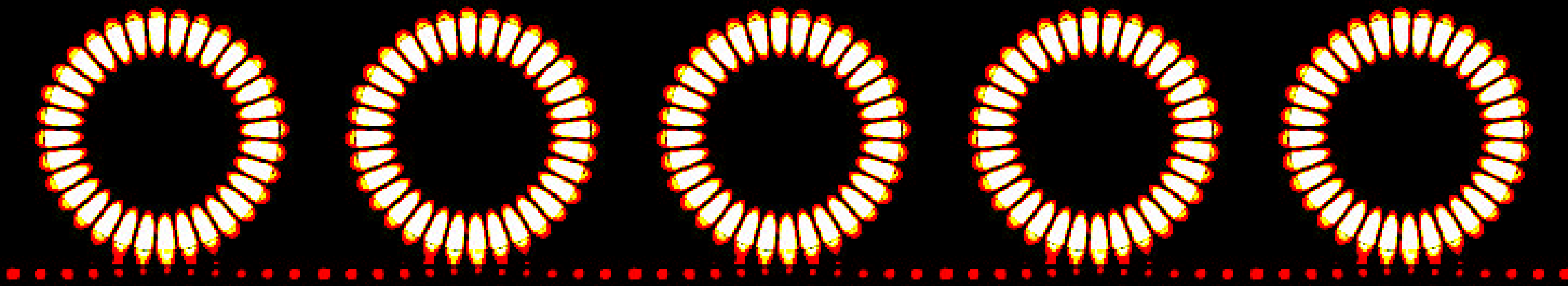
Exotic Nonlinear Pulse Propagation Effects in Microresonators



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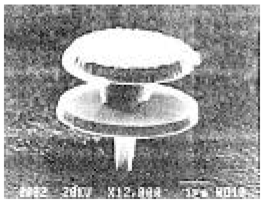
Nonlinear Optics

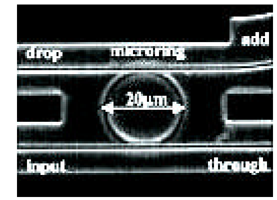
Motivation

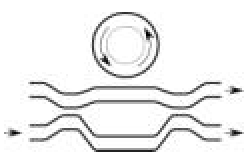
Microresonators are natural building blocks for integrated photonics



Applications have thus far included:

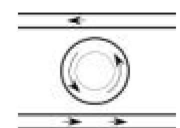
- Whispering Gallery Lasers (Slusher, Vahala, Lefevre, Chang)
- Add-drop filters (Ho, Little, Dapkus)
- Dispersion Compensators (Madsen, Lenz)
- Optical Delay Lines (Madsen, Slusher)
- Chemical / Biological Sensing (Arnold, Driessen)
- Cavity QED (Ilchenko, Imamoglu)

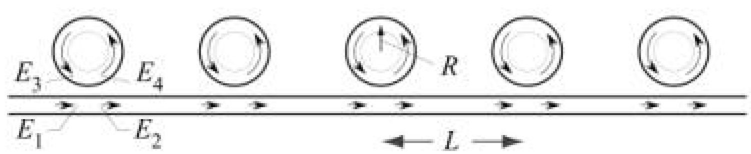




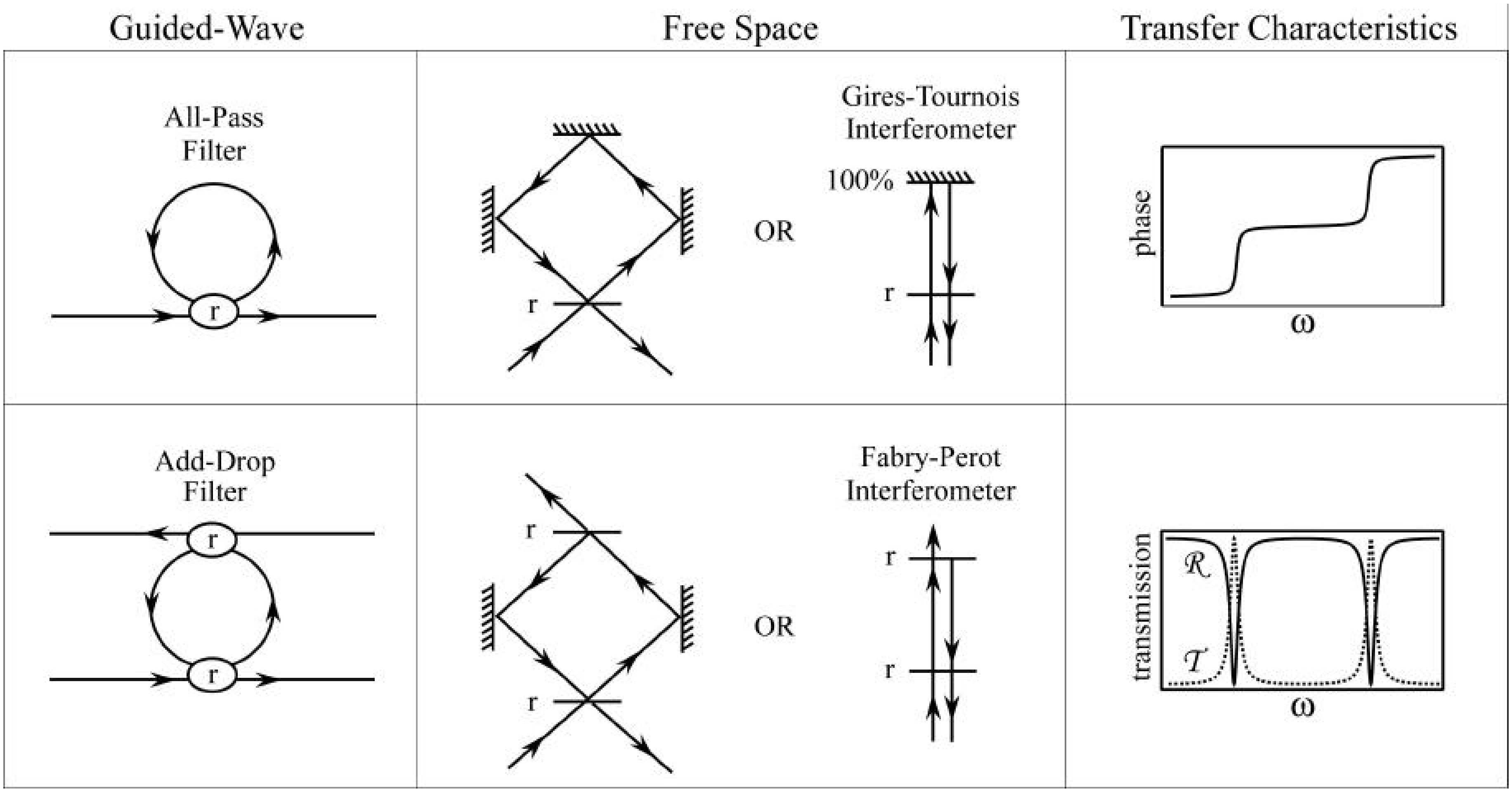
Applications that hold promise:

- All-Optical Switching / Logic (Ho, Boyd)
- Engineerable/tunable nonlinear waveguides



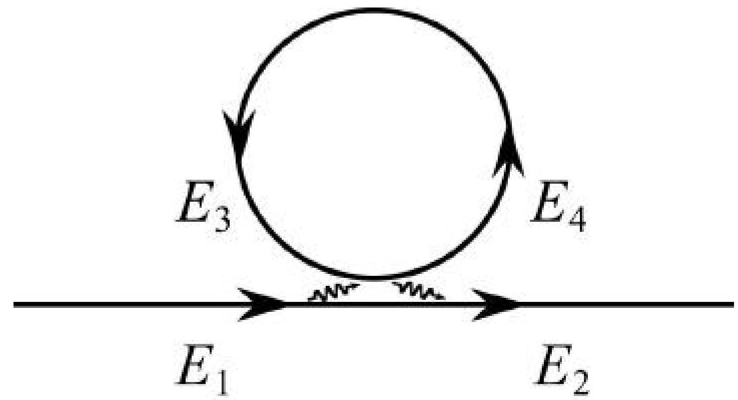


Ring Resonators



All-Pass Ring Resonators

Assuming negligible attenuation, a ring resonator coupled in this manner is, unlike a Fabry-Perot, a fully-transmissive system - there is no mechanism for reflection or existence of a “drop” port.



