Slow and fast light: fundamentals and applications

Robert W. Boyd [a]

[a] The Institute of Optics and Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, USA

PLEASE SCROLL DOWN FOR ARTICLE

Taylor & Francis
Taylor & Francis Group

TOPICAL REVIEW

Slow and fast light: fundamentals and applications

Robert W. Boyd*

*The Institute of Optics and Department of Physics and Astronomy,
University of Rochester, Rochester, New York 14627, USA*

We review progress in the development of methods for controlling the group velocity of light. These methods allow one to create situations in which the group velocity of light is much smaller than the velocity of light in vacuum *c*, in which the group velocity is greater than *c*, or in which the group velocity is negative. We present a survey of methods for establishing extreme values of the group velocity, concentrating especially on methods that can work in room temperature solids. We also describe some of the applications of slow and fast light that are currently under development.

Keywords: slow light; fast light; group velocity; interferometry; laser radar; causality

1. Introduction

For the past decade or more, the optical physics community has been fascinated by the related phenomena of slow and fast light [1–3]. These names are taken to refer to situations in which the group velocity (roughly, the velocity at which light pulses propagate through a material system) is very much different from the vacuum speed of light *c*. Several of the early stunning demonstrations of slow and fast light made use of atomic media [4–7]. More recently, it has been realized that extreme values of the group velocity can also be realized in room-temperature solid-state materials, which are more suited for many practical applications. In this article, we review some of the physical mechanisms that can be used to induce slow- and fast-light effects in room temperature solids [8–10], and we describe the some of the propagation effects that can thereby be observed [11]. We also present some ideas for applications of slow light within the fields of quantum electronics and photonics [12–14].

2. Slow and fast light fundamentals

The terms *slow* and *fast* light conventionally refer to the group velocity of a light wave. The group velocity is the velocity most closely related to the velocity at which the peak of a light pulse moves through an optical material [3], and is given by the standard result

$$v_g = c/n_g \quad n_g = n + \omega \frac{dn}{d\omega}, \tag{1}$$

where *n* is the refractive (phase) index and ω is the angular frequency of the carrier wave of the light field. One refers to light as being *slow light* for $v_g \ll c$, *fast light* for $v_g > c$ [15], and backwards light for v_g negative. Extreme values of the group velocity (that is, v_g appreciably different from *c*) invariably rely on the dominance of the second contribution to the group index of Equation (1). This contribution of course results from the frequency dependence of the refractive index, and for this reason extreme values of the group velocity are usually associated with the resonant or near-resonant response of material systems. This point is illustrated in Figures 1 and 2. The left-hand column of Figure 1 shows why slow light is expected in the wings of an absorption line and fast light is expected near line center. The right-hand column shows that just the opposite is expected for a gain line. Figure 2 shows the expected behavior for the case of a hole burned into a gain or loss feature. We treat this situation because just this sort of spectral behavior is observed in many nonlinear optical interactions. We also note that spatial dispersion, that is, the non-locality in space of the medium response, is another mechanism that can lead to slow light, as has been predicted [16] and observed [17].

