Exploring Energy-Time Entanglement Using Geometric Phase

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Background: Geometric Phase

- Geometric Phase (Berry's Phase)

M.V. Berry, Proc. R. Soc. A **392**, 45 (1984)
J. Anandan, Nature **360**, 307 (1992)

acquired by a system in an eigenstate when transported around a closed circuit by varying some parameters in its Hamiltonian

- Pancharatnam's Phase

S. Pancharatnam, Proc. Indian Acad. Sci. A **44**, 247 (1956)
R. Bhandari, Phys. Rep. **280**, 1 (1997)

acquired by a light field when its state of polarization is taken through a closed circuit on the Poincare sphere

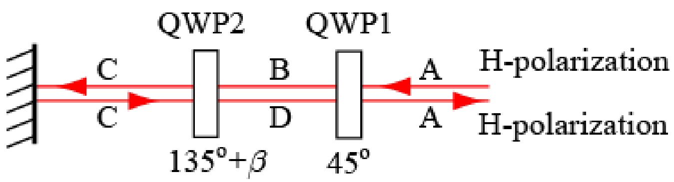
- Connection

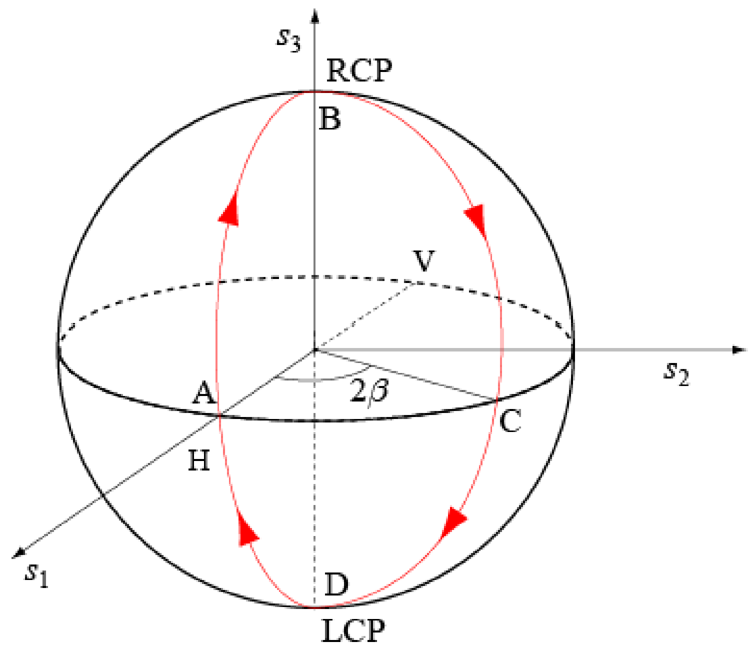
M.V. Berry, J. Mod. Opt. **34**, 1401 (1987)
Ramaseshan and Nityananda, Curr. Sci. **55**, 1225 (1986)

Pancharatnam's phase is the geometric phase in polarization optics.

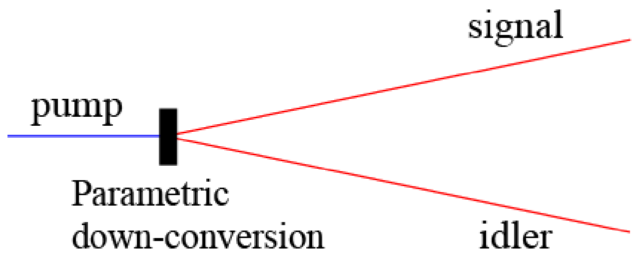
- Example:

Tomita and Chiao, PRL **57**, 937 (1986)
Bhandari and Samuel, PRL **60**, 1211 (1988)
Chyba, Wang, Mandel, and Simon, Opt. Lett. **13**, 562 (1988)





