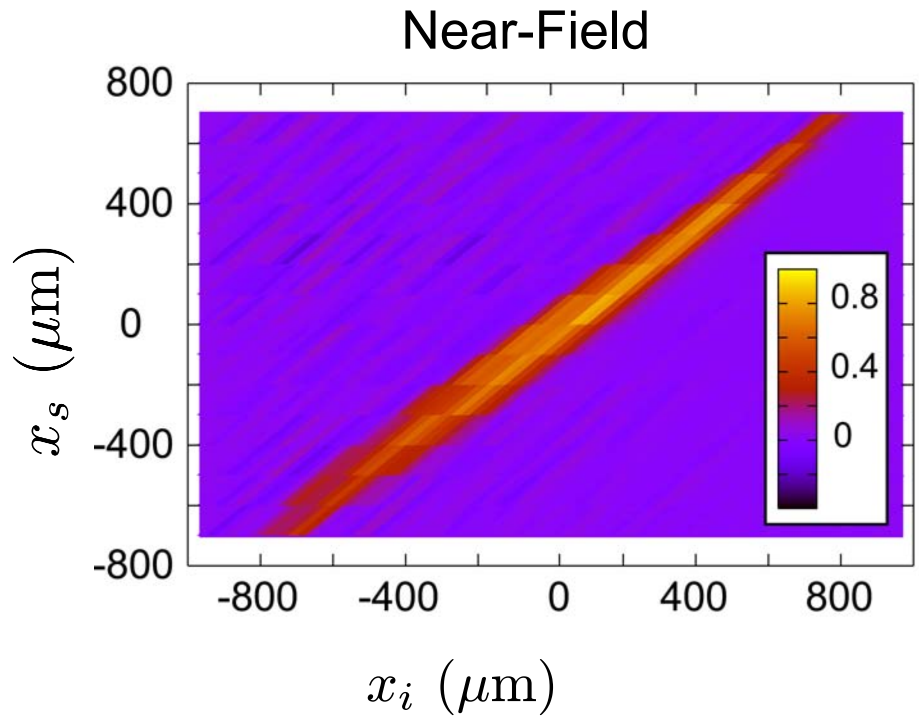
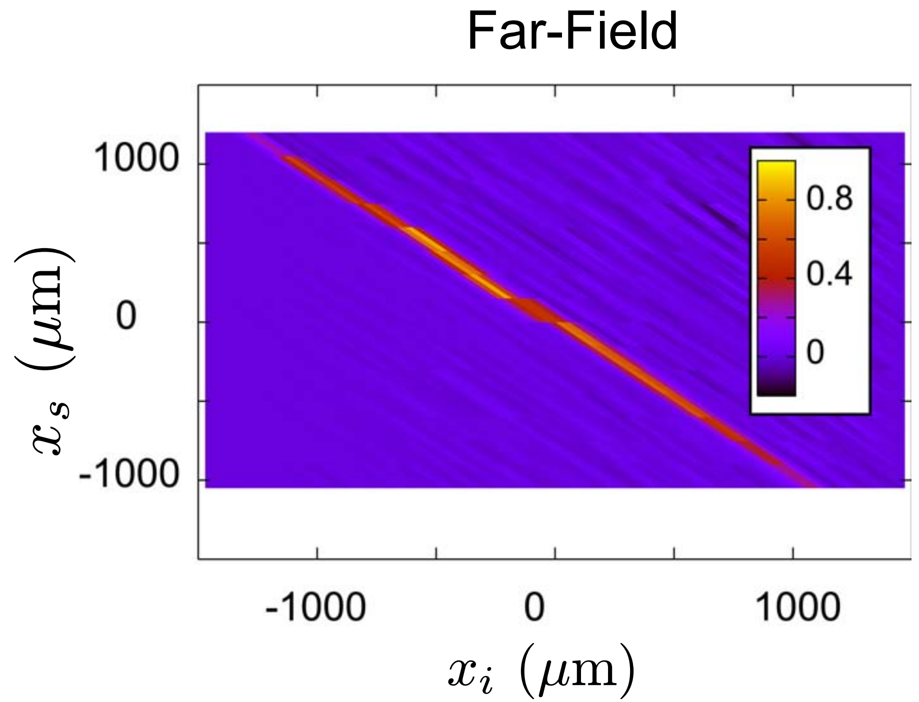
Near- and Far-Field Correlations





- Normalized coincidence rates plotted versus slit position in the signal and idler arms.

- Strong correlations in the near- and far-fields typical of entangled light beams. The Fedorov ratios are 9.28 and 28.52 respectively.

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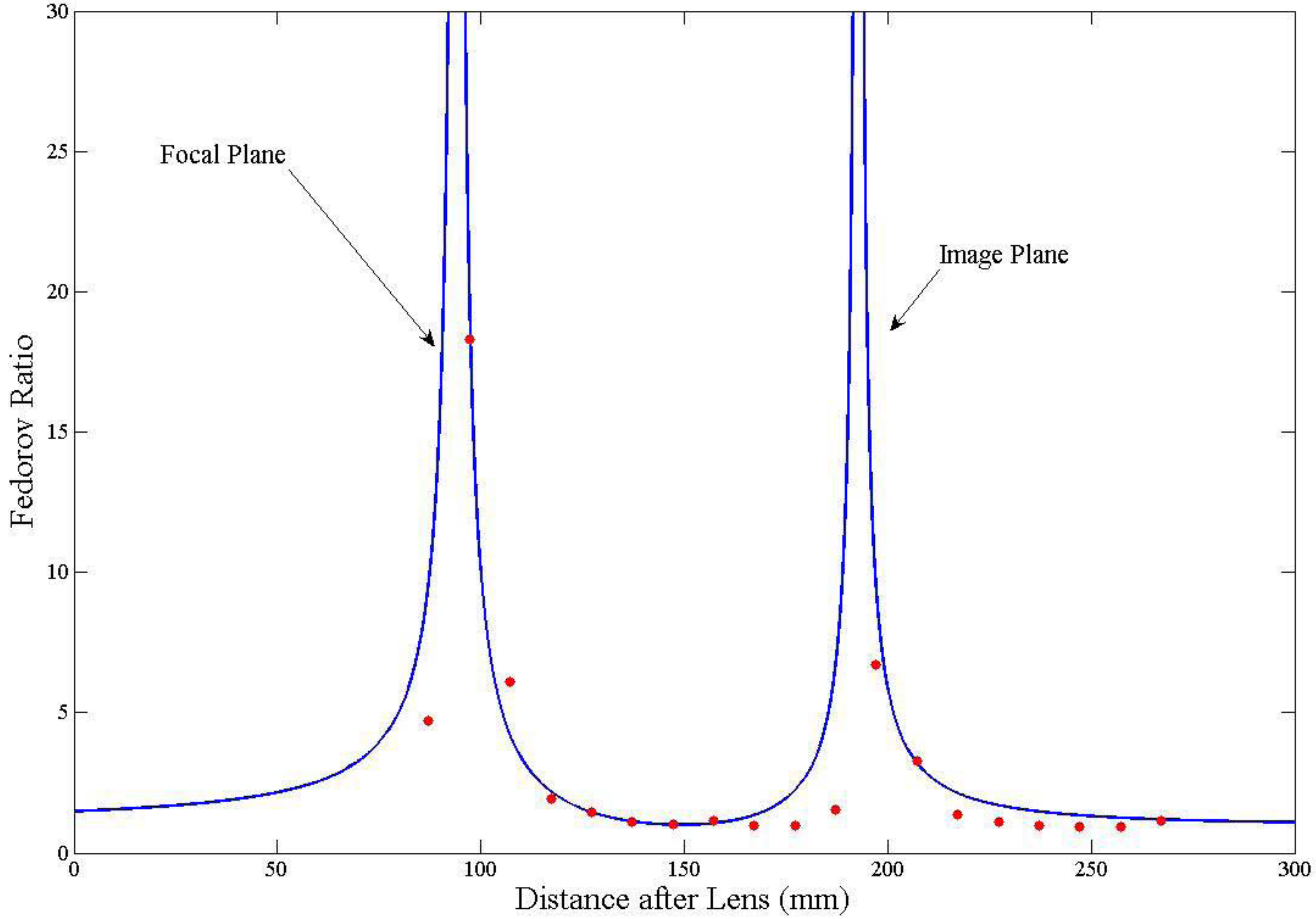
Propagation of the Fedorov Ratio



