Analysis of Raman Systems using the Bloch Sphere

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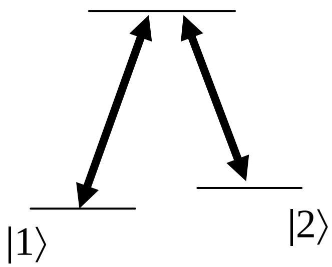
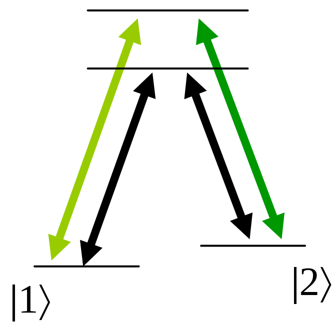
Raman Systems

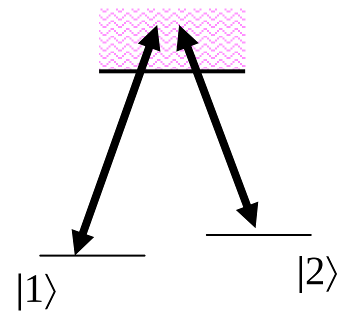
- Raman scattering is an interaction between a pair of optical fields and a pair of states of similar energy
- We consider two ground states coupled by one or more two-photon (Raman) transitions:



Lambda system



Double-Lambda system

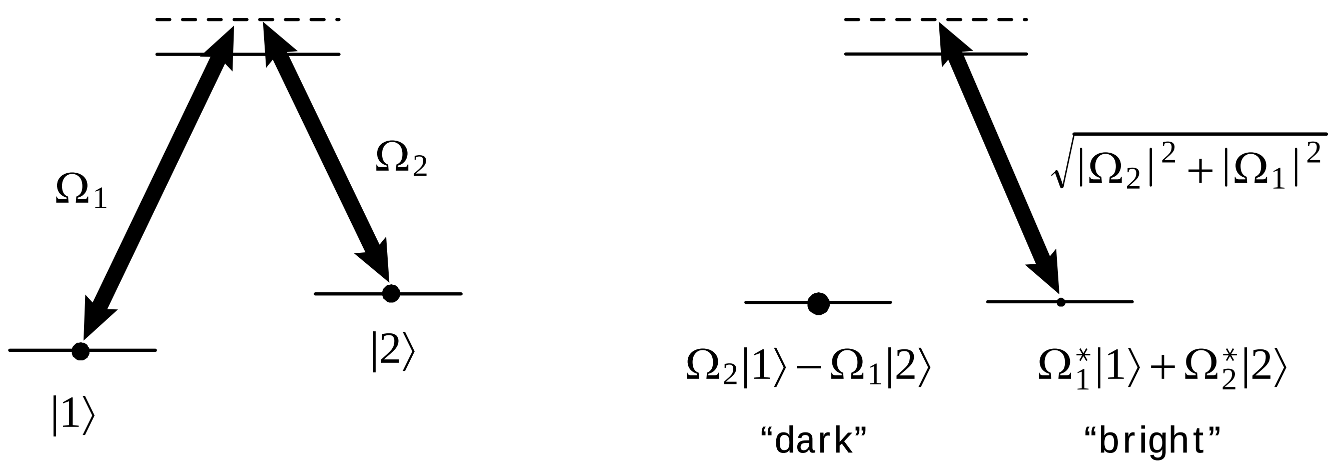


Double excitation of manifold

- Many interesting phenomena involve the formation of coherence between the ground states
  - ultra-narrow (~10's of Hz) spectroscopy[1]
  - optical switching[2] via induced transparency (EIT)[3] or absorption (EIA)[4]
  - extremely strong nonlinear optical processes[5-7]

The Dark/Bright State Basis

- Raman systems can sometimes be understood in terms of a "dark state" and a "bright state"



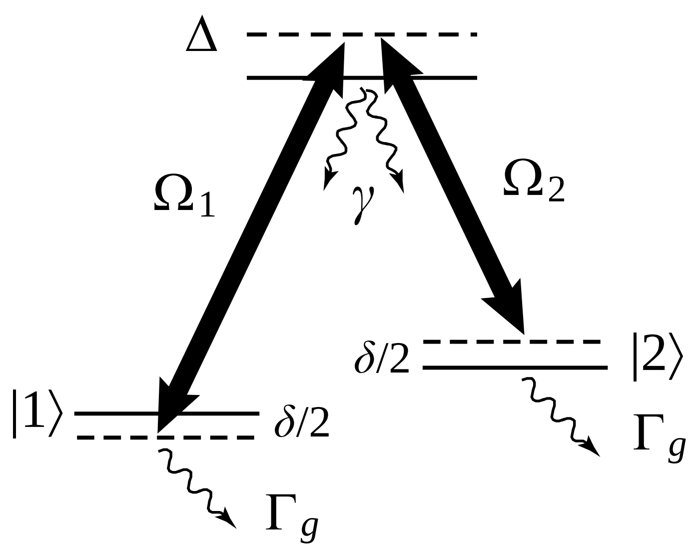
- However, the dark/bright basis is not helpful when
  - the two-photon detuning δ is non-zero, or
  - multiple excited states provide multiple channels for Raman transitions

The Goal

Simplify the analysis of Raman systems and develop intuition

How?