Early investigations of extreme propagation effects include those of Basov et al. [18], Faxvog et al. [19], and Chu and Wong [20]. More recent interest in slow light was motivated strongly by the experiment of Hau et al. [4] in which slow light (velocities as small as 17 m/s) were observed in an ultracold sodium vapor. This report was soon followed by that of
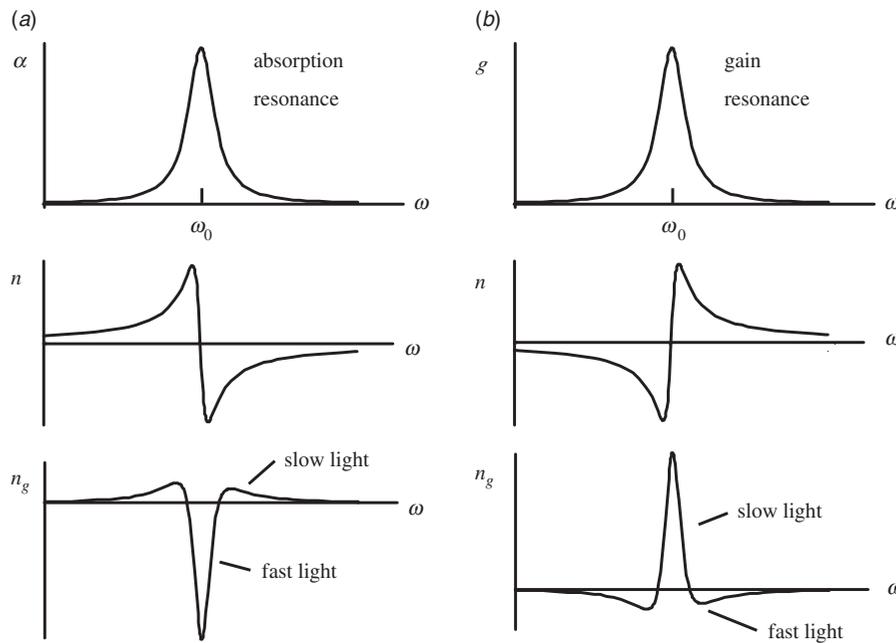
Figure 1. Origin of slow and fast light for an isolated absorption resonance (*a*) and gain resonance (*b*).
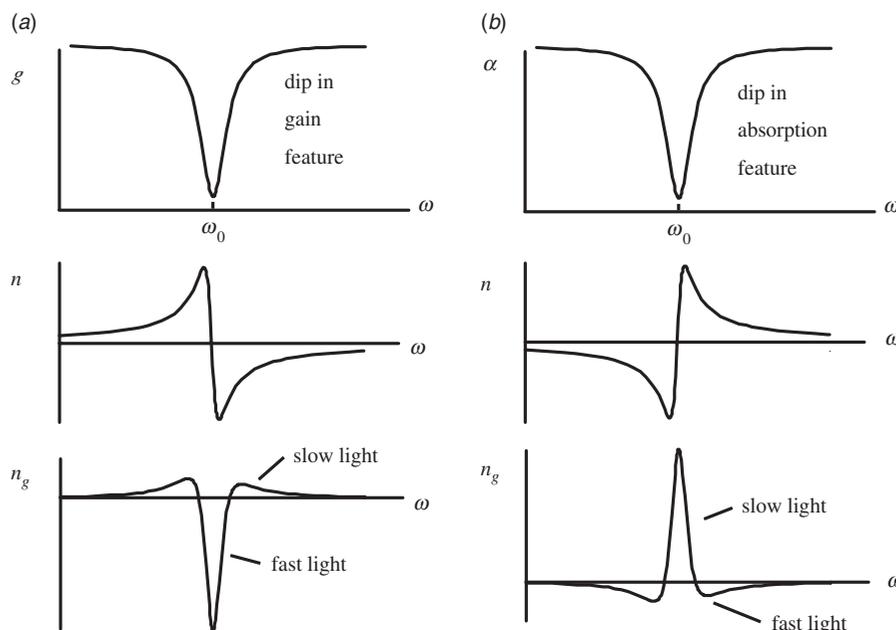
Figure 2. Origin of slow and fast light for a dip in a gain feature (*a*) and an absorption feature (*b*).

Kash et al. [5], who showed that ultra-slow light speeds could also be obtained in a hot atomic vapor of rubidium. This observation dispelled the notion that the use of ultracold atoms was essential to ultraslow light propagation. Since the time of these early experiments, there have been many reports [6,7,21] of slow- and fast-light phenomena under a variety of circumstances.

3. Slow light in a room temperature solid

Many (but by no means all!) potential applications of slow light are much more readily addressed by slow light methods based on the use of room temperature solids, rather than by the use of crystals at cryogenic temperatures [20,22], hot atomic vapors [5], or ultra-cold atomic ensembles [4,7]. For this reason, there has

been enormous emphasis over the past few years in the development of methods for obtaining extreme group velocities based on the use of room temperature solids. Some of the methods that have been developed for producing slow light in room temperature solids include the use of stimulated Raman scattering in optical fibers [23], the use of photonic crystals [24], and exciton resonances [25]. In the view of the present author, three methods involving room-temperature solids have emerged that are particularly well suited for use in applications of slow light. These methods are reviewed in this section.

3.1. Slow light via stimulated Brillouin scattering (SBS)

Stimulated Brillouin scattering (SBS) is a process in which an applied laser field scatters from a retreating sound wave to create a down-shifted Stokes wave [26,27], as illustrated in part (*a*) of Figure 3. The sound wave is itself created by the interaction of the laser beam with the Stokes beam through the process of electrostriction. Thus, the Stokes beam and the sound wave are mutually reinforcing, leading to the generation of a very strong Stokes wave. In fact, ignoring pump depletion, the intensity I_L of the Stokes waves grows exponentially, as described by the equations

$$\frac{dI_S}{dz} = -gI_LI_S \quad g = \frac{\gamma_e^2\omega^2}{nvc^3\rho_0\Gamma_B}, \tag{2}$$