Fedorov Ratio measured at different longitudinal points.

Next step: use interferometry to measure the phase of the two-photon wavefunction to demonstrate that entanglement has "migrated" to the phase of the wavefunction.

Theory of Propagation through Turbulence

The propagated field in a turbulent medium is given by

$$\hat{E}^{(+)}(\vec{x}, z) = e^{ikz} \int d\vec{x}' \; h(\vec{x}, \vec{x}', z) e^{i\phi(\vec{x}')} \hat{E}^{(+)}(\vec{x}', 0)$$

Note:

1. Turbulent medium is described by the statistical character of $\phi(\vec{x}')$.
2. The medium is replaced by a single "phase screen" accounting for all the phase fluctuation incurred in the propagation to z, i.e., $\phi(\vec{x}') = k \int_0^z n(\vec{x}', z') dz'$
3. Fluctuating phase: $\overline{e^{i[\phi(\vec{x}') - \phi(\vec{y}')]}} = e^{-(1/2) D_s(|\vec{x}' - \vec{y}'|)}$
   Phase structure function $D_s(|\vec{x}' - \vec{y}'|) = \alpha |\vec{x}' - \vec{y}'|^{5/3}$ (Kolmogorov)

Now take ensemble average when calculating four-point correlation function:

$$G(\vec{x}_s, \vec{y}_s; \vec{x}_i, \vec{y}_i) \equiv \langle \Psi | \overline{\hat{E}^{(-)}(\vec{y}_i, z_i) \hat{E}^{(-)}(\vec{y}_s, z_s) \hat{E}^{(+)}(\vec{x}_s, z_s) \hat{E}^{(+)}(\vec{x}_i, z_i)} | \Psi \rangle$$

Quantification of Entanglement

Biphoton density matrix - approximate r^(5/3) dependence of D by r^(6/3)

$$G_0(\vec{x}_s,\vec{y}_s;\vec{x}_i,\vec{y}_i) = e^{-\frac{1}{2}\mathcal{D}_s(|\vec{x}_s-\vec{y}_s|)} e^{-\frac{1}{2}\mathcal{D}_i(|\vec{x}_i-\vec{y}_i|)} \Psi(\vec{x}_s,\vec{x}_i)\Psi^*(\vec{y}_s,\vec{y}_i)$$

with $$\Psi(\vec{x}_s,\vec{x}_i) = N \exp\left[-\frac{B}{2}(\vec{x}_s-\vec{x}_i)^2\right] \exp\left[-\frac{A}{2}(\vec{x}_s+\vec{x}_i)^2\right]$$

$$D_s(r) = 3.44\left(\frac{r}{r_0}\right)^{6/3}$$

r_0 – the length scale of turbulence structure

For continuous variable entanglement

The second moments of the variables provide useful information about the degree of entanglement. #

Measures of entanglement (these are mixed states; can't use Schmidt and Fedorov)

1. EPR uncertainty
2. Entanglement of formation

Hyllus & Eisert, New J. Phys. **8**, 51 (2006)

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Effect of Turbulence on Entanglement

<u>EPR uncertainty</u>

$$\Delta = \sqrt{\Delta^2(x_s - x_i) + \Delta^2(p_s + p_i)}$$

Gaussian state: $\Delta < 1$ entangled

$\Delta \geq 1$ disentangled

We find #

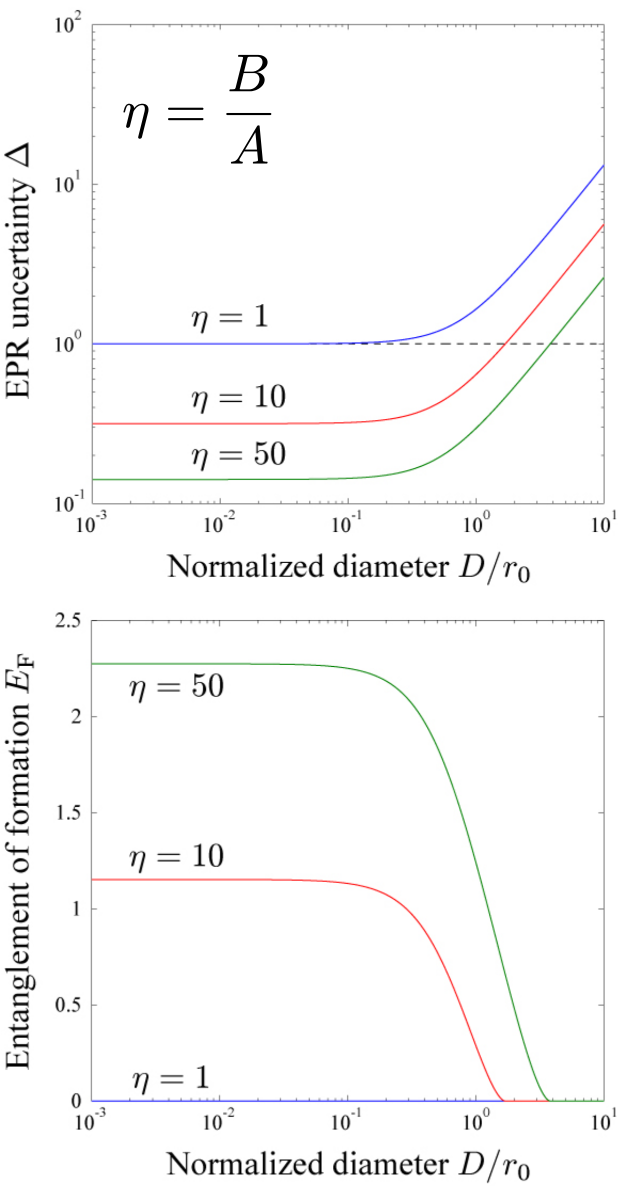
$$\Delta = \sqrt{\frac{(1+\eta^{-1}) + 3.44(D/r_0)^2}{1+\eta}}$$