By developing a *basis independent* representation of two states interacting via two fields

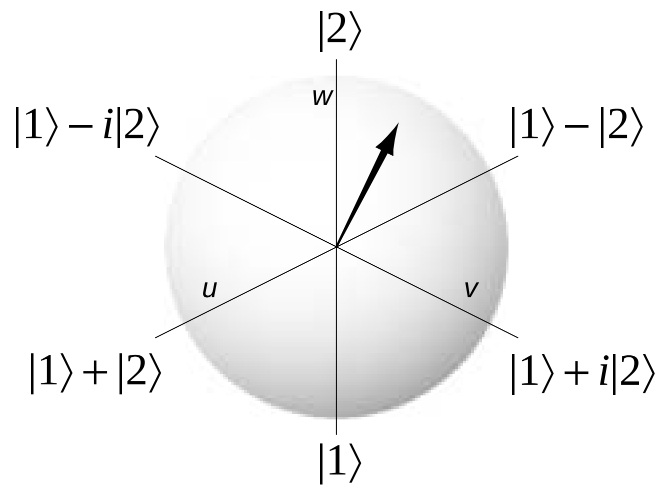


Δ	One-photon detuning
δ	Raman detuning
Ω_1 Ω_2	Rabi frequencies
γ	Decay rate of optical coherence
Γ_g	Decay rate of ground states

The Bloch Sphere Representation

We represent the density matrix for states $|1\rangle$ and $|2\rangle$ by a Bloch vector[8-10]:

$$\vec{\rho} \equiv \begin{pmatrix} u \\ v \\ w \end{pmatrix} \equiv \begin{pmatrix} 2\,\mathrm{Re}\,\rho_{21} \\ 2\,\mathrm{Im}\,\rho_{21} \\ \rho_{22} - \rho_{11} \end{pmatrix}$$

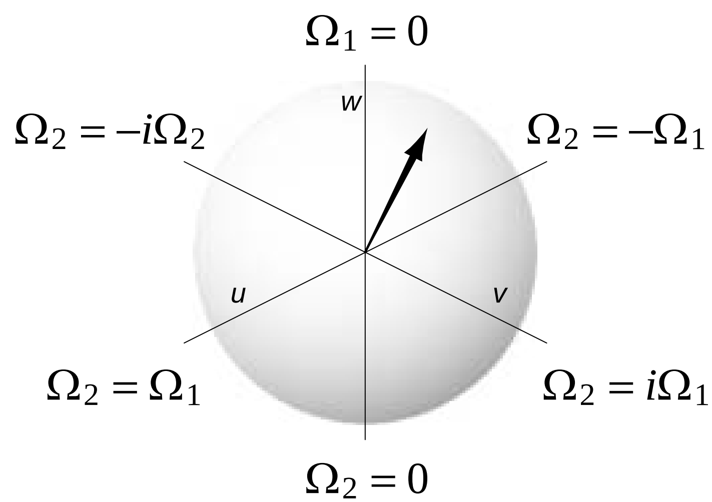


- points on the surface of the unit sphere are pure states
- opposing points on the surface correspond to orthogonal states
- the origin corresponds to a 50/50 incoherent mixture (any basis)

The Mutual Intensity Vector

We represent the pair of fields with a "mutual intensity vector" (analogous to Stokes vector)

$$\vec{I} \equiv \begin{pmatrix} I_u \\ I_v \\ I_w \end{pmatrix} \equiv \begin{pmatrix} 2\,\mathrm{Re}\langle \Omega_2^* \Omega_1 \rangle \\ 2\,\mathrm{Im}\langle \Omega_2^* \Omega_1 \rangle \\ |\Omega_2|^2 - |\Omega_1|^2 \end{pmatrix}$$



- $|I| = |\Omega_1|^2 + |\Omega_2|^2$ for mutually coherent fields
- $|I| < |\Omega_1|^2 + |\Omega_2|^2$ for partially coherent fields