where γ_e is the electrostrictive constant, v is the velocity of sound, ρ_0 is the mean mass density of the material, Γ_B is the Brillouin linewidth and I_L is the intensity of the pump laser. However, there will necessarily (as can be seen from Kramers–Kronig relations, for instance) also be a contribution to the refractive index associated with the SBS gain. Furthermore, just as there is gain for a field detuned to the Stokes sideband of the laser field, there will be induced attenuation for a field at the anti-Stokes sideband. Consequently, there will be a region of slow light at the Stokes resonance and a region of fast light at the anti-Stokes resonance. These features are illustrated in part (*b*) of Figure 3. Simple scaling laws show that the induced time delay ΔT for the slow-light situation is of the order of $g\,I_LL/\Gamma_B$, where L is the length of the interaction region. Slow light based on this process was first observed in optical fibers at a wavelength of 1550 nm by Song et al. [28] and Okawachi et al. [10]. A limitation to the usefulness of this process is that the Brillouin linewidth for typical optical fibers is only 30 to 50 MHz. This linewidth sets the characteristic frequency bandwidth over which slow-light effects can be observed, and a bandwidth of 30 to 50 MHz is much too small for
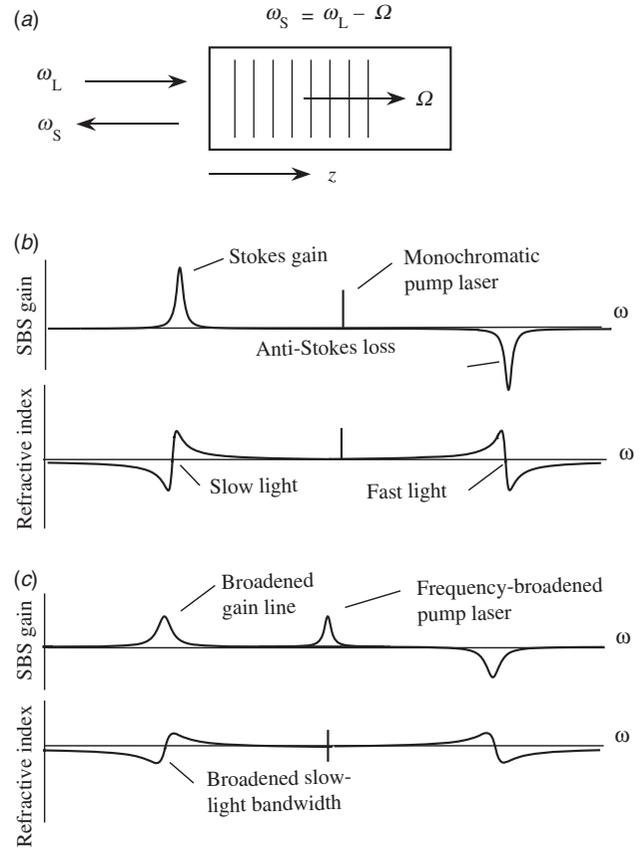

Figure 3. (*a*) Schematic representation of the SBS process. (*b*) Origin of slow and fast light through SBS and a monochromatic pump field. (*c*) Origin of slow and fast light through SBS and a broadened pump field.

many applications in optical telecommunications. Several procedures have been introduced to broaden this linewidth. One method is to use multiple pump frequencies to produce multiple overlapping gain lines. This idea has been implemented for the case of double [29] and triple [13] gain lines to simultaneously increase the bandwidth and fractional delay of SBS slow light by factors of the order of two. Another method is to broaden the linewidth of the laser that pumps the SBS process by adding noise to the current that drives the laser. This procedure was first implemented by Herráez et al. [30] who broadened the intrinsic 35 MHz linewidth of single-mode silica fiber to approximately 325 MHz. This methods was later extended to achieve a 12 GHz bandwidth [31] and to delay 100-ps-long pulses by up to 3 pulse widths [32].

3.2. Slow light via coherent population oscillations (CPO)

The second preferred method for producing slow light in room-temperature solids is based on the process of
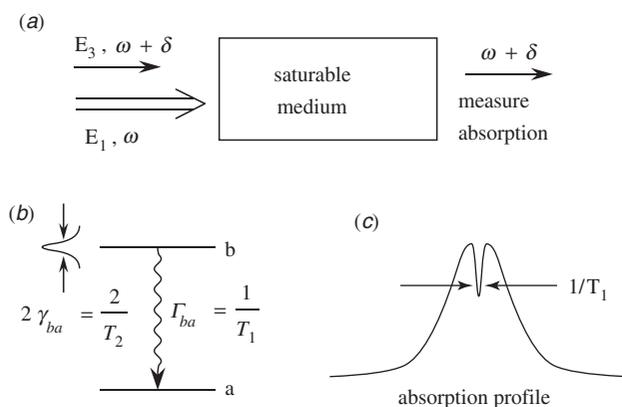
Figure 4. (*a*) Configuration used to study CPO. (*b*) Energy level diagram of a typical saturable absorber. (*c*) Absorption profile in the presence of CPO.