<u>Entanglement of formation for Gaussian states</u>

– how much entanglement is needed to construct the state

$$E_F = c_+ \log c_+ - c_- \log c_-$$

where $c_\pm = \frac{1}{4}\left(\Delta^{-1/2} \pm \Delta^{1/2}\right)^2$

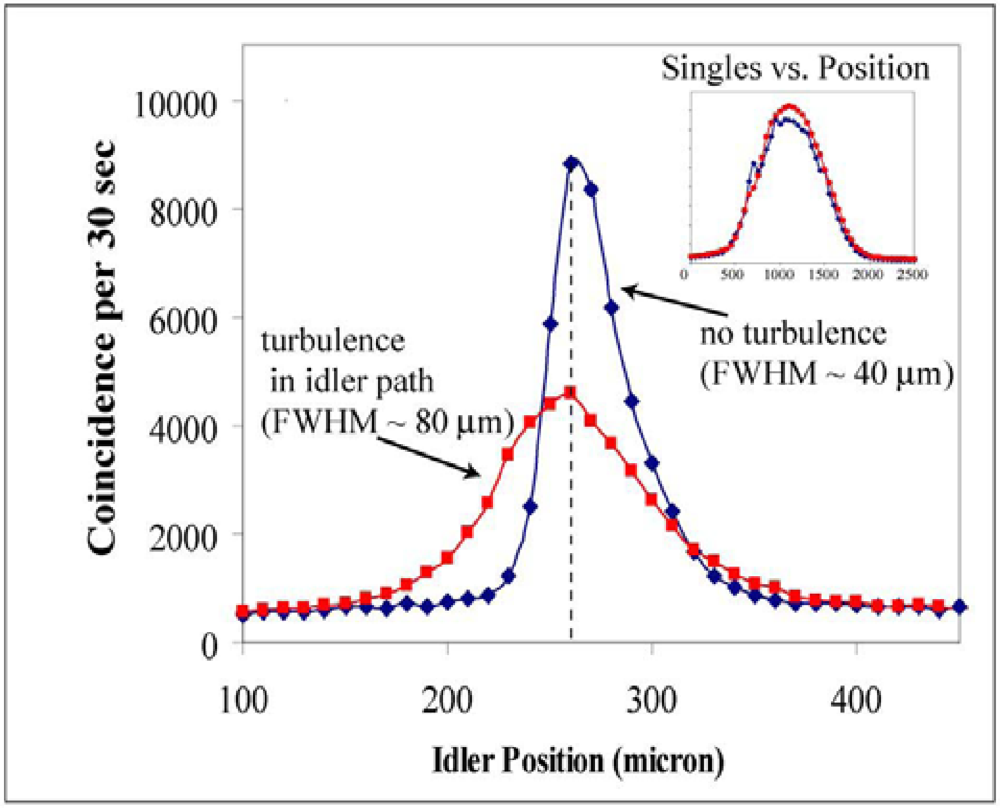


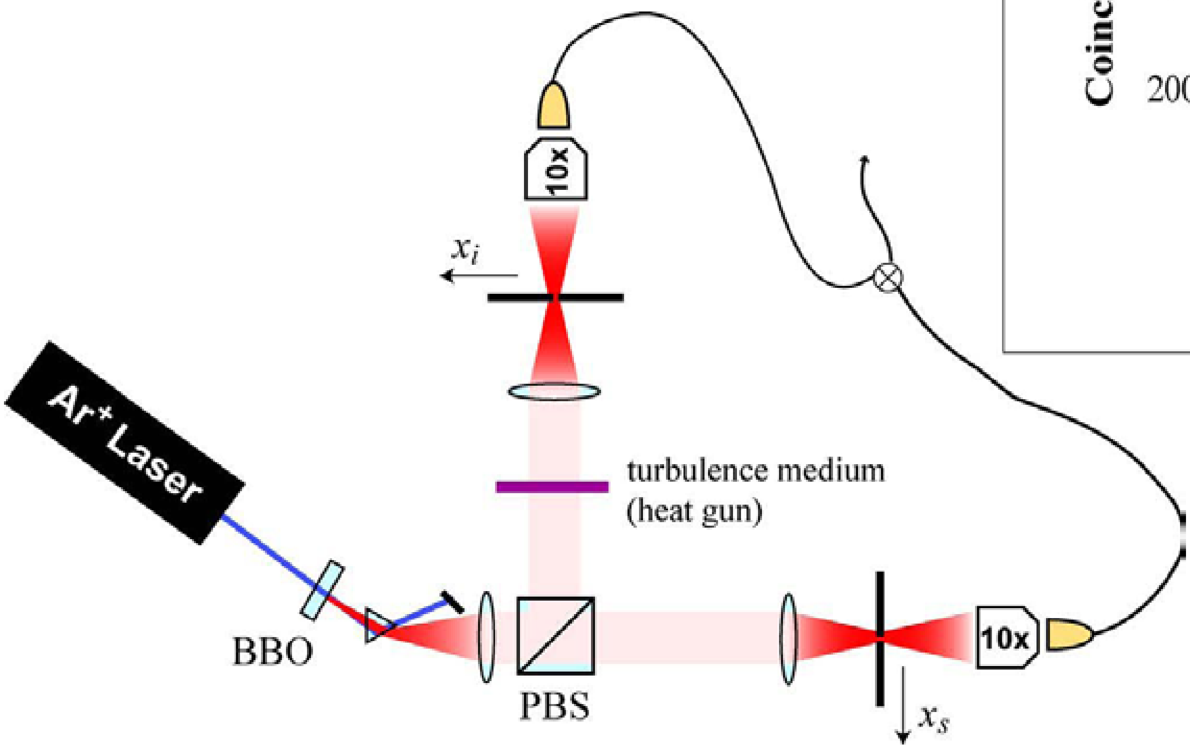
Giedke *et al.*, PRL **91**, 107901 (2003)

Preliminary Results

Turbulence medium:

- heat gun
  (easy to implement)
- Kolmogorov phase screen
  (quantitative degree of turbulence)





Next: Can we use adaptive optics to prevent the loss of entanglement due to turbulence?