Equation of Motion for the State Vector

Under conditions of interest*,

$$\frac{d}{dt}\vec{\rho} = (\vec{R} - R\vec{\rho}) + \left(R + \Gamma_g\right)\vec{T} \times \vec{\rho} - \Gamma_g \vec{\rho}$$

where

$$\vec{R} \equiv -\frac{1}{4}\frac{\gamma}{\Delta^2 + \gamma^2}\vec{I}, \qquad (R = \left|\vec{R}\right| \text{ is the total excitation rate})$$

$$\vec{T} \equiv \left(\frac{\delta}{R + \Gamma_g}\right)\vec{w} - \left(\frac{\Delta}{R + \Gamma_g}\right)\frac{\vec{R}}{\gamma}$$

- $\vec{R} - R\vec{\rho}$ describes optical pumping into the dark state
- $\vec{T} \times \vec{\rho}$ describes precession of the bright/dark basis
- $-\Gamma_g \vec{\rho}$ describes relaxation of the ground states

* $|\delta| \ll |\Delta + i\gamma|$, $\Gamma_g \ll \gamma$, $|d\vec{\rho}/dt| \ll |\Delta + i\gamma|$ Slight changes in the definitions of R and $\vec{R}$ are also required to accommodate saturating fields or mutually incoherent fields.

Discussion: Bright and Dark states

From the equation of motion, optical pumping vanishes when $\vec{R} - R\vec{\rho} = 0$. Therefore,

> The point $\vec{R}/R$ corresponds to the dark state.

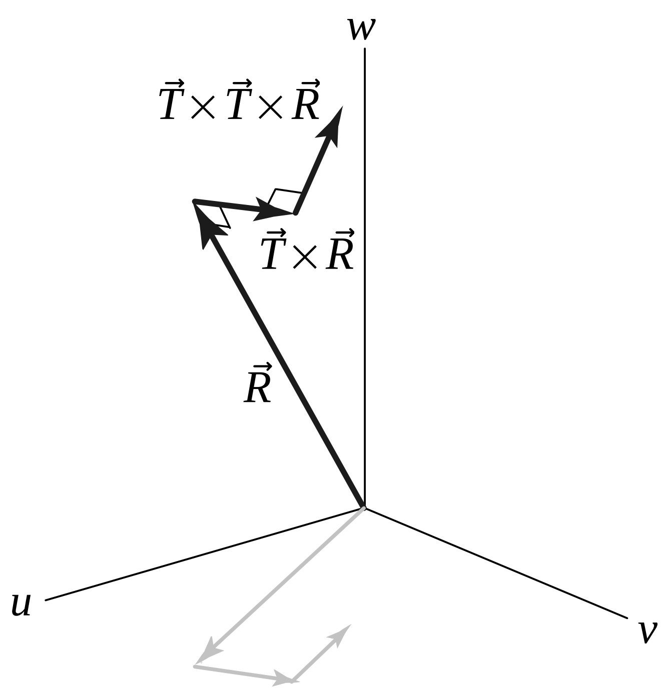
The absorption rate is proportional to the projection of $\vec{\rho}$ onto the bright state $-\vec{R}/R$.

The relative phase of the two absorption pathways is given by

$$\begin{aligned}\phi &\equiv \arg(\rho_{21}) - \arg(\Omega_2^* \Omega_1) \\ &= \text{angle between } \vec{\rho} \text{ and } \vec{R}/R \text{ in the } u\text{-}v \text{ plane}\end{aligned}$$