COUPLING

coupling matrix

$$\begin{bmatrix} E_4(w) \\ E_2(w) \end{bmatrix} = \begin{bmatrix} r & it \\ it & r \end{bmatrix} \begin{bmatrix} E_3(w) \\ E_1(w) \end{bmatrix}$$

+

FEEDBACK

internal phase

$$E_3(w) = t\ e^{in_{\text{eff}} w\, 2pR/c} E_4(w)$$

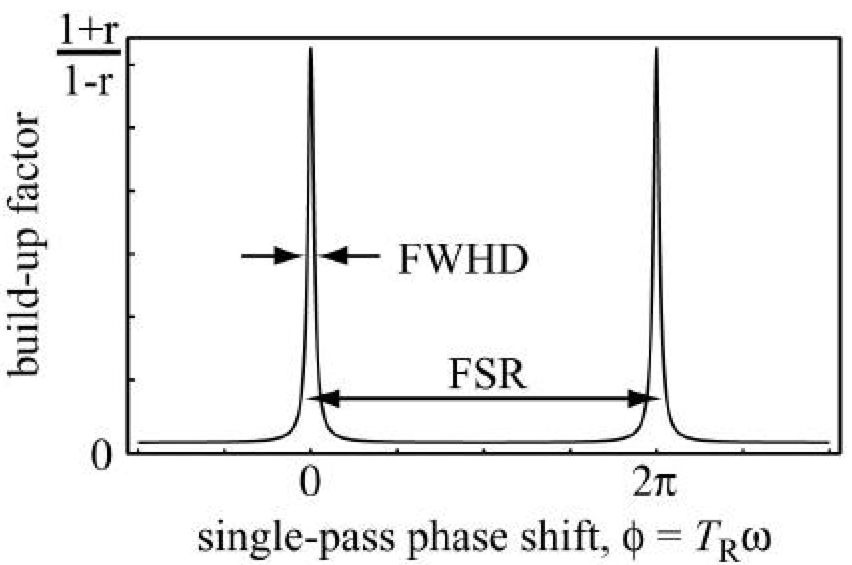
internal transmission

Intensity Build-Up

- Near resonance, the circulating field experiences a coherent build-up of intensity.
- Optical energy is stored and effectively compressed within the resonator volume.

$$E_4 = rE_3 + itE_1 = r\left(t\ e^{if} E_4\right) + itE_1 = \frac{it}{1 - rt\ e^{if}} E_1$$

$$\mathcal{B} = \frac{|E_4|^2}{|E_1|^2} = \frac{1 - r^2}{1 - 2rt\ \cos f + r^2 t^2}$$



Finesse

$$\mathcal{F} = \frac{\text{FSR}}{\text{FWHD}} = \frac{2}{1 - r}$$

Quality Factor

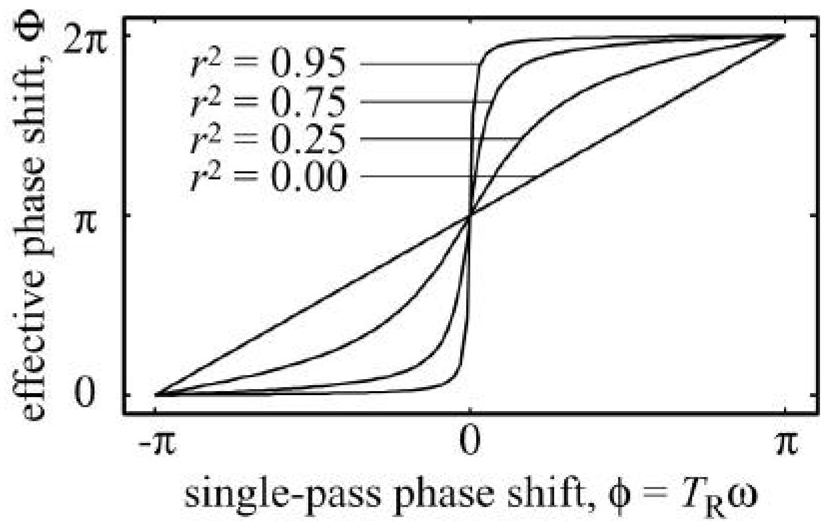
$$Q = \frac{n_0}{\Delta n} = \frac{n 2pR}{l} \mathcal{F}$$

Phase Sensitivity / Group Delay

- The effective phase shift is sensitively dependent on frequency near resonance.
- The slope of the curve is related to the group delay of a pulse envelope traversing the resonator. The maximum slope is exactly equal to the peak intensity build-up factor (lossless case).

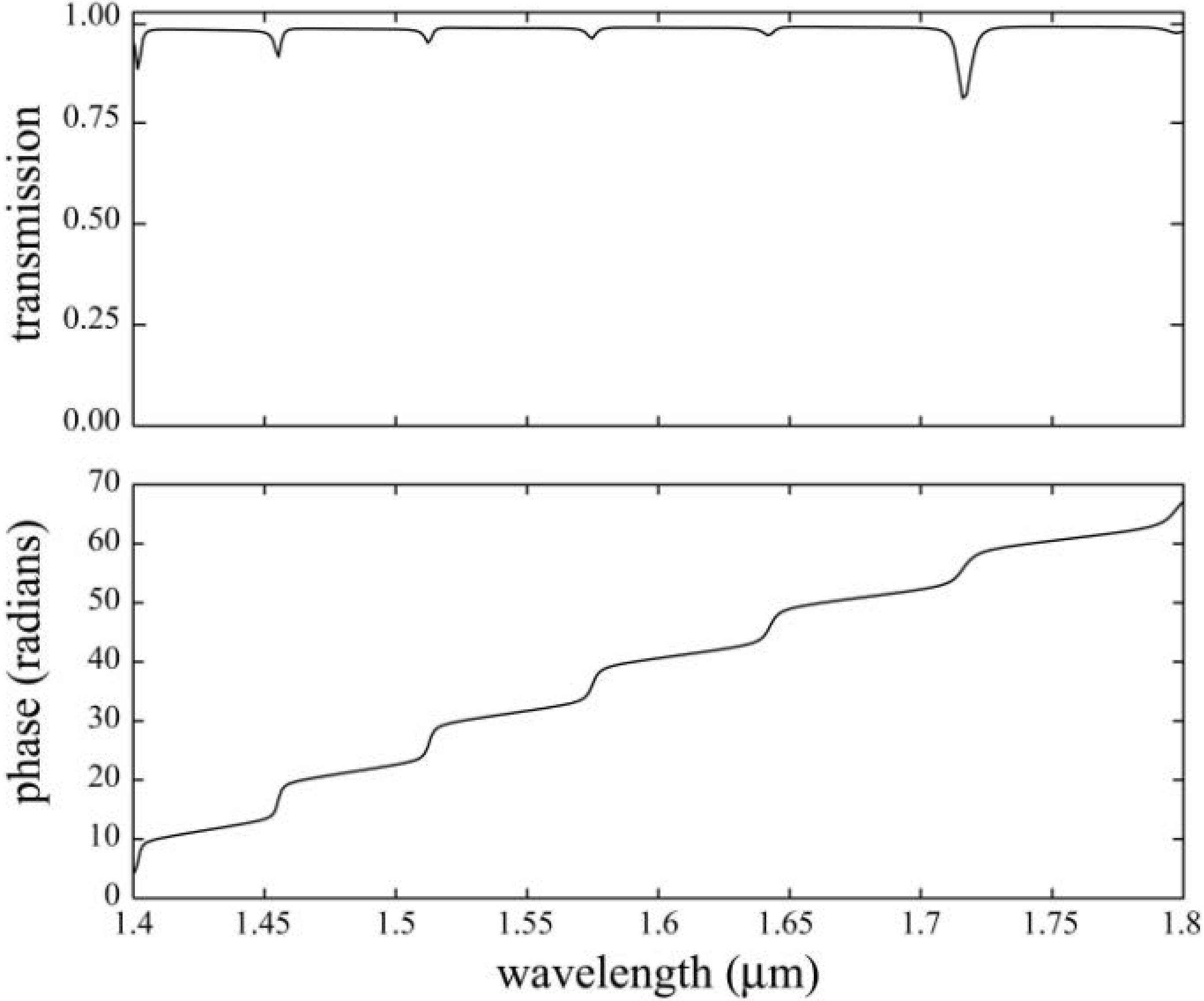
$$E_2 = rE_1 + itE_3 = rE_1 + it\left(t\ e^{if} \frac{it}{1 - rt\ e^{if}} E_1 \right) = \frac{r - t\ e^{if}}{1 - rt\ e^{if}} E_1$$

$$\mathcal{T} = \frac{\left|E_2\right|^2}{\left|E_1\right|^2} = \frac{r^2 - 2rt\ \cos f + t^2}{1 - 2rt\ \cos f + r^2 t^2} \underset{t=1}{\longrightarrow} 1 \qquad \Phi = \arg\left(\frac{r - t\ e^{if}}{1 - rt\ e^{if}} \right)$$



Violates Kramers Kronig relations, but not Hilbert relations or causality

FDTD Simulation Results



Enhanced Nonlinear Phase

Change in effective phase with respect to input power:

$$\frac{d\Phi}{dP_1} = \frac{d\Phi}{df}\,\frac{df}{dP_C}\,\frac{dP_C}{dP_1} \xrightarrow{\text{near resonance}} \Delta\Phi = \frac{4}{p^2}\mathcal{F}^2\,\frac{p}{P_p}\,\Delta P_1$$

increased phase sensitivity

single-pass switching threshold

coherent build-up of power

finesse-squared enhancement

Bandwidth:

$$\Delta n = \frac{n_0}{Q} = \frac{c}{n2pR\mathcal{F}}$$

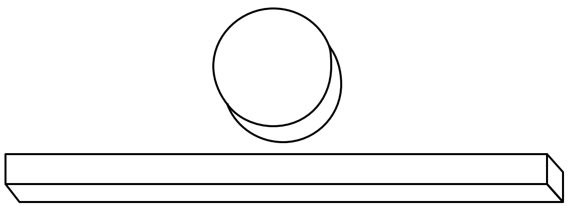
bandwidth compromised, but only linearly

Unfortunately, the bandwidth is reduced. But fortunately the tradeoff is not a balanced one. The nonlinear enhancement scales quadratically while the bandwidth is reduced linearly!