coherent population oscillations (CPO). This process has been studied since the 1960s [33–35] and has successfully been exploited for slow- and fast-light research [8,9,11,36–42]. The idea behind CPO is illustrated in Figure 4. If a pump beam at frequency ω and a detuned probe beam at frequency $\omega+\delta$ co-propagate through a saturable absorber, the ground state population will be induced to oscillate in time at frequency δ. Detailed calculation then shows that the absorption experienced by the probe beam will be decreased as a consequence of these population oscillations over a frequency range of the order of $1/T_1$, where T_1 is the population relaxation time of the saturable absorber. If T_1 is much greater than T_2, where T_2 is the conventional dipole dephasing time, a well-defined hole will be induced in the absorption profile of the saturable absorber, even if the absorption profile is homogeneously broadened in the conventional sense. This spectral hole then leads through the usual Kramers–Kronig relations to a rapid spectral variation of the refractive index and thus to a strong slow-light effect. Detailed calculation shows that the resulting group index is given by [8,43]

$$n_g = \frac{1}{2}\alpha_0 T_1 c \frac{I}{(1+I)^3} \qquad (3)$$

where I is the intensity of the pump wave normalized by the saturation intensity of the material medium.

In the first reported experimental study of slow light based on CPO [8], saturation of the strong green absorption band of ruby was used to produce the slow light effect. Group velocities as low as $60\,\text{m/s}$ corresponding to a group index of 5×10^6 were observed. A follow-up experiment made use of the nonlinear optical properties of the crystal alexandrite [9]. Alexandrite is a saturable absorber at some frequencies but an inverse saturable absorber at others. At frequencies at which alexandrite is an inverse saturable absorber, it displays fast and backwards light as a result of the CPO effect. In one situation, a velocity of $-800\,\text{m/s}$ was measured.

The occurrence of negative group velocities leads to intriguing conceptual questions. To address these matters, still another experiment was performed, this one using an erbium-doped fiber amplifier (EDFA). When driven into saturation, an EDFA is a saturable amplifier, and thus shows fast light by means of the CPO effect [36] for the reason shown in Figure 2(*c*). The key feature of this experiment was to study the time evolution of an optical pulse as a function of position within the optical fiber. It was found [11] that the peak of the pulse actually did propagate in the backwards direction within the fiber, in agreement with the standard meaning of the group velocity. These results are summarized in Figure 5.

The CPO effect can also be understood in terms of a time-domain description [44], as shown in Figure 6. Here we consider a saturable absorber with non-instantaneous response. We see that the leading edge of the pulse will thus experience more absorption than the trailing edge of the pulse, and that consequently the peak of the pulse will be shifted to later times. Despite the opinions of others [45], the present author believes that the time domain and frequency domain treatments of CPO are equivalent, with the frequency domain treatment being more convenient for most circumstances.

3.3. Tunable time delays based on group velocity dispersion

The simplest method for controlling the velocity of light is to make use of transmission through a medium with a large dispersion in the group velocity [46,47]. By varying the frequency of the carrier wave of the signal, the time delay can thereby be controlled directly. In practice, this method is often implemented by starting with a signal at a prescribed carrier frequency, shifting the carrier frequency using a nonlinear optical conversion method, passing this frequency-shifted signal through a highly dispersive medium, and finally converting the carrier frequency back to the original frequency. This method is often called the conversion/dispersion (C/D) method for this reason. Although the C/D method is not really a true slow-light method, it is often included in discussions of slow light because it is related to slow light by its reliance on highly dispersive materials and because it is useful under many of the same conditions for which slow light is useful.
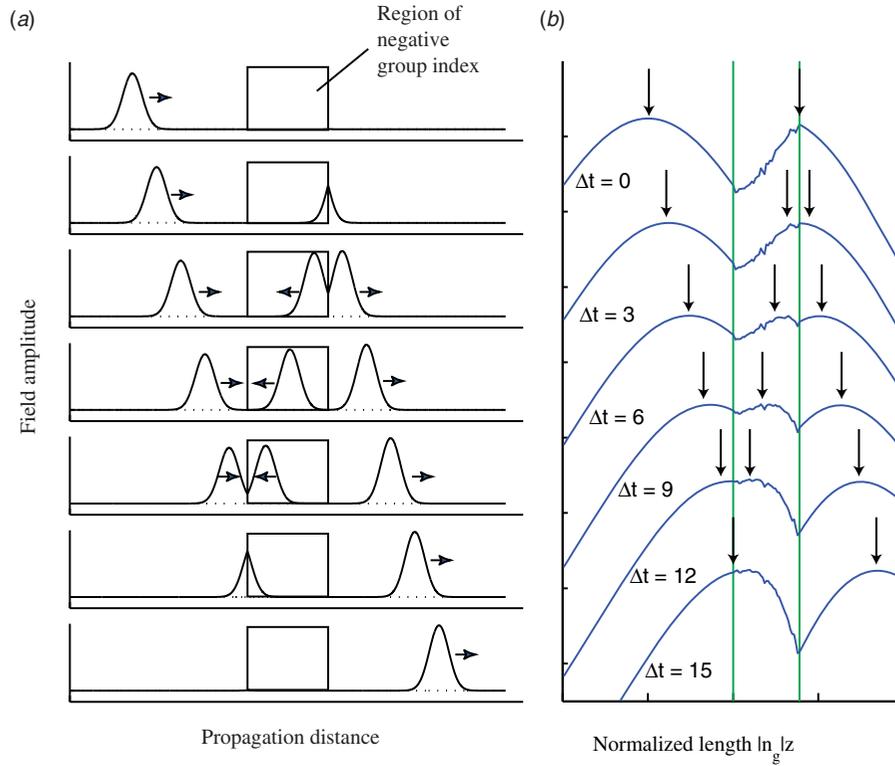
Figure 5. (*a*) Conceptual prediction and (*b*) experimental results (adapted from [9]) showing the backwards propagation of the peak of a light pulse. (The color version of this figure is included in the online version of the journal.)
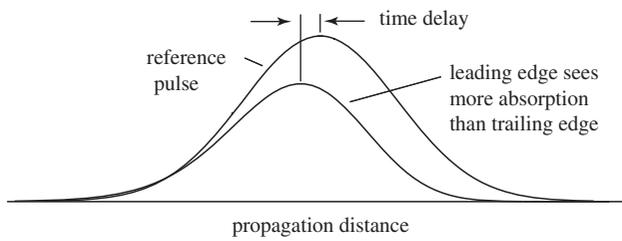