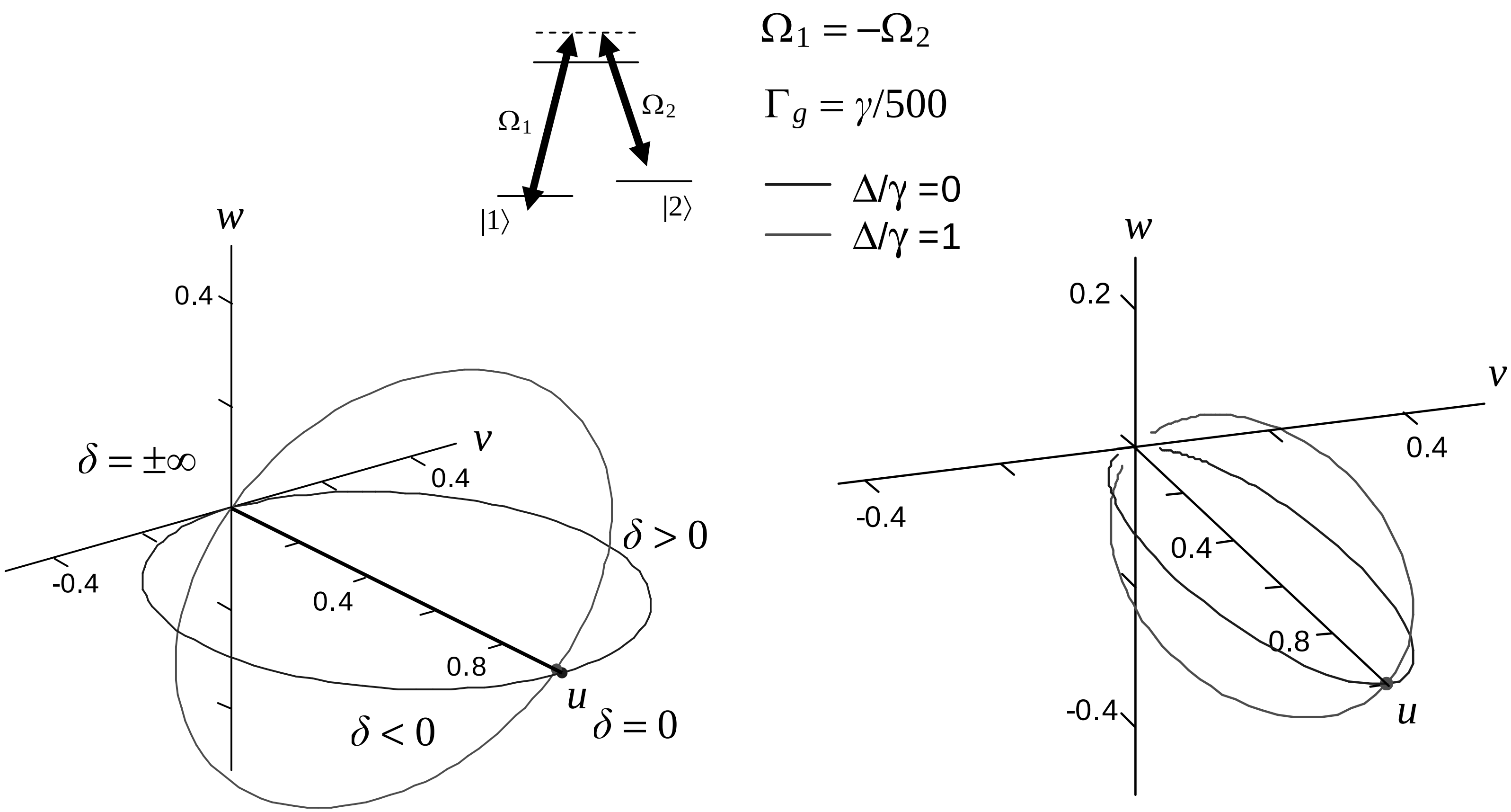
Steady-State Solution

$$\vec{\rho} = \left[\vec{R} + \frac{\vec{T} \times (\vec{R} + \vec{T} \times \vec{R})}{1 + |\vec{T}|^2} \right] \left(R + \Gamma_g \right)^{-1}$$



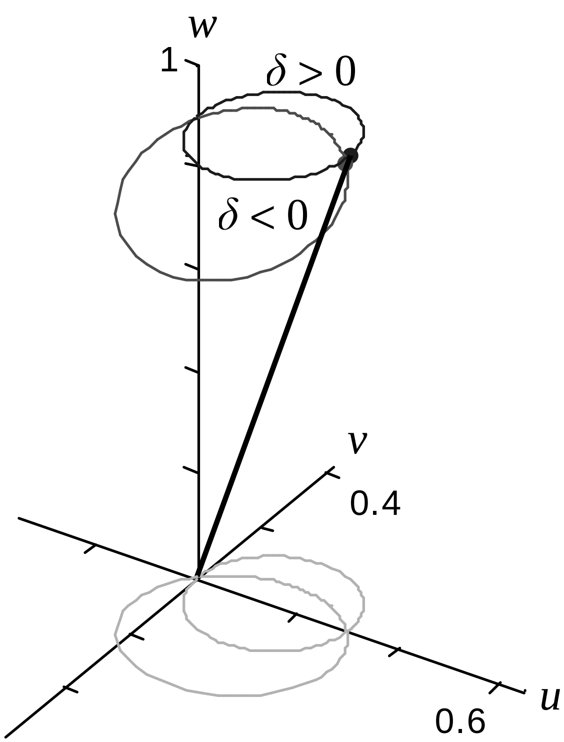
- With zero Raman detuning ($\delta = 0$), $\vec{\rho}$ is parallel to the intensity vector and has magnitude $R/(R + \Gamma_g)$
- $\vec{T} \times \vec{R}$ advances ($\delta < 0$) or retards ($\delta > 0$) the phase of ρ_{21}
- $\vec{T} \times \vec{T} \times \vec{R}$ decreases the magnitude of ρ_{21} and increases ($\Delta\delta < 0$) or decreases ($\Delta\delta > 0$) the inversion