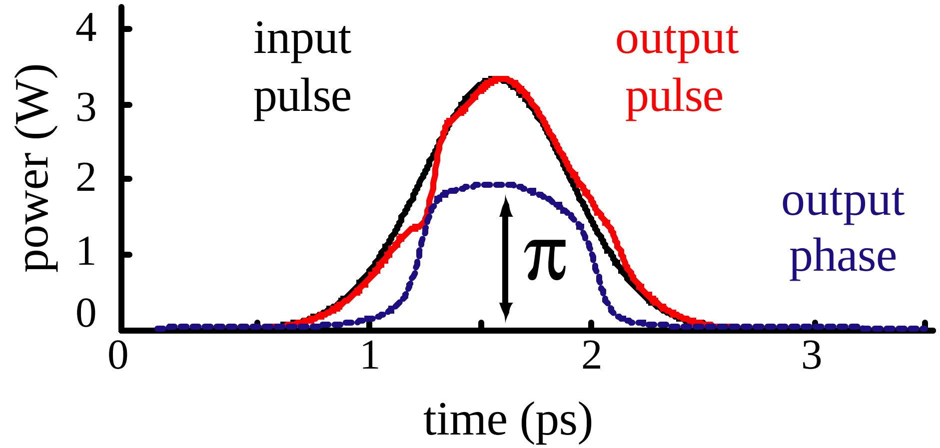
"Enhanced All-Optical Switching Using a Nonlinear Fiber Ring Resonator"
J. E. Heebner and R. W. Boyd, Optics Letters, 24, pp.847-849, (1999)

Enhanced All-Optical Switching

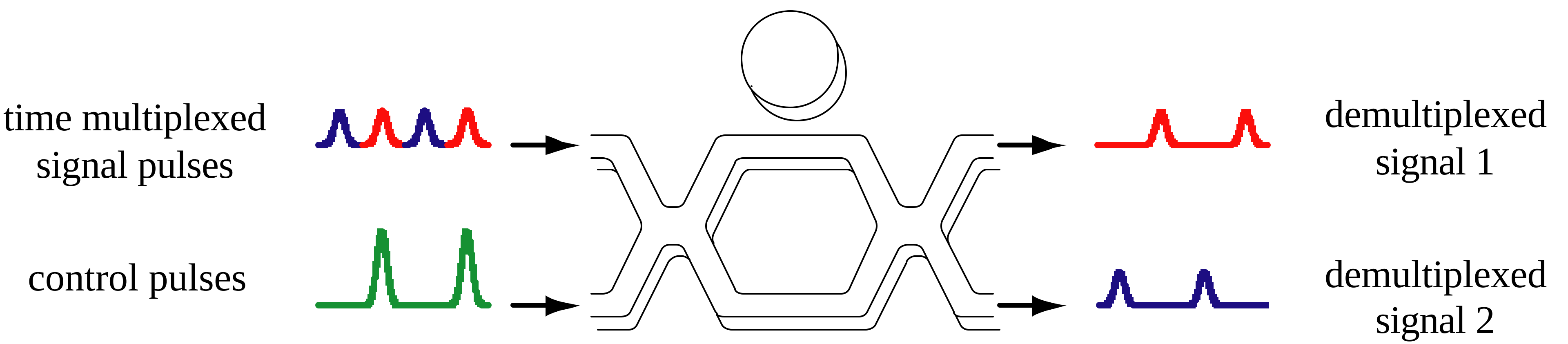
Pulse Transfer characteristics for a single side-coupled resonator driven to π NL phase shift



Enhanced Self-Phase Modulation (SPM) (rigorous simulation)



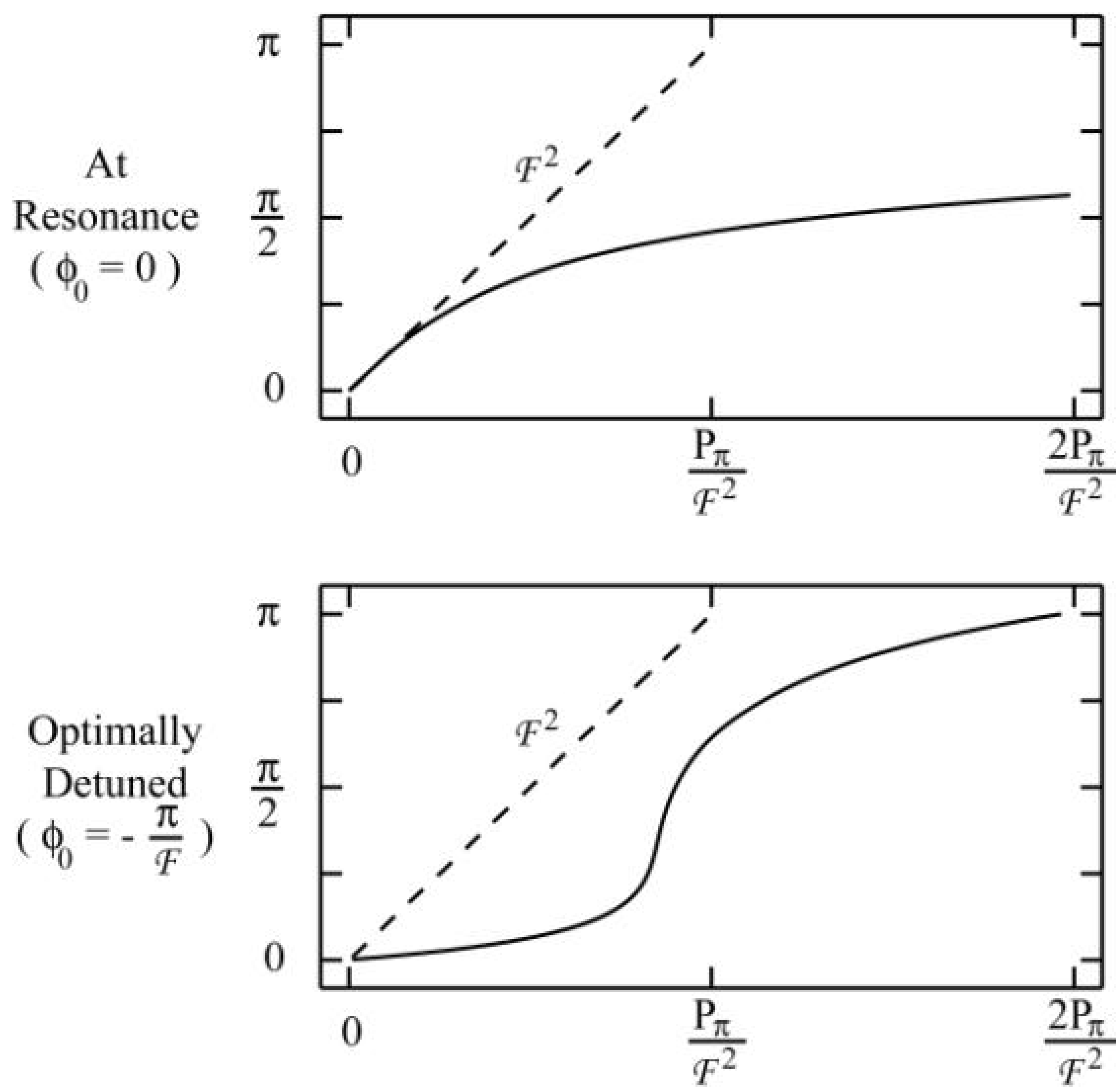
Cross Phase Modulation (XPM) could similarly be enhanced for integrated, chip-level light by light switching:



Saturation / "Pulling"