Background: Energy-Time Entanglement



$$\omega_p = \omega_s + \omega_i$$

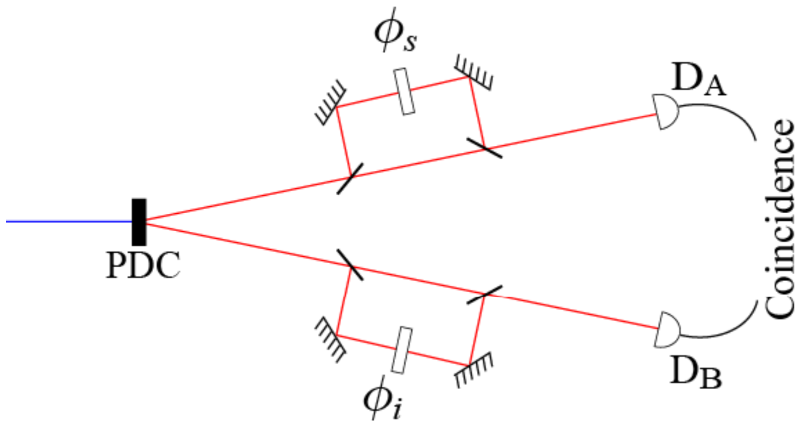
Energy conservation

$$|\psi\rangle = \int d\omega \phi(\omega) |\omega\rangle_s |\omega_p - \omega\rangle_i$$

$$|\psi\rangle = \int dt f(t) |t\rangle_s |t\rangle_i$$

Energy-Time Entanglement

- Bell inequality for Energy and Time



Franson, PRL **62**, 2205 (1989)

$$|\psi\rangle = \frac{1}{\sqrt{2}} [\ |l\rangle_s |l\rangle_i + |s\rangle_s |s\rangle_i\ ]$$

State

$$R_{\mathrm{AB}} = C[1 + \cos(\phi_s + \phi_i)]$$

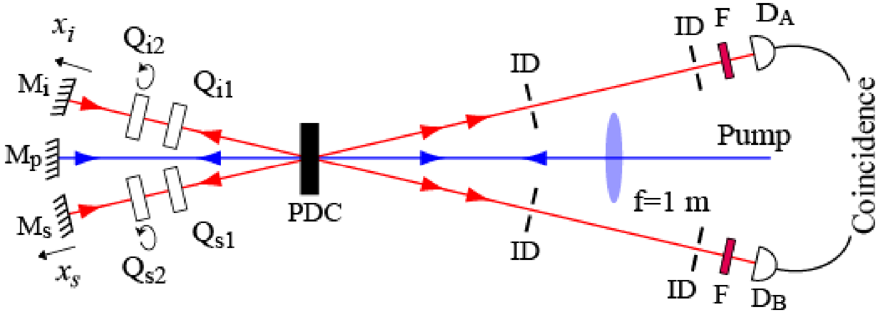
Coincidence count rate

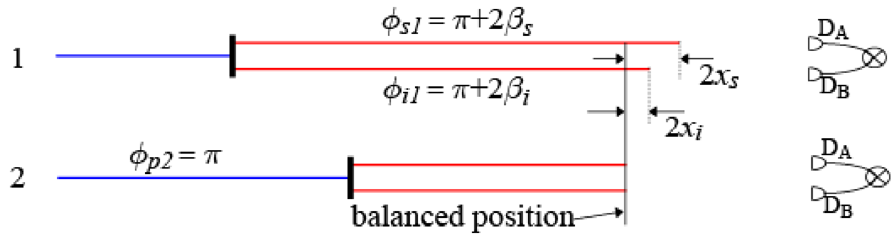
Violation of CHSH Bell inequality (as entanglement witness)

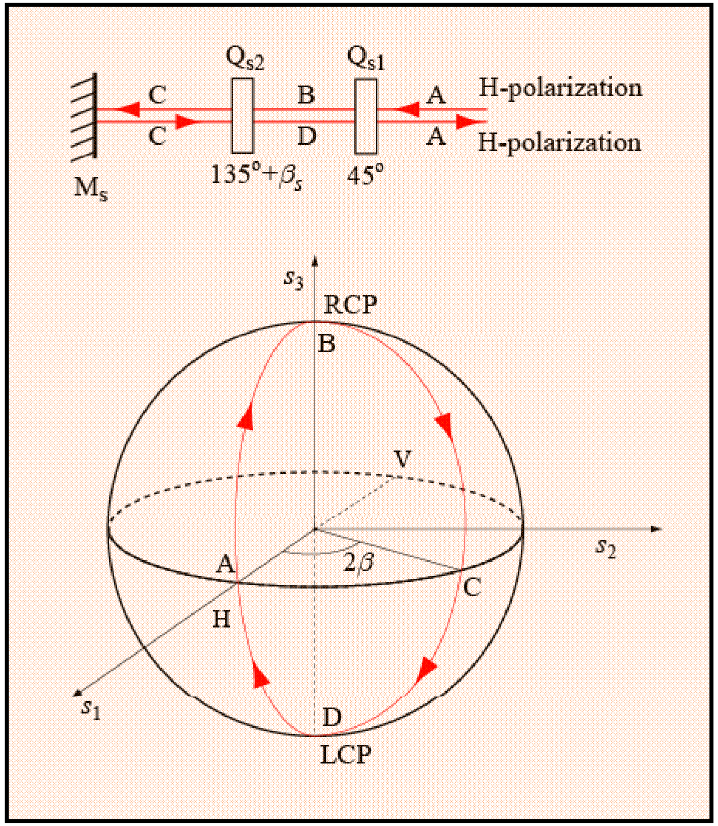
Brendel et al., PRL **66**, 1142 (1991)
Kwiat et al., PRA **47**, R2472 (1993)
Strekalov et al., PRA **54**, R1 (1996)
Barreiro et al., PRL **95**, 260501 (2005)