Coherence and Indistinguishability in Two-Photon Interference

Anand Kumar Jha, Malcolm N. O'Sullivan-Hale,
Kam Wai Chan, and Robert W. Boyd

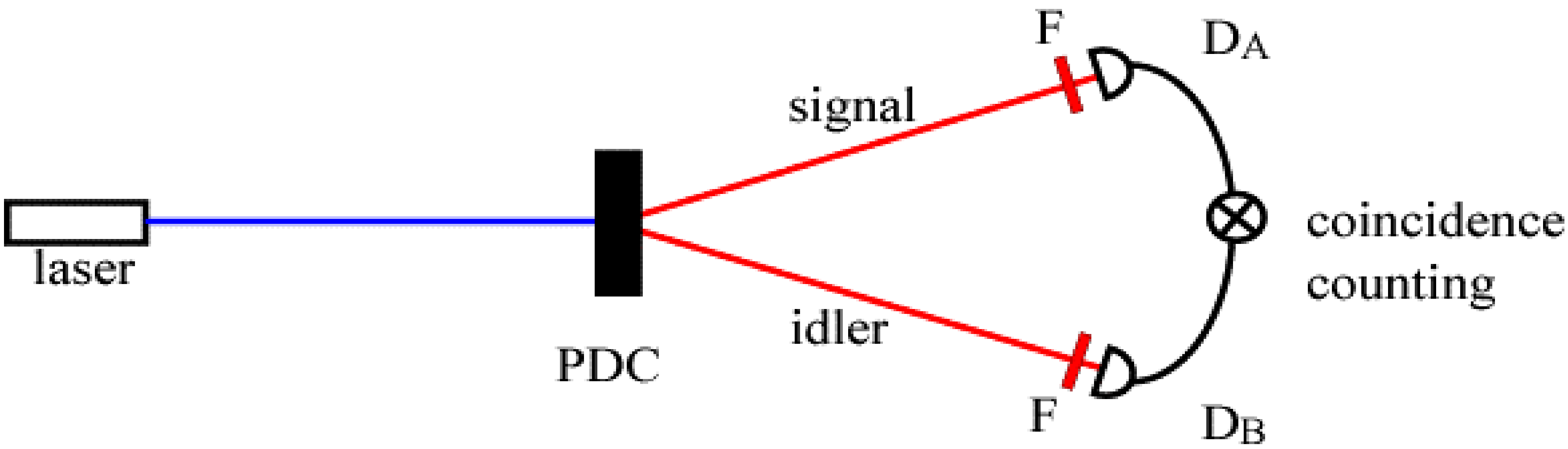
Institute of Optics, University of Rochester

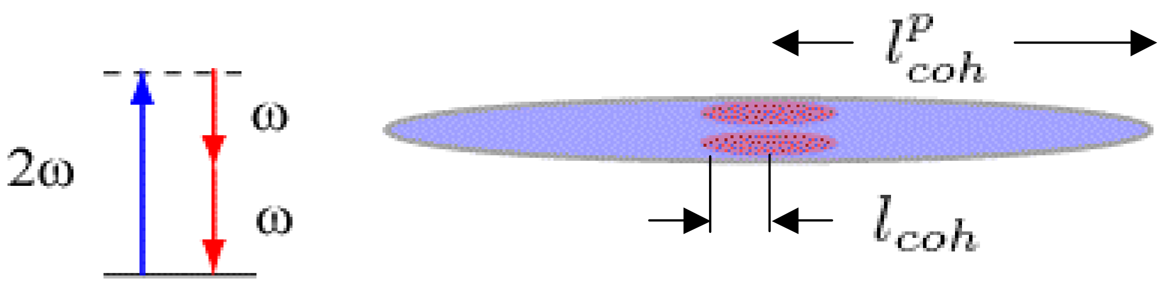
http://www.optics.rochester.edu/~boyd

What are the relevant degrees of freedom of a biphoton?

What are the generic features of two-photon interference?

Biphotons Are Created by Parametric Downconversion (PDC)





Length of two-photon wavepacket ~ coherence length of pump laser ~ 10 cm

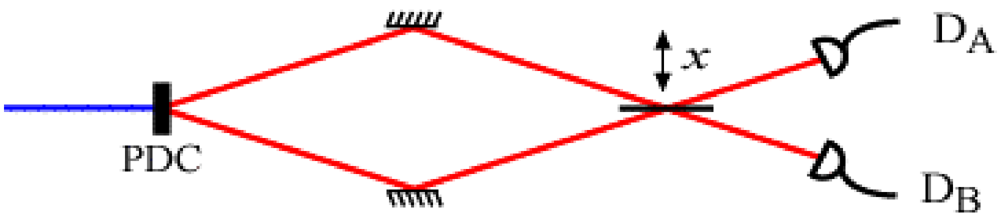
Coherence length of signal/idler photons ~ c/Δω ~ 100 μm.

Individual photons are entangled and can be made indistinguishable.

Two-Photon Interference -- How to Understand?