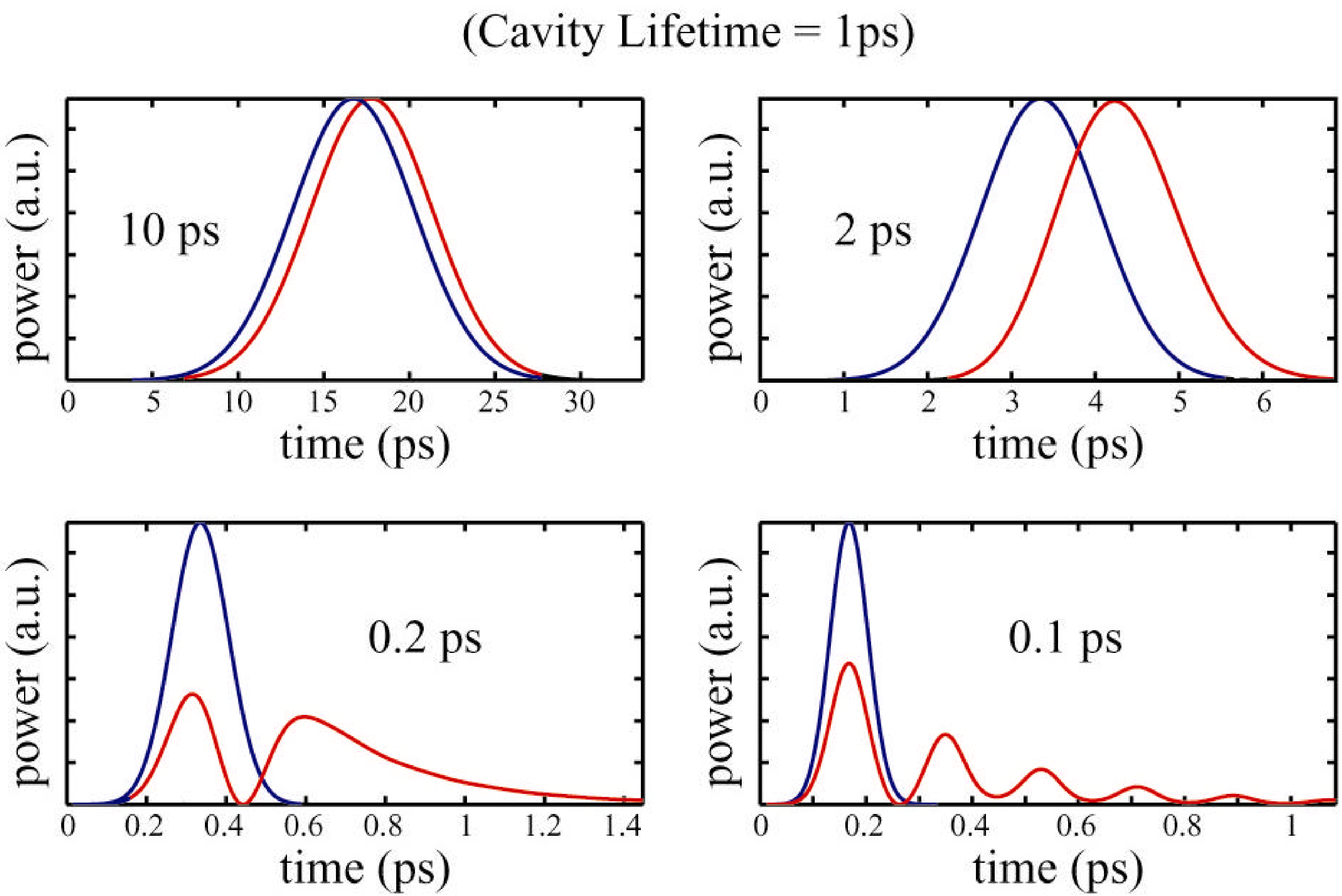
- While the transmitted nonlinear phase shift is enhanced, the enhancement drops off as the resonator is power-detuned away from resonance
- With an initially resonant resonator, this saturation effect prevents a π phase shift from being extracted
- However, if the resonator is initially red-detuned, a π phase shift is readily achievable within a factor of 2 of the finesse-squared prediction

"Extractable" phase shift Vs. input power

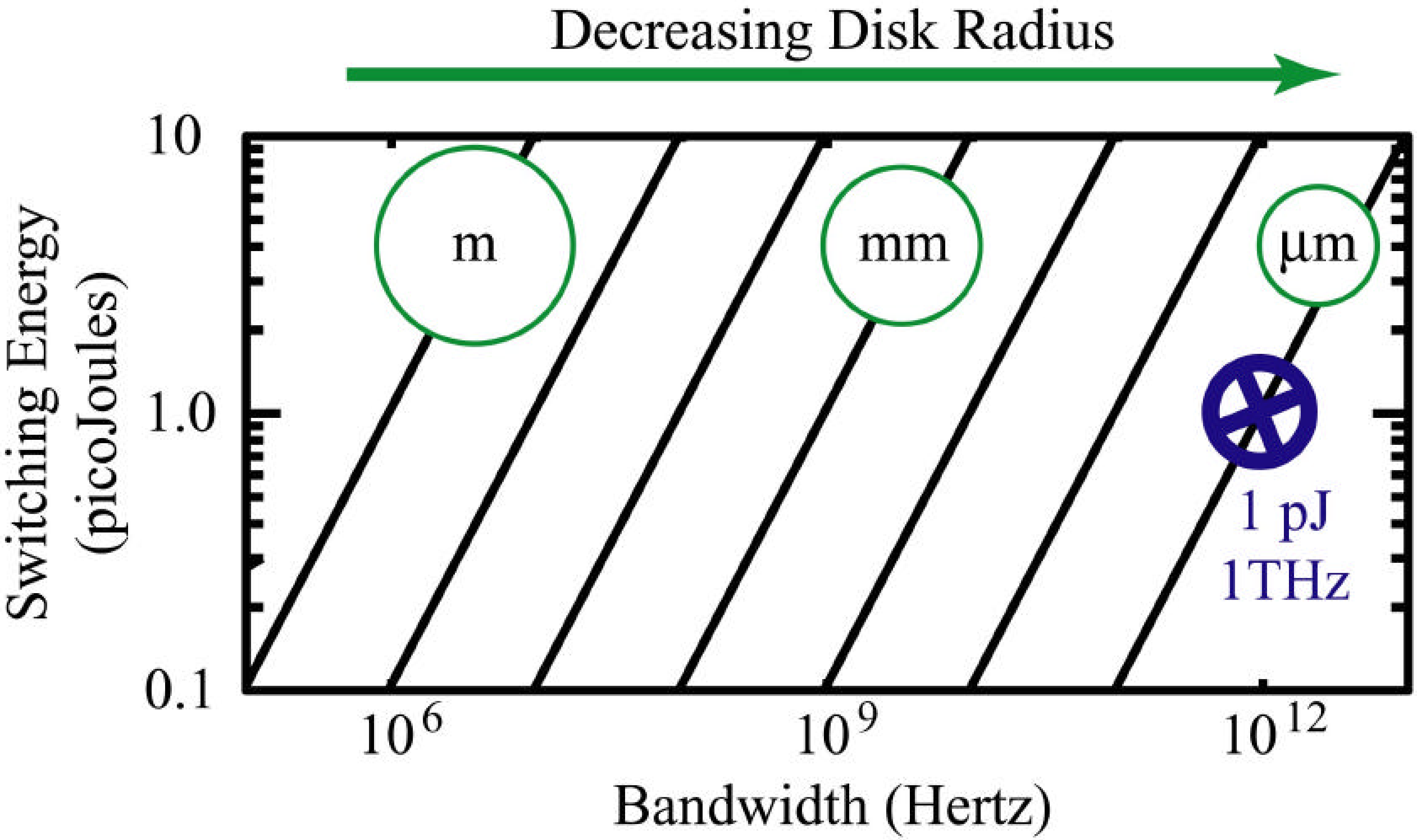


Pulse Response

Pulses propagating through a resonator must be longer than the cavity lifetime or else the resonator output will "ring"