Dynamic phase based violation

Geometric phase based violation







State:

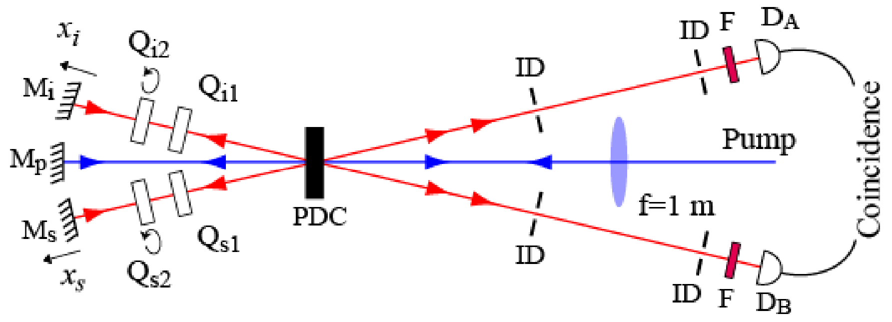
$$|\psi\rangle = \frac{1}{\sqrt{2}}[\ |l\rangle_s |l\rangle_i + |s\rangle_s |s\rangle_i\ ]$$

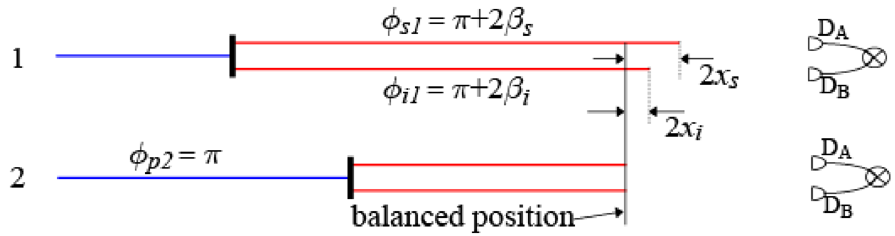
Coincidence count rate:

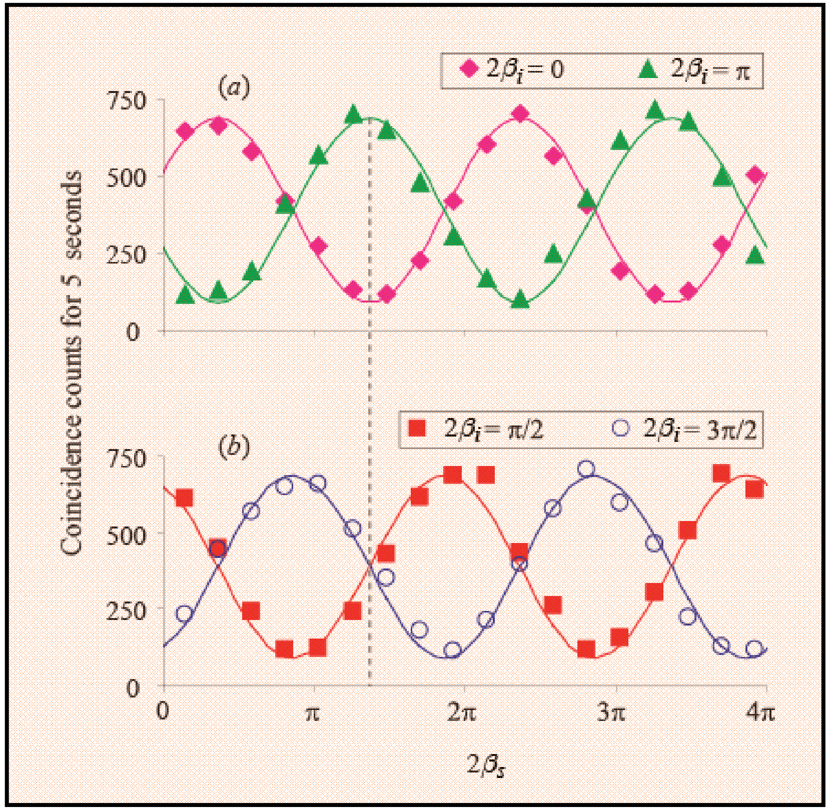
$$R_{\mathrm{AB}} = C\{1 - \cos[\underbrace{k_0(x_s + x_i)}_{\text{dynamic phase}} + \underbrace{2\beta_s + 2\beta_i}_{\text{geometric phase}}]\}$$