Figure 6. Time-domain explanation of slow light by means of CPO.

Figure 7. Use of slow light as a buffer to increase the throughput of an optical switch.

4. Applications of slow and fast light

In this section we review three current applications of slow and fast light.

4.1. Applications of slow and fast light in telecommunication

Slow light has direct usefulness in the field of optical telecommunication for applications such as buffering and regeneration. Figure 7 shows how a slow-light buffer might be used to increase the throughput of a telecommunication system. Part (*a*) of the figure shows two data packets arriving simultaneously at an *N*-port

by *N*-port optical switch. If these two data packets are intended for the same output port, a problem occurs because the switch cannot handle the two data packets simultaneously. In the worse case, one of the data packets is simply dropped and must be retransmitted at a later time. This procedure of course simply makes the problem of system overload worse, as certain data

packets have to be transmitted more than once. A more desirable resolution is to construct a controllable delay line that can act as a buffer for a complete data packet, as shown in part (*b*) of the figure. In this scenario, one data packet is directed into the buffer and is released only after the other data packet has cleared the switch.

To implement this idea, the delay line needs to be able to store as many bits of information (data pulses) as are contained in a data packet. This number is determined by the system architecture. In most implementations, at least 1024 bits of data would be contained in each data packet. In fundamental terms, the number of stored pulses is known as the delay-bandwidth product of the delay line. There have been a number of analyses of the theoretical limit to how large the delay-bandwidth-product of a slow-light delay line can be [48–55]. The general conclusion of these analyses seems to be that there is no fundamental limit to how large the delay-bandwidth product can be, although there can be serious practical problems involved in obtaining a large value of the delay-bandwidth product. In fact, to date, the largest value of the delay-bandwidth product obtained using true slow light is the value of 80 pulse widths reported by Camacho et al. [56]. However, the C/D method has been able to achieve a storage capacity of 440 bit slots, or approximately 880 pulse widths, which is large enough to prove useful in actual communication systems.

Another application of slow and fast light is in the area of data pulse regeneration. An example of such a procedure is shown in Figure 8. In an optical communication system, each pulse needs to be centered in its time widow. However, due to environmental effects and optical sources of noise, individual pulses might become displaced from the centers of their time windows. The use of slow- and fast-light methods could provide a useful means for the real-time centering of pulses in their time windows. For both buffering and regeneration, it is crucial that the pulse shape does not become distorted as a result of the slow- or fast-light effect. Various methods have been demonstrated for minimizing pulse-shape distortion [29,57].

4.2. Applications of slow and fast light in interferometry

The performance of many types of interferometers can be dramatically improved by placing a highly dispersive material within the interferometer. Here we review some of the resulting modifications in device performance.
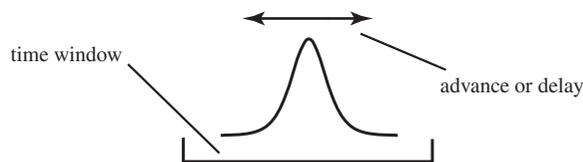

Figure 8. Use of slow and fast light to center an optical pulse in its time window.
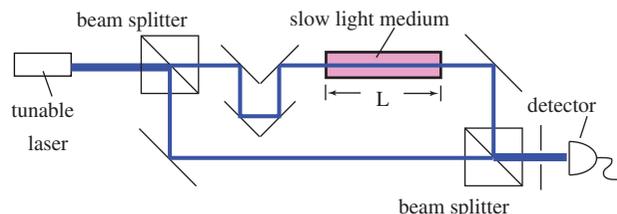

Figure 9. Use of slow-light to modify the properties of an interferometer. (The color version of this figure is included in the online version of the journal.)