Pulse Energy / Bandwidth Tradeoff



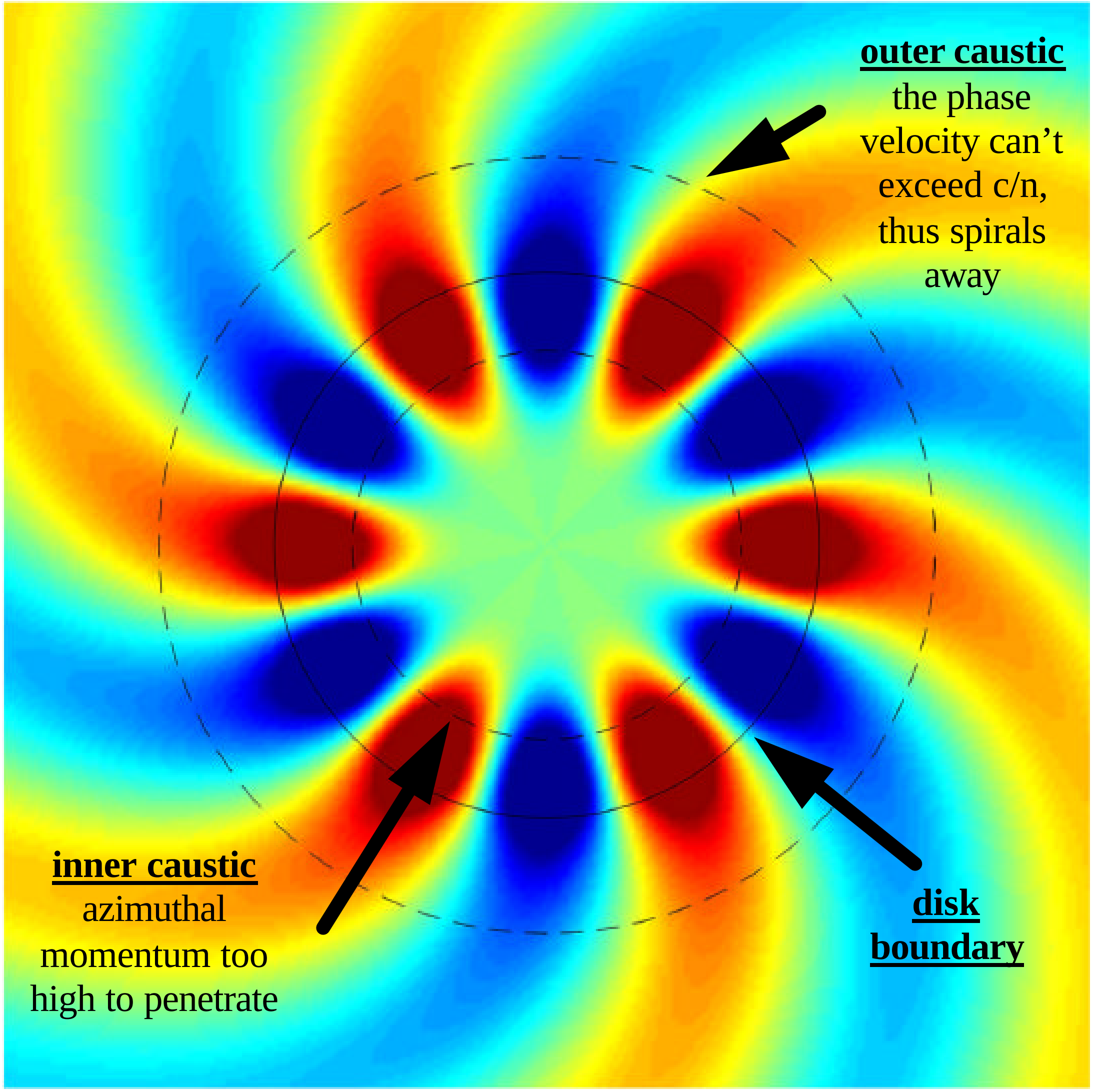
Optical Whispering Gallery Modes

Field plot of weakly confined WGM

- azimuthal mode number: m = 6
- index contrast: n_1/n_2 = 2:1
- polarization: out of plane
- Q-factor: 61

Guidance is between disk edge and ***inner caustic***

Bending radiation loss is due to coupling to cylindrical continuum existing beyond ***outer caustic***

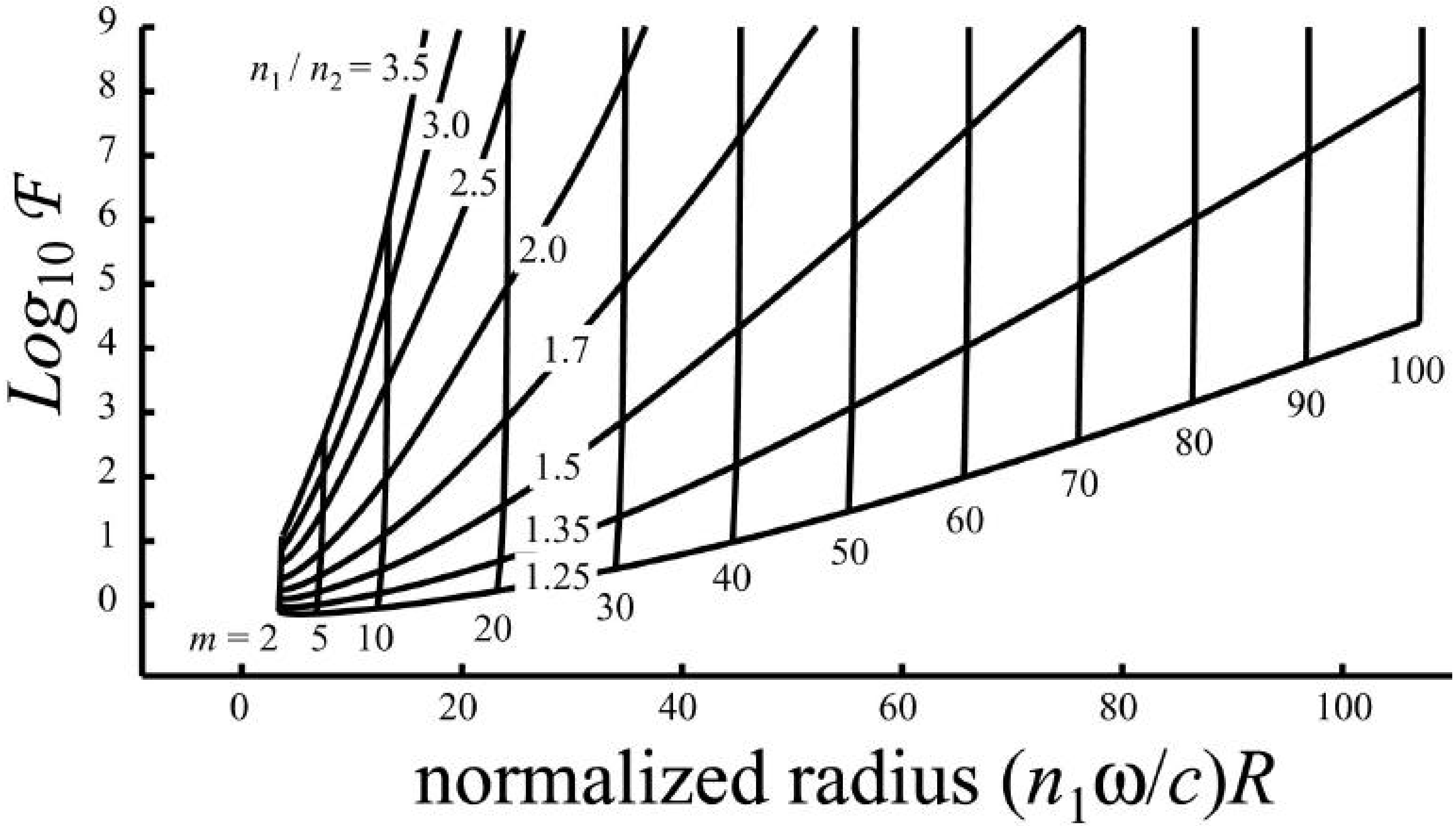


WGM Dispersion Relation / Bending Loss

The bending-loss-limited finesse vs. normalized radius is plotted for a variety of index contrasts.

The plot is generated by numerically solving the complex whispering gallery dispersion relation:

$$\frac{\tilde{n}_1 J'_m\left(\tilde{k}_1 R\right)}{J_m\left(\tilde{k}_1 R\right)} = \frac{\tilde{n}_2 H'_m\left(\tilde{k}_2 R\right)}{H_m\left(\tilde{k}_2 R\right)}$$



(For most semiconductors at 1.55μm, $n_1\omega/c$~12)

All-Optical Switching

- Channel rates exceeding 40 Gbit/s are difficult to achieve with high-speed electronics.
- Significantly higher channel rates (> 100 Gbit/s) require optical time division multiplexing, switching and/or logic which in turn rely on optical nonlinearities.
- Semiconductor excited carrier nonlinearities are strong but limited by recombination time (~10 ps)
- The optical Kerr effect / AC Stark effect are non-material-resonant third-order nonlinearities that possess femtosecond response and are ideal

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Kerr Effects:

Self-phase modulation (SPM) $P^{(3)}(w_1) = 3c^{(3)}E(w_1)E^*(-w_1)E(w_1)$

Cross-phase modulation (XPM) $P^{(3)}(w_1) = 6c^{(3)}E(w_2)E^*(-w_2)E(w_1)$

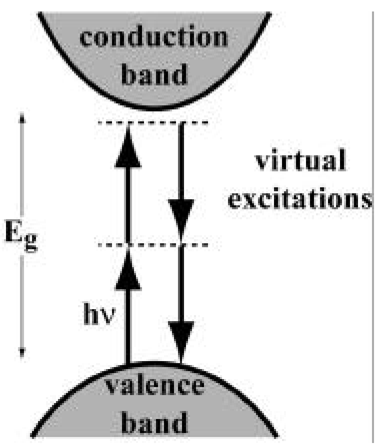
Intensity dependent refractive index $n = n_0 + n_2 I_{\text{self}} + 2n_2 I_{\text{cross}}$

Optical Switching Materials

- **Strong nonlinearity** – the refractive nonlinearities in semiconductors can be 2-3 orders of magnitude larger than in silica glass, due to a smaller bandgap (dependence on bandgap is to the –4 power)
- **Fast, sub-picosecond response** – If the photon energy is slightly less than the half–gap energy, two-photon absorption may be avoided, leaving a reasonably strong nonlinearity. [Sheik-Bahae, Hagan,Van Stryland]
- **Good NL figure of merit (NLFOM)** – If carrier generation via two-photon absorption is avoided, a fast (femtosecond response) bound nonlinearity remains.