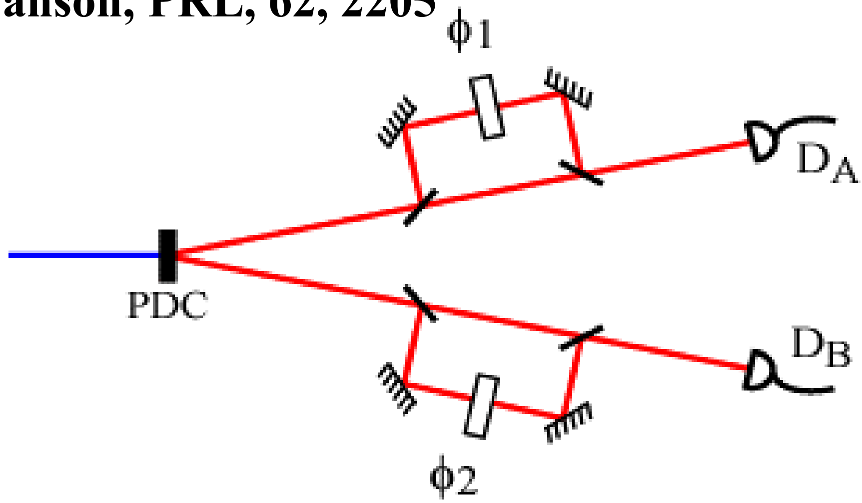
- **Hong-Ou-Mandel effect (1987)**

PRL, 59, 2044



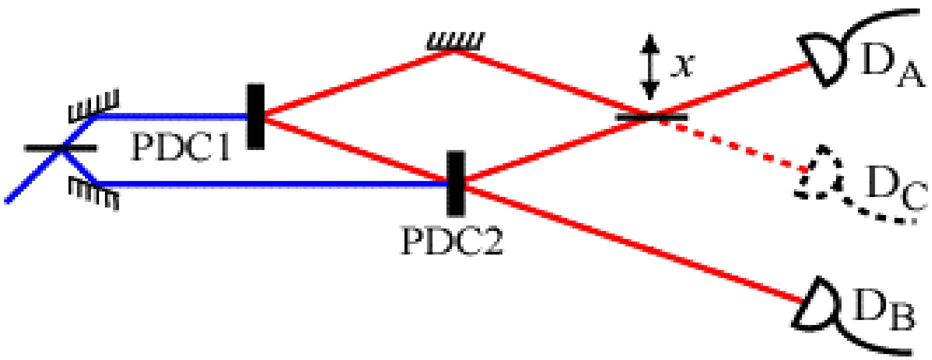
- **Bell Inequality for position and time (1989)**

Franson, PRL, 62, 2205



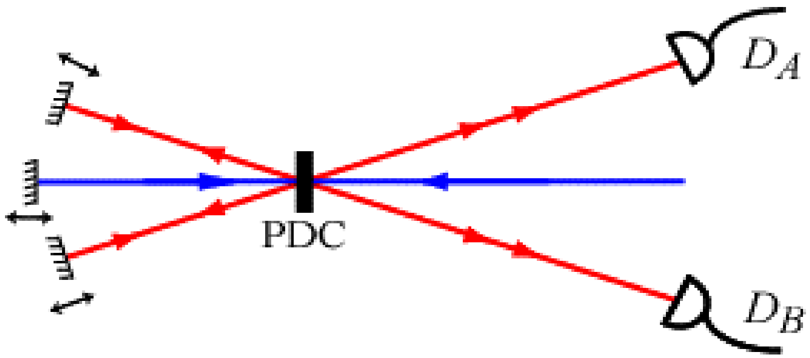
- **Induced Coherence (1991)**

Zou et al. PRL, 67, 318

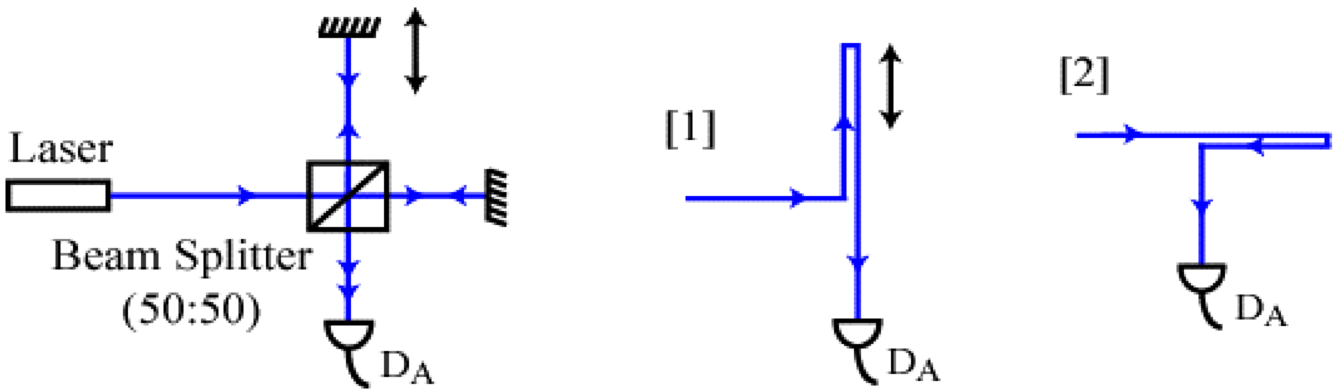


- **Frustrated two-photon creation (1994)**

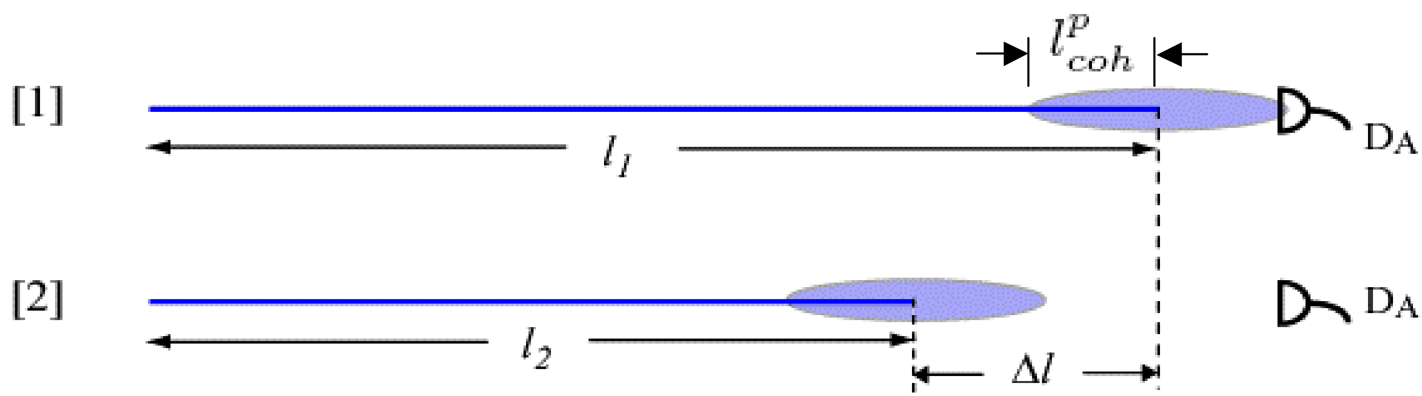
Herzog et al. PRL, 72, 629



Single-Photon Interference: "A photon interferes only with itself" - Dirac



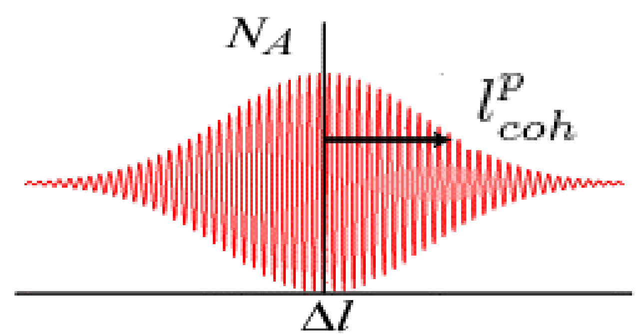
Add probability amplitudes for alternative pathways [1] and [2]