Let us first consider the situation in which a highly dispersive material is placed in one arm of a Mach-Zehnder interferometer, as shown in Figure 9. We consider what happens as one measures the output of the interferometer as the input wavelength is scanned. We note that the output of the interferometer depends on the phase difference between the two arms of the interferometer. If, for simplicity, we assume that the pathlengths of the two arms differ only by means of the pathlength through the slow-light medium, we find that the phase difference is given by $\Delta\phi = n\omega L/c$. Thus, the rate at which the phase difference changes as the input frequency is changed is given by

$$\frac{d\Delta\phi}{d\omega} = \frac{d}{d\omega}\frac{n\omega L}{c} = \frac{L}{c}\left(n + \omega\frac{dn}{d\omega}\right) = \frac{Ln_g}{c}. \qquad (4)$$

This quantity is a measure of the spectral sensitivity of the spectrometer, and we see that the spectral sensitivity scales with the group index of the material within the interferometer.

A slow light spectrometer of this sort was constructed by Shi et al. [13], and it achieved an increase in spectral sensitivity by a factor of approximately 2. More recently, Shi et al. [12] have demonstrated an improved design in which the sensitivity was enhanced by a factor of 100. More advanced modeling of slow-light interferometers [58] and the demonstration of a slow-light interferometer based on the properties of photonic crystals [59] have recently been reported.

An entirely different concept has been introduced by Shahriar and co-workers [60,61]. They consider the situation in which a fast-light medium is placed within the interferometer. In this case, the spectral sensitivity of the interferometer is actually decreased, but the
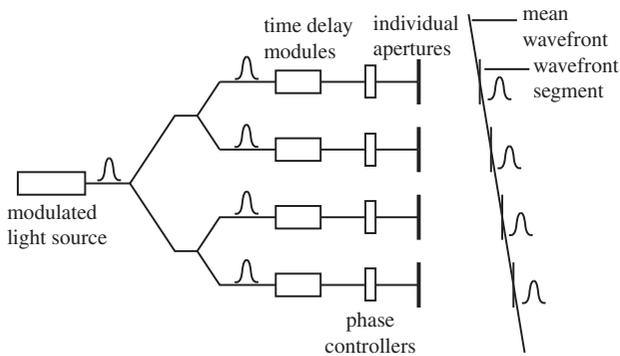
Figure 10. Use of slow light in a phased- and synchronized-array laser-radar system. Beam steering is accomplished by adjusting the phase difference between successive apertures, and slow-light methods are used to ensure that the pulses leaving each aperture arrive simultaneously in the far field.

sensitivity of the interferometer to changes in the positions of the end mirrors is dramatically enhanced. Interferometers of this sort hold promise for applications such as the detection of gravitational waves and for rotation sensors based on laser gyros.

4.3. *Applications of slow light for laser radar*

Another application of slow light has recently emerged in the context of laser radar. For many applications of laser radar, it would be desirable to steer the launched laser beam electro-optically rather than mechanically. Under many conditions, it would also be desirable to use a number of small sub-apertures rather than a single (and necessarily large and expensive) aperture. A standard procedure is to use a phased array of emitters [62] to steer the output beam. However, if one needs to launch short pulses, it is necessary also to ensure that the pulses leaving each sub-aperture are synchronized so that they arrive simultaneously at a target in the far field. To achieve these additional conditions, slow-light methods are expected to prove extremely useful. The concept of such a design is shown in Figure 10. Here we see that by controlling the phase shift imposed on each channel it is possible to steer the output beam. However, to ensure synchronization of the output pulses, it is also necessary to control the group delay of each channel. The author and his students are currently constructing such a system in their laboratory.

5. Summary

Slow light methods have advanced dramatically in recent years. Many of the fundamental aspect of slow and fast light are currently well understood. Thus, slow

light research has turned to the equally exciting task of developing applications of this new technology.

References

[1] Boyd, R.W.; Gauthier, D.J. *Progress in Optics*; Elsevier: Amsterdam, 2002; p 497.

[2] Khurgin, J.B.; Tucker, R.S., Eds. *Slow Light: Science and Applications*; CRC Press: Boca Raton, 2008.

[3] In fact, there are several other definitions of the velocity of light that, refer to various other properties of a light field. These definitions are reviewed for instance by Milonni, P.W. *Fast Light*, *Slow Light*, *and Left-Handed Light*; IOP Publishing, 2005, p 58.

[4] Hau, L.V.; Harris, S.E.; Dutton, Z.; Behroozi, C.H. *Nature* **1999,** *397,* 594.

[5] Kash, M.M; Sautenkov, V.A.; Zibrov, A.S.; Hollberg, L.; Welch, G.R.; Lukin, M.D.; Rostovtsev, Y.; Fry, E.S.; Scully, M.O. *Phys. Rev. Lett.* **1999,** *82,* 5229.