$Al_{0.2-0.4}Ga_{0.8-0.6}As$ and chalcogenide glasses (e.g. $AsSe_3$) satisfy these requirements [Stegeman, Slusher].

energy diagram



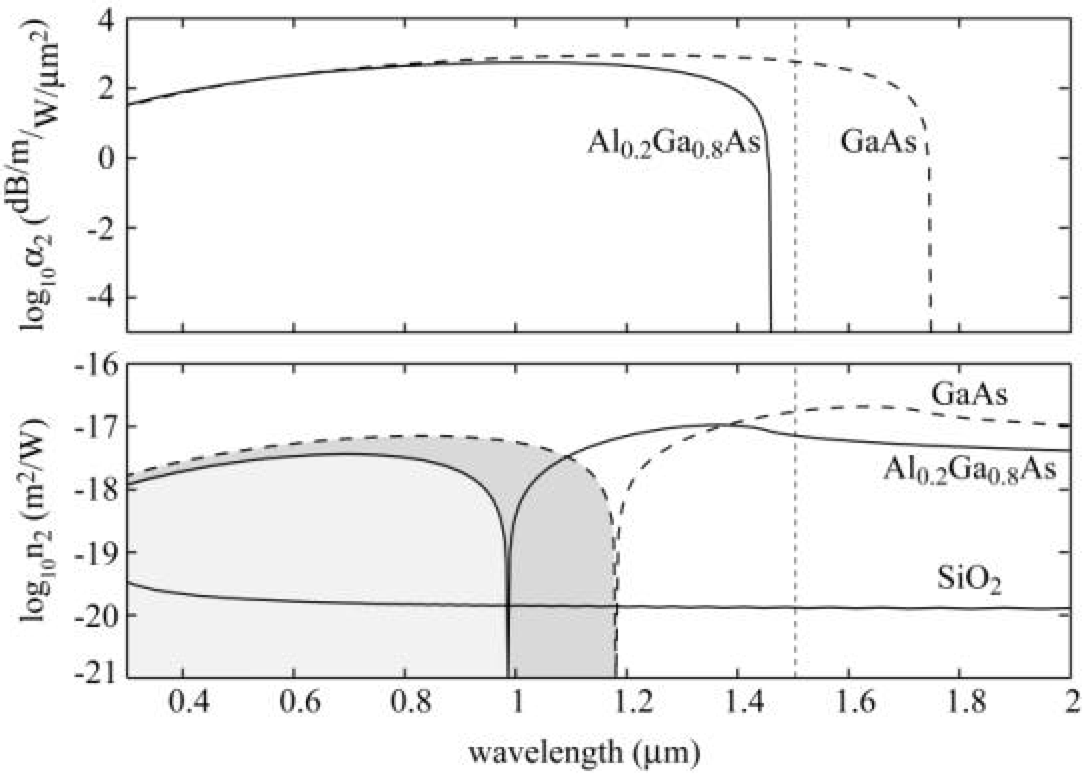
two-photon absorption

↓

Hilbert transform

↓

nonlinear refractive index



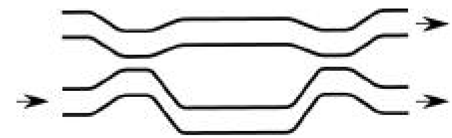
Switching Thresholds

…but there are still problems

Nonlinear phase shift: $\Delta f_{NL} = \frac{2p\ n_2}{l A_{eff}} P\Delta L = gP\Delta L$

Switching threshold: $P_p = \frac{l\ A_{eff}}{2n_2\Delta L} = \frac{p}{g\Delta L}$

Unbalanced NL MZ



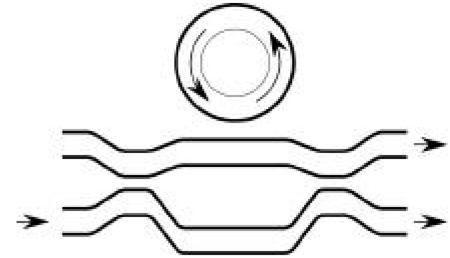
ΔL ~ 3cm

	n_2 =	γ =	1pJ,1ps pulse requires:
Silica SMF:	3?10^{-20} m^2/W	0.0025 W^{-1}m^{-1}	1.25 km
AlGaAs, $AsSe_3$:	1.5?10^{-17} m^2/W	100 W^{-1}m^{-1}	3 cm

3 cm is TOO LONG for photonic LSI!

A microresonator has the potential of 1000X reduction to 30 microns

Microresonator-Enhanced

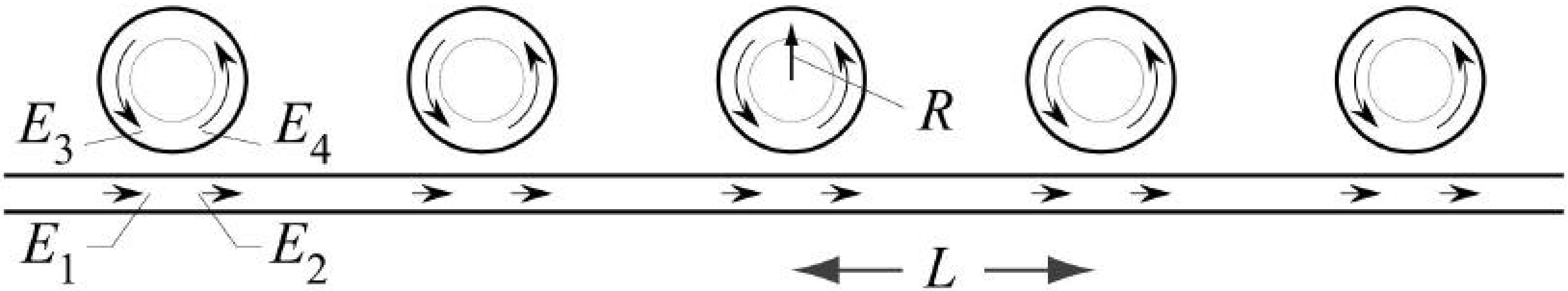


$2\pi R$ ~ 30μm, ΔL ~ 0

SCISSORs

Side Coupled Integrated Spaced Sequence Of Resonators

By coupling resonators to an ordinary optical waveguide, the propagation parameters governing nonlinear pulse propagation may be dramatically modified, leading to exotic and controllable nonlinear pulse evolution