Solutions for Pump-Pump Configuration

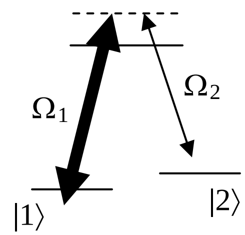


weak excitation $(I = 0.4\gamma^2)$ strong excitation $(I = 20\gamma^2)$

Solutions for Pump-probe Configuration



weak excitation $(I = 0.4\gamma^2)$

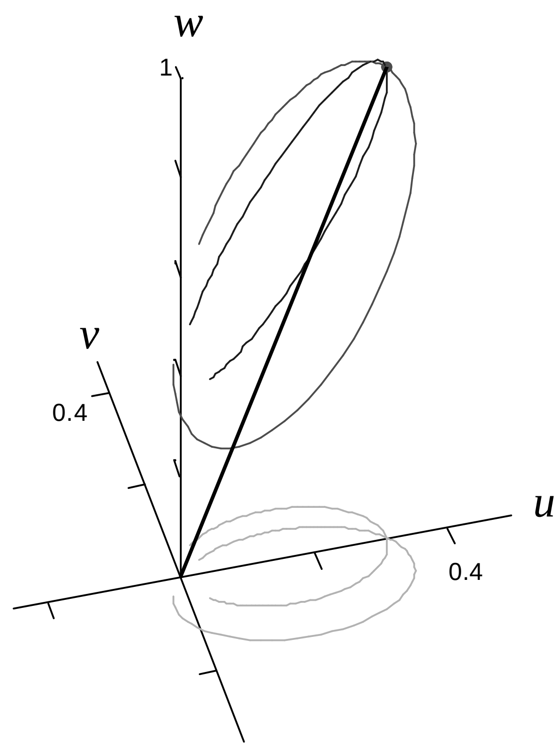


$\Omega_1 = -6\Omega_2$

$\Gamma_g = \gamma/500$

— $\Delta/\gamma = 0$

— $\Delta/\gamma = 1$



strong excitation $(I = 20\gamma^2)$

Discussion of General Solutions

As parametric functions of δ, the solutions are ...

- circular for weak excitation
- elliptical for strong excitation ($\vec{\rho}$ shrinks toward the origin as population is removed to the excited state)

- "flat" for $\Delta = 0$ (only coherence changes with δ)
- tilted for $\Delta \neq 0$ (both coherence and inversion change with δ)

- Symmetric about $\delta = 0$ for $|\Omega_1| = |\Omega_2|$ or $\Delta = 0$
- Asymmetric about $\delta = 0$ for $|\Omega_1| \neq |\Omega_2|$ and $\Delta \neq 0$

Example: Quick Results for EIT

Threshold for EIT

The peak of EIT occurs at $\delta = 0$. Then $\vec{\rho} = \frac{\vec{R}}{R + \Gamma_g}$. The transparency is reduced by ½ when $\vec{\rho}$ is reduced half-way to 0. Therefore, the threshold for EIT is

$$|\Omega_1|^2 + |\Omega_2|^2 \gtrsim 4\frac{\Gamma_g(\Delta^2 + \gamma^2)}{\gamma}.$$

Spectral width of EIT

At single-photon resonance ($\Delta = 0$), the angle ϕ between $\vec{\rho}$ and the dark state $\vec{R}/R$ is

$$\phi = \tan^{-1}\frac{\delta}{R + \Gamma_g}.$$

Transparency is reduced by ½ when the component along $\vec{R}/R$ is reduced by ½ $\Rightarrow \phi = \pi/4$. Therefore, the EIT width is

$$\mathrm{FWHM} = 2\left(\Gamma_g + R\right)$$

Summary

- The optical properties of a Raman system are largely determined by the state of the two ground levels.
- The state of the two ground levels can be represented conveniently as a Bloch state vector within a unit sphere.
- Each pair of optical fields which interacts with these two levels can be represented by an intensity vector.
- By adiabatically eliminating the excited level(s), a simple, direct solution of the state vector can be obtained in terms of the intensity vector(s) and the detunings of the fields.
- The solutions take simple geometric forms, allowing one to develop intuition about the behavior of Raman systems.
- Fundamental quantities of interest, such as the EIT width and field intensity required for transparency, follow easily from the direct solution.

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