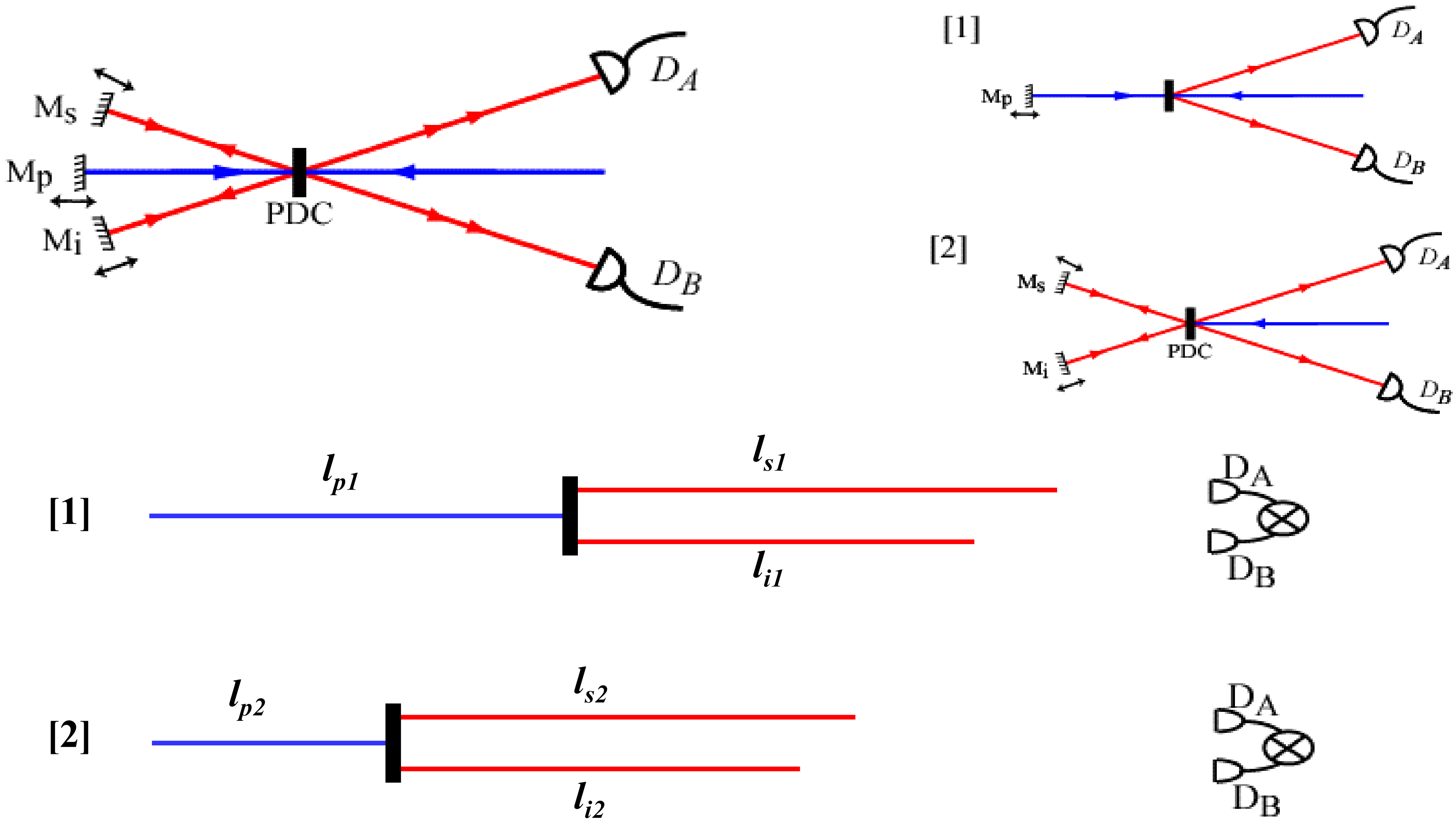
(unfolded paths)

Necessary condition for one-photon interference

$$\Delta l \;<\; l^p_{coh}$$

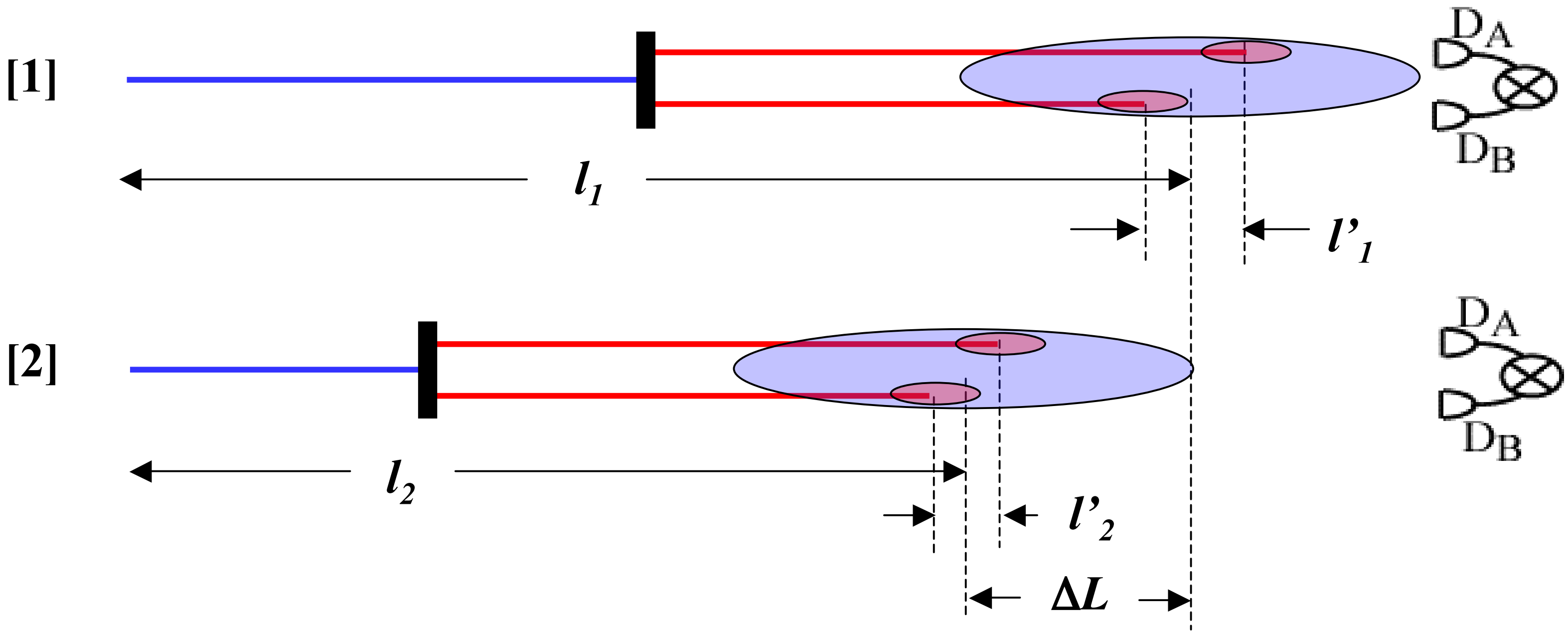


What about biphoton interference? (Generic setup)



Probability amplitudes for pathways [1] and [2] add to produce interference.