[6] Budker, D.; Kimball, D.F.; Rochester, S.M.; Yashchuk, V.V. *Phys. Rev. Lett.* **1999,** *83,* 1767.

[7] Inouye, S.; Löw, R.F.; Gupta, S.; Pfau, T.; Görlitz, A.; Gustavson, T.L.; Pritchard, D.E.; Ketterle, W. *Phys. Rev. Lett.* **2000,** *85,* 4225–4228.

[8] Bigelow, M.S.; Lepeshkin, N.N.; Boyd, R.W. *Phys. Rev. Lett.* **2003,** *90,* 113903.

[9] Bigelow, M.S.; Lepeshkin, N.N.; Boyd, R.W. *Science* **2003,** *301,* 200.

[10] Okawachi, Y.; Bigelow, M.S.; Sharping, J.E.; Zhu, Z.; Schweinsberg, A.; Gauthier, D.J.; Boyd, R.W.; Gaeta, A.L. *Phys. Rev. Lett.* **2005,** *94,* 153902.

[11] Gehring, G.M.; Schweinsberg, A.; Barsi, C.; Kostinski, N.; Boyd, R.W. *Science* **2006,** *312,* 895–897.

[12] Shi, Z.; Boyd, R.W.; Camacho, R.M.; Vudyasetu, P.K.; Howell, J.C. *Phys. Rev. Lett.* **2007,** *99,* 240801.

[13] Shi, Z.; Boyd, R.W.; Gauthier, D.J.; Dudley, C.C. *Optics Lett.* **2007,** *32,* 915–917.

[14] Zhu, Z.; Gauthier, D.J.; Boyd, R.W. *Science* **2007,** *318,* 7238.

[15] Wang, L.J.; Kuzmich, A.; Dogariu, A. *Nature* **2000,** *406,* 277.

[16] Kocharovskaya, O.; Rostovtsev, Y.; Scully, M.O. *Phys. Rev. Lett.* **2001,** *86,* 628.

[17] Strekalov, D.; Matsko, A.B.; Yu, N.; Maleki, L.J. *J. Mod. Opt.* **2004,** *51,* 16.

[18] Basov, N.G.; Ambartsumyan, R.V.; Zuev, V.S.; Kryukov, P.G.; Letokhov, V.S. *Sov. Phys. Dokl.* **1966,** *10,* 1039.

[19] Faxvog, F.R.; Chow, C.N.Y.; Bieber, T.; Carruthers, J.A. *Appl. Phys. Lett.* **1970,** *17,* 192.

[20] Chu, S.; Wong, S. *Phys. Rev. Lett.* **1982,** *48,* 738.

[21] Mikhailov, E.E.; Sautenkov, V.A.; Novikova, I.; Welch, G.R. *Phys. Rev. A* **2004,** *69,* 063808.

[22] Turukhin, A.V.; Sudarshanam, V.S.; Shahriar, M.S.; Musser, J.A.; Ham, B.S.; Hemmer, P.R. *Phys. Rev. Lett.* **2001,** *88,* 023602.

[23] Sharping, J.E.; Okawachi, Y.; Gaeta, A.L. *Opt. Express* **2005,** *13,* 6092.

[24] Vlasov, Y.A.; O'Boyle, M.; Hamann, H.F.; McNab, S.J. *Nature* **2005,** *438,* 65.

[25] Chang, S.-W.; Chuang, S.-L.; Ku, P.-C.; Chang-Hasnian, C.J.; Palinginis, P.; Wang, H. *Phys. Rev. B* **2004,** *70,* 235333.

[26] Chiao, R.Y.; Townes, C.H.; Stoicheff, B.P. *Phys. Rev. Lett.* **1964,** *12,* 592–595.

[27] Boyd, R.W. *Nonlinear Optics*; Academic Press: Amsterdam, 2008; Chapter 9.

[28] Song, K.W.; Herraez, M.G.; Thévenaz, L. *Opt. Express* **2005,** *13,* 9758.

[29] Stenner, M.D.; Neifeld, M.A.; Zhu, Z.; Dawes, A.M.C.; Gauthier, D.J. *Opt. Express* **2005,** *13,* 9995.

[30] Herráez, M.G.; Song, K.Y.; Thévenaz, L. *Opt. Express* **2006,** *14,* 1395–1400.

[31] Zhu, Z.; Dawes, A.M.C.; Gauthier, D.J.; Zhang, L.; Willner, A.E. *J. Lightwave Technol.* **2007,** *25,* 201–206.

[32] Cabrera-Granado, E.; Calderón, O.G.; Melle, S.; Gauthier, D.J. *Opt. Express* **2008,** *16,* 16032–16042.

[33] Schwarz, S.E.; Tan, T.Y. *Appl. Phys. Lett.* **1967,** *10,* 4–7.