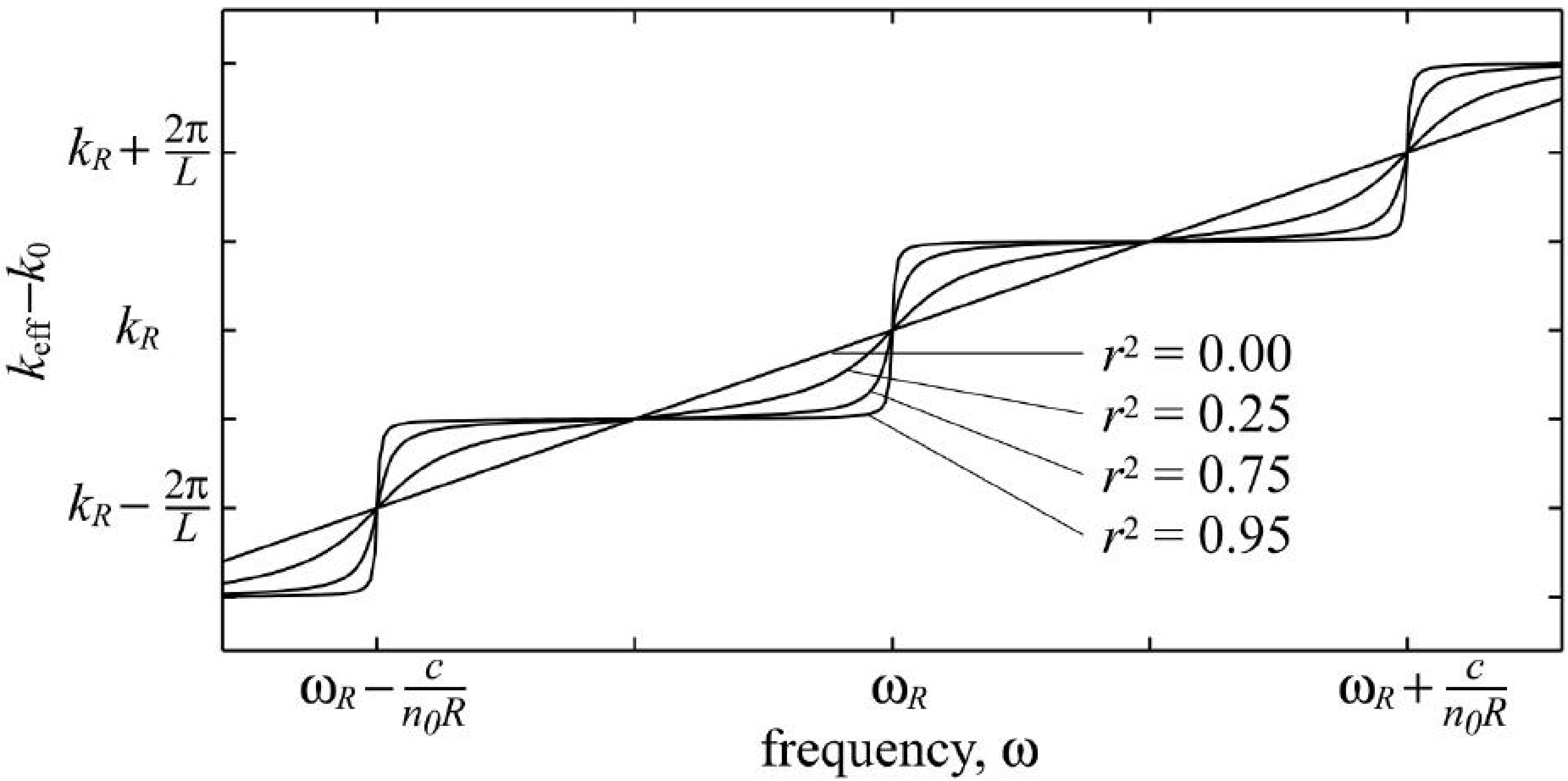


Feedback is intra-resonator, not inter-resonator

Thus, there is NO PHOTONIC BANDGAP!

Nevertheless, the system exhibits many properties similar to PBGs (eg. Bragg gratings) such as reduced group velocities, induced dispersion, and enhanced nonlinearities.

SCISSOR Dispersion Relation



Resonator Induced Dispersion

Resonator induced dispersion can be 5-7 orders of magnitude greater than material dispersion in silica!

Pulse dispersion is independent of finesse for finesse > 10.

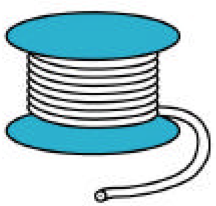
Thus, while an ultra-high finesse is required for propagating ultra-slow light an ultra-high finesse is *not required* for dispersing or delaying a pulse arbitrarily.

N resonators allow one to tailor dispersion profile with 2N degrees of freedom (coupling strength & radius)

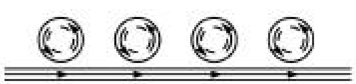
A single resonator can roughly delay a pulse by one pulse width or disperse a pulse by one dispersion length.

1) SMF-28 Silica Fiber

$\beta_2 = 20\ \text{ps}^2/\text{km}$



2) SCISSOR

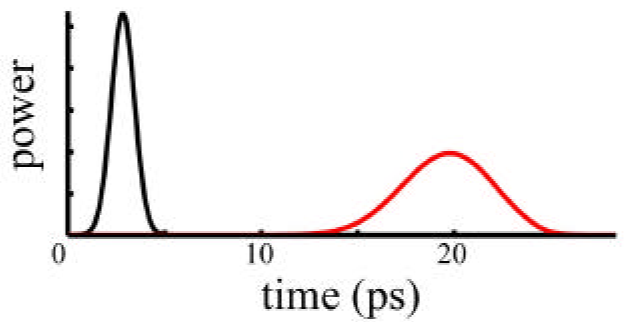


$$\beta_2 = \frac{T^2}{L}\frac{-2r(1-r^2)\sin\phi_0}{(1-2r\cos\phi_0+r^2)^2} \xrightarrow[\phi_0=\phi_D]{} \frac{3\sqrt{3}}{4\pi^2}\frac{\Im^2 T^2}{L}$$

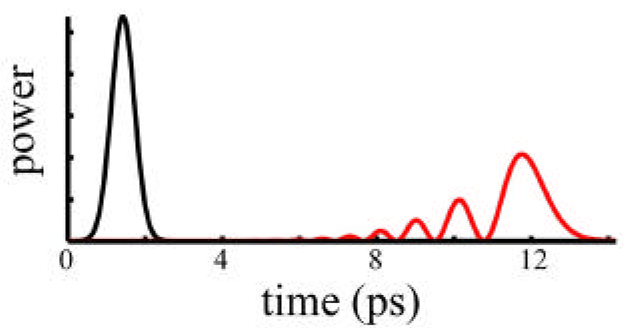
$= 20\ \text{ps}^2/\text{mm}$

$\Im = 20$
$L = 10\ \mu\text{m}$
$1/T = 20$ THz

Group Velocity Dispersion (β_2)



Higher Order Dispersion (β_3)



Derivation of Envelope Equation

Only the phase is modified in the frequency domain

$$E_2(\omega) = e^{i\Phi(\omega)} E_1(\omega) \approx e^{i\Phi(\omega_0)} \left\{ 1+i\left[\Phi(\omega) - \Phi(\omega_0)\right]\right\} E_1(\omega)$$

Expand effective phase in a Taylor's series where internal phase is a perturbation

$$\Phi(\omega) - \Phi(\omega_0) \approx \Phi'(\omega_0)\,\Delta\phi + 1/2\,\Phi''(\omega_0)\,\Delta\phi^2 + \ldots$$

Include linear and nonlinear perturbations

$$\Delta\phi = T\Delta\omega + \gamma L \mathcal{B}\,|E_1|^2$$

Expand transmitted field with detuning and nonlinearity as perturbations

$$E_2(\omega) = e^{i\Phi(\omega_0)} \left\{ 1 + i\mathcal{B}\left[\, T\Delta\omega + \gamma L \mathcal{B}\,|E_1|^2 \,\right] + i/2\ \mathcal{B}'\left[\, T\Delta\omega + \gamma L \mathcal{B}\,|E_1|^2 \,\right]^2 + \ldots \right\} E_1(\omega)$$

Fourier Transform to time domain to relate output pulse envelope to that of input

$$A_2(t) = A_1(t) - \mathcal{B}\,T\,{}^{\partial}\!/_{\partial t}\,A_1(t) + i\gamma L \mathcal{B}^2 |A_1(t)|^2 A_1(t) - i/2\ \mathcal{B}' T^2\,{}^{\partial^2}\!/_{\partial t^2}\,A_1(t) + \ldots$$

Next take the continuum limit of distributed resonators...

Nonlinear Schrodinger Equation (NLSE) Limit

NLSE:

$$\frac{\partial A}{\partial z} = -\frac{ib_2}{2}\frac{\partial^2 A}{\partial t^2} + ig\left|A\right|^2 A$$

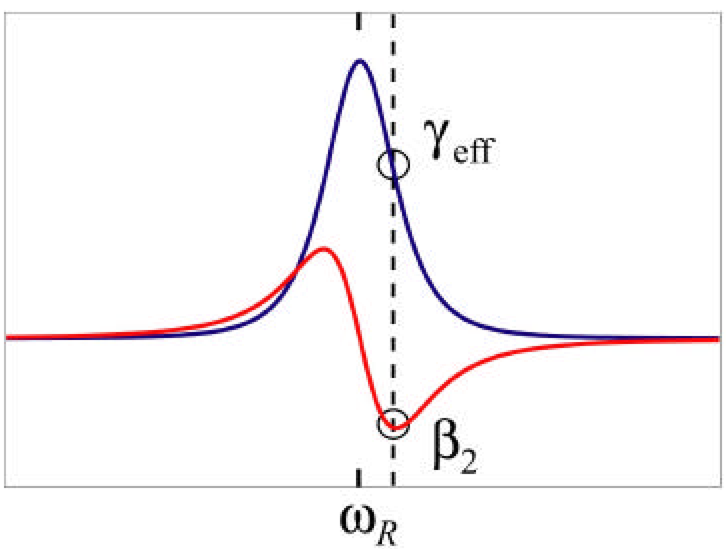
Fundamental Soliton Solution:

$$A(z,t) = A_0 \operatorname{sech}(t/T_P)\, e^{i\frac{1}{2}g\left|A_0\right|^2 z}$$

soliton amplitude

$$A_0 = \sqrt{\frac{|\beta_2|}{\gamma\, T_P^2}} = \sqrt{\frac{T_R^2}{\sqrt{3}\,\gamma\, 2\pi R\, T_P^2}}$$

adjustable by controlling ratio of transit time T_R to pulse width T_P



An enhanced nonlinearity may be balanced by an induced anomalous dispersion at some detuning from resonance to form solitons

The characteristic length scale for nonlinear pulse evolution (soliton period) may as small as the distance between resonator units!