Biphotons Can Interfere Only If They Are Indistinguishable



$\Delta L = l_1 - l_2$ ≡ **Biphoton path-length difference**

$\Delta L' = l'_1 - l'_2$ ≡ **Biphoton path-length asymmetry difference**

$$N_{AB} \propto \quad 1 - \gamma' \left(\Delta L'\right) \gamma \left(\Delta L\right) \cos\left(k_0 \Delta L\right)$$

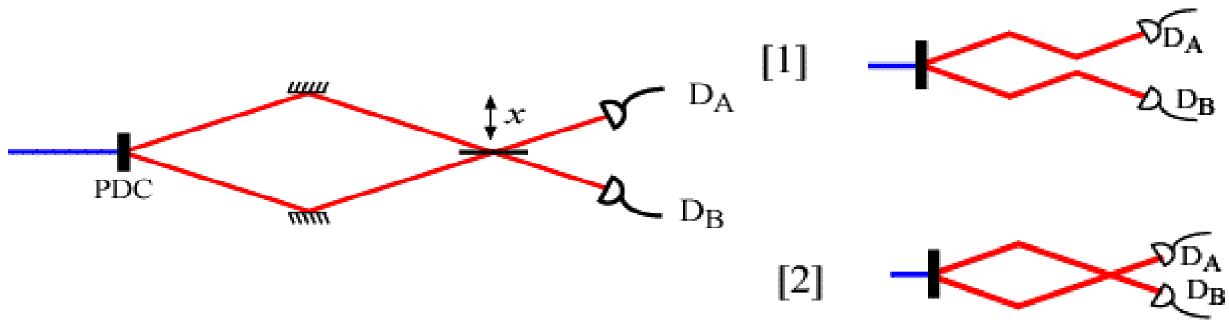
$$\gamma\left(\Delta L\right) = \exp\left[-\frac{1}{2}\left(\frac{\Delta L}{l^p_{coh}}\right)^2\right] \qquad \gamma'\left(\Delta L'\right) = \exp\left[-\frac{1}{2}\left(\frac{\Delta L'}{l_{coh}}\right)^2\right]$$

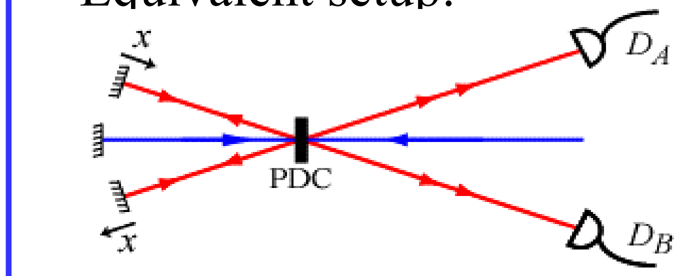
Conditions for two-photon interference:

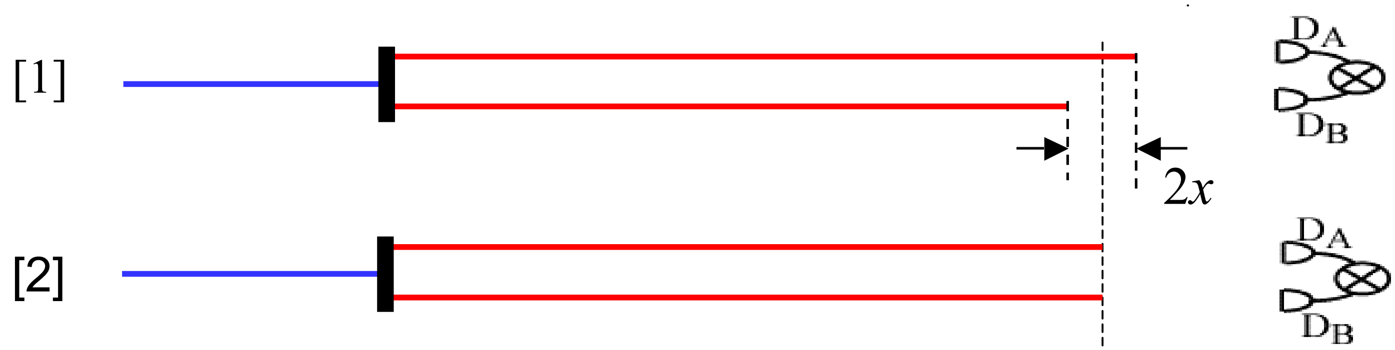
$$\Delta L \; < \; l^p_{coh}$$

$$\Delta L' \; < \; l_{coh}$$

Hong-Ou-Mandel Experiment



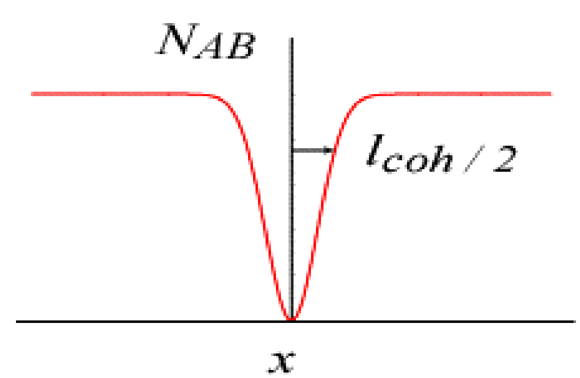




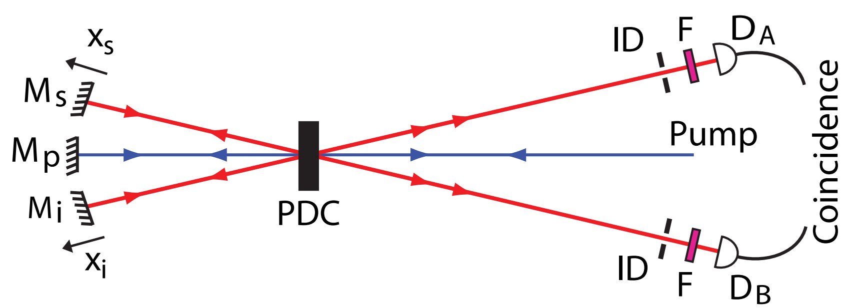
$\Delta L = 0 \quad \Delta L' = 2x$

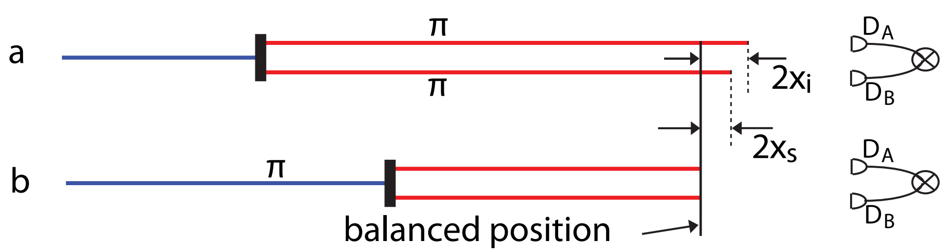
$$N_{AB} \propto \; 1 - \gamma'(\Delta L')\,\gamma(\Delta L)\cos(k_0 \Delta L)$$

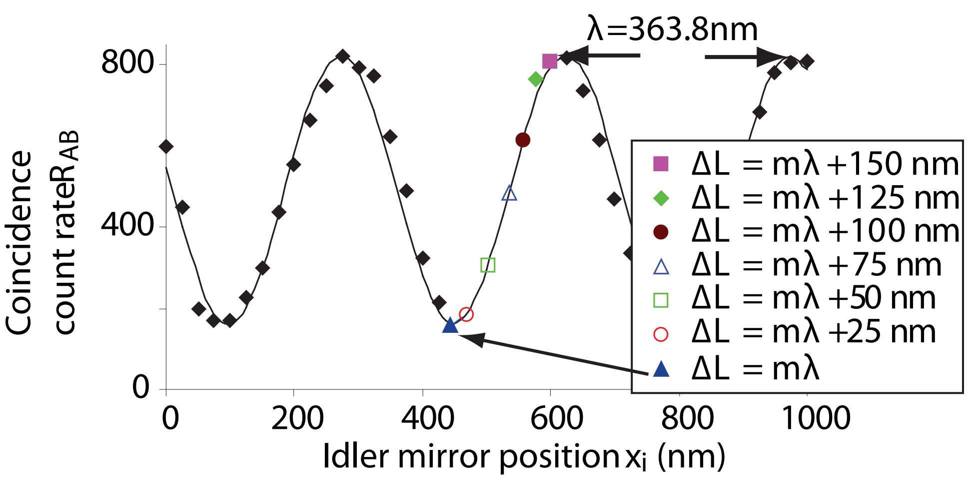
$$N_{AB} \propto \; 1 - \gamma'(2x)$$

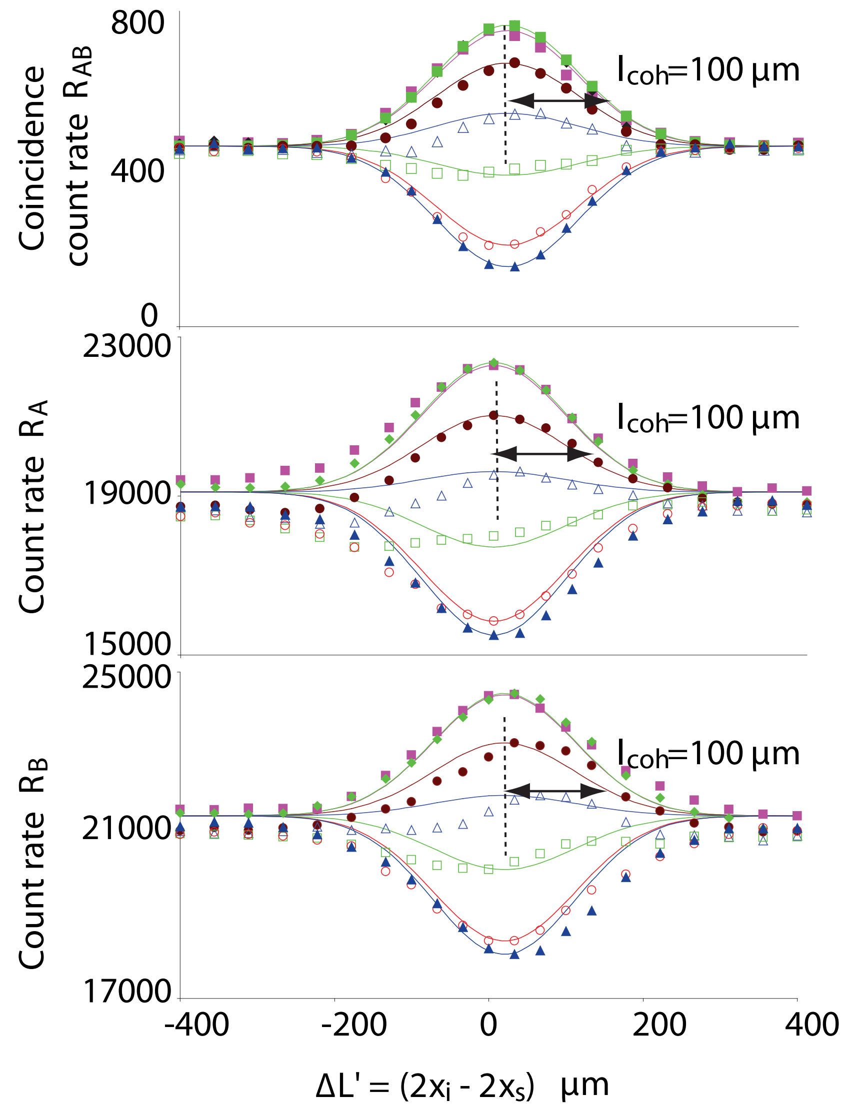


Our Experiment: Generalization of the Hong-Ou-Mandel Effect









We see either a dip or a hump (depending on the value of ΔL) in both the single and coincidence count rates as we scan $\Delta L'$.

Why is interference seen in single-detector count rate?

Path-length difference is much larger than single-photon coherence length; this is not conventional (Young's) interference!

Note that: $$R_{\text{X}} = \sum_i R_{\text{XY}_i}$$

R_{X} = single detector count rate R_{XY_i} = coincidence count rate

But for our setup, the twin of the photon detected at A can end up only at B.

Thus:

$$R_{\text{A}} = R_{\text{AB}}$$