[34] Hillman, L.W.; Boyd, R.W.; Krasinski, J.; Stroud, Jr., C.R. *Opt. Commun.* **1983,** *45,* 416–419.

[35] Malcuit, M.S.; Boyd, R.W.; Hillman, L.W.; Krasinski, J.; Stroud, Jr., C.R. *J. Opt. Soc. Am. B* **1984,** *1,* 73–75.

[36] Schweinsberg, A.; Schweinsberg, Lepeshkin, N.N.; Bigelow, M.S.; Boyd, R.W.; Jarabo, S. *Europhysics Lett.* **2006,** *73,* 218–224.

[37] Ku, P.-C.; Sedgwick, F.; Chang-Hasnain, C.J.; Palinginis, P.; Li, T.; Wang, H.; Chang, S.-W.; Chuang, S.-L. *Opt. Lett.* **2004,** *29,* 2291–2293.

[38] Palinginis, P.; Sedgwick, F.; Crankshaw, S.; Moewe, M.; Chang-Hasnain, C. *Opt. Express* **2005,** *13,* 9909–9915.

[39] Mørk, J.; Kjær, R.; van der Poel, M.; Yvind, K. *Opt. Express* **2005,** *13,* 8136–8145.

[40] Su, H.; Chuang, S.L. *Appl. Phys. Lett.* **2006,** *88,* 061102.

[41] Su, H.; Chuang, S.L. *Opt. Lett.* **2006,** *31,* 271–273.

[42] Su, H.; Kondratko, P.; Chuang, S.L. *Opt. Express* **2006,** *14,* 4800–4807.

[43] Piredda, G.; Boyd, R.W. *J. European Opt. Soc.* **2007,** *2,* 07004.

[44] Basov, N.G.; Ambartsumyan, R.V.; Zuev, V.S.; Kryukov, P.G.; Letokhov, V.S. *Sov. Phys. JETP* **1966,** *23,* 16–22.

[45] Zapasskii, V.S.; Kozlov, G.G. *Opt. Spectrosc.* **2006,** *100,* 419–424.

[46] Sharping, J.; Okawachi, Y.; van Howe, J.; Xu, C.; Wang, Y.; Willner, A.; Gaeta, A. *Opt. Express* **2005,** *13,* 7872–7877.

[47] Wang, Y.; Yu, C.; Yan, L.; Willner, A.E.; Roussev, R.; Langrock, C.; Fejer, M.M.; Sharping, J.E.; Gaeta, A.L. *Photonics Technology Lett. IEEE* **2007,** *19,* 861–863.

[48] Boyd, R.W.; Gauthier, D.J.; Gaeta, A.L.; Willner, A.E. *Phys. Rev. A* **2005,** *71,* 023801.

[49] Tucker, R.S.; Ku, P.-C.; Chang-Hasnain, C.J. *Electron. Lett.* **2005,** *41,* 208.

[50] Macke, B.; Ségard, B. *Phys. Rev. A* **2008,** *78,* 013817.

[51] Miller, D.A.B. *Phys. Rev. Lett.* **2007,** *99,* 203903.

[52] Khurgin, J.B. *Optics Lett.* **2006,** *31,* 948.

[53] Tucker, R.S.; Ku, P.-C.; Chang-Hasnain, C.J. *J. Lightwave Technol.* **2005,** *23,* 4046.

[54] Matsko, A.; Strekalov, D.; Maleki, L. *Opt. Express* **2005,** *13,* 2210–2223.

[55] Boyd, R.W.; Narum, P. *J. Mod. Optic.* **2007,** *54,* 2403–2411.

[56] Camacho, R.M.; Pack, M.V.; Howell, J.C.; Schweinsberg, A.; Boyd, R.W. *Phys. Rev. Lett.* **2007,** *98,* 153601.

[57] Shin, H.; Schweinsberg, A.; Gehring, G.; Schwertz, K.; Chang, H.J.; Boyd, R.W.; Park, Q-H.; Gauthier, D.J. *Opt. Lett.* **2007,** *32,* 906–908.

[58] Shi, Z.; Boyd, R.W. *J. Opt. Soc. Am. B* **2008,** *25,* C136–C143.

[59] Chamanzar, M.; Momeni, B.; Adibi, A. *Opt. Lett.* **2009,** *34,* 220–222.

[60] Pati, G.S.; Salit, M.; Salit, K.; Shahriar, M.S. *Phys. Rev. Lett.* **2007,** *99,* 133601.

[61] Shahriar, M.S.; Pati, G.S.; Tripathi, R.; Gopal, V.; Messall, M.; Salit, K. *Phys. Rev. A* **2007,** *75,* 053807.

[62] Frigyes, I.; Seeds, A.J. *IEEE Trans. Micro. Theory Tech.* **1995,** *43,* 2378–2386.