Jha, O'Sullivan, Chan, and Boyd, PRA **77**, 021801(R) (2008)

Geometric phase based violation







State:

$$|\psi\rangle = \frac{1}{\sqrt{2}}[\ |l\rangle_s|l\rangle_i + |s\rangle_s|s\rangle_i\ ]$$

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$$V = 77\%\ (> 70.7\%)$$

$$S = 2\sqrt{2}V$$

$$= 2.18 \pm 0.04\ (> 2.0)$$

Violation by 5 standard deviations

Jha, O'Sullivan, Chan, and Boyd, PRA **77**, 021801(R) (2008)

Jha, Malik, and Boyd, PRL **101**, 180405 (2008)

Conclusions

- Energy-Time Entanglement can be explored using geometric phase

- Potential benefits for quantum information science

    - Geometric phase is non-dispersive (wavelength independent)

    - Ease in introducing small phase shifts

Acknowledgments

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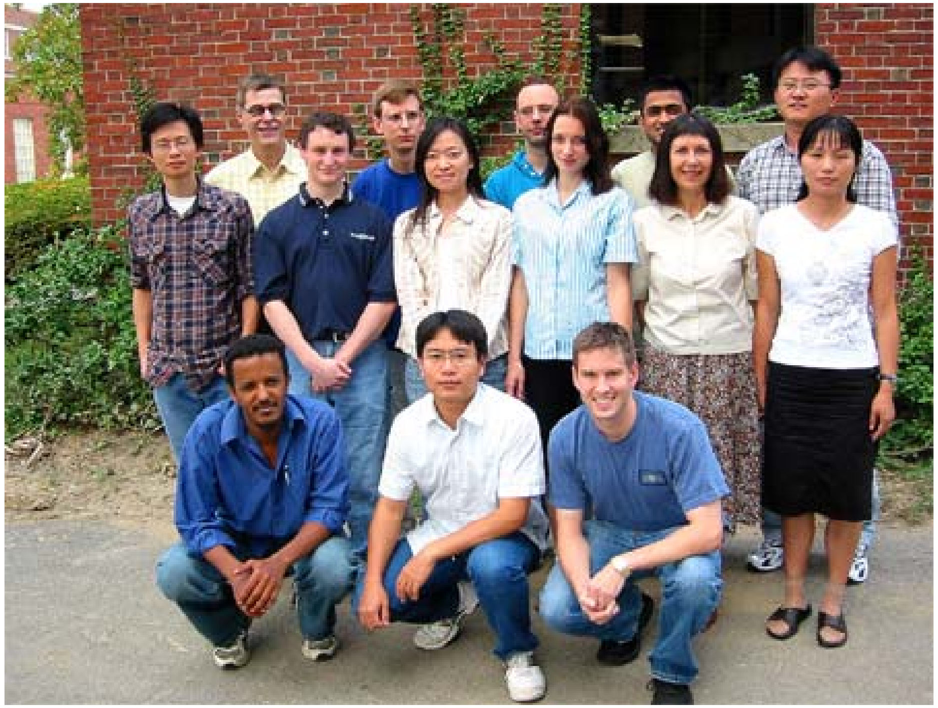

http://www.optics.rochester.edu/~boyd