"Slow light, induced dispersion, enhanced nonlinearity, and optical solitons in a resonator-array waveguide"
J. E. Heebner, R. W. Boyd, and Q. Park, Phys. Rev. E, 65 (2002)

SCISSOR Solitons

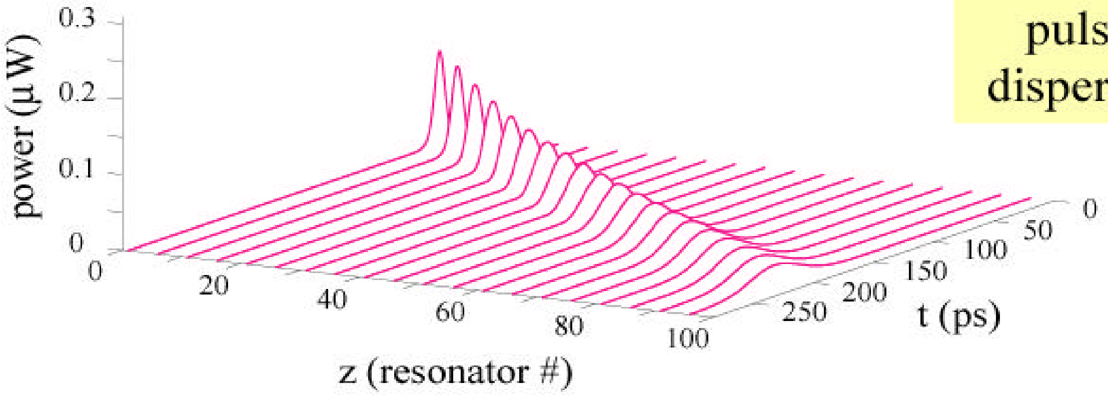
5 μm diameter resonators with a finesse of 30

SCISSOR may be constructed from 100 resonators spaced by 10 μm for a total length of 1 mm

soliton may be excited via a 10 ps, 125mW pulse

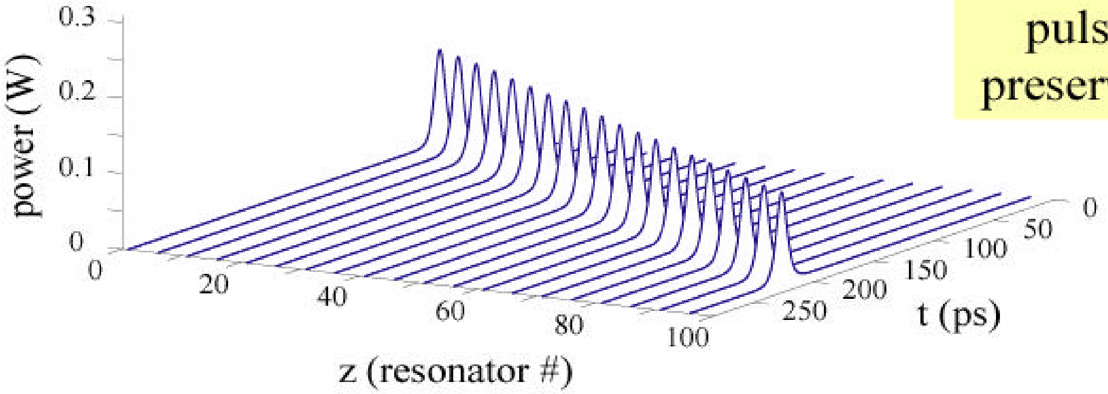
simulation assumes a chalcogenide/AlGaAs-like nonlinearity

Weak Pulse



pulse disperses

Fundamental Soliton

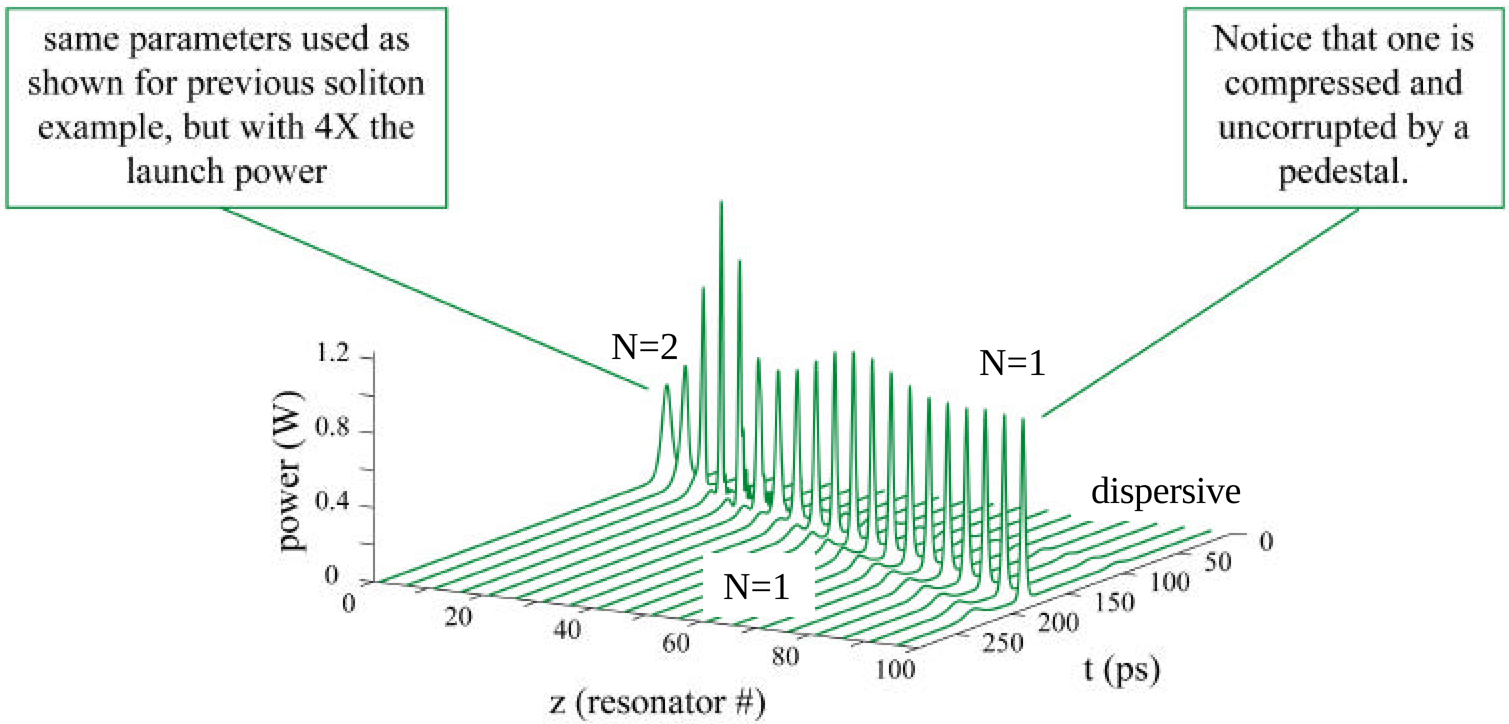


pulse preserved

Soliton Splitting & Pulse Compression

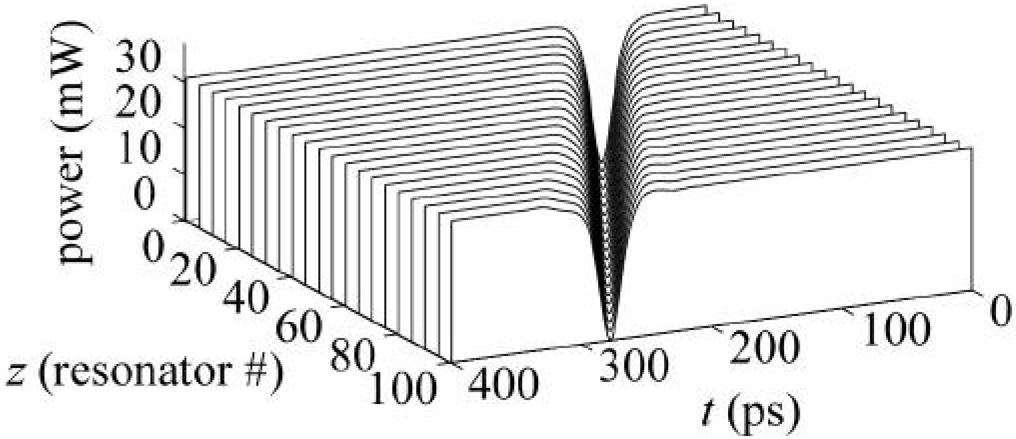
The dispersive nature of the nonlinear enhancement (self-steepening) leads to an intensity-dependent group velocity which splits an N-order soliton into N fundamental solitons of differing peak intensities and widths.

Here, a 2^{nd} - order "breathing" soliton splits into 2 fundamental solitons:



"SCISSOR Solitons & other propagation effects in microresonator modified waveguides"
J. E. Heebner, R. W. Boyd, and Q. Park, JOSA B, 19 (2002)

Other Exotic Nonlinear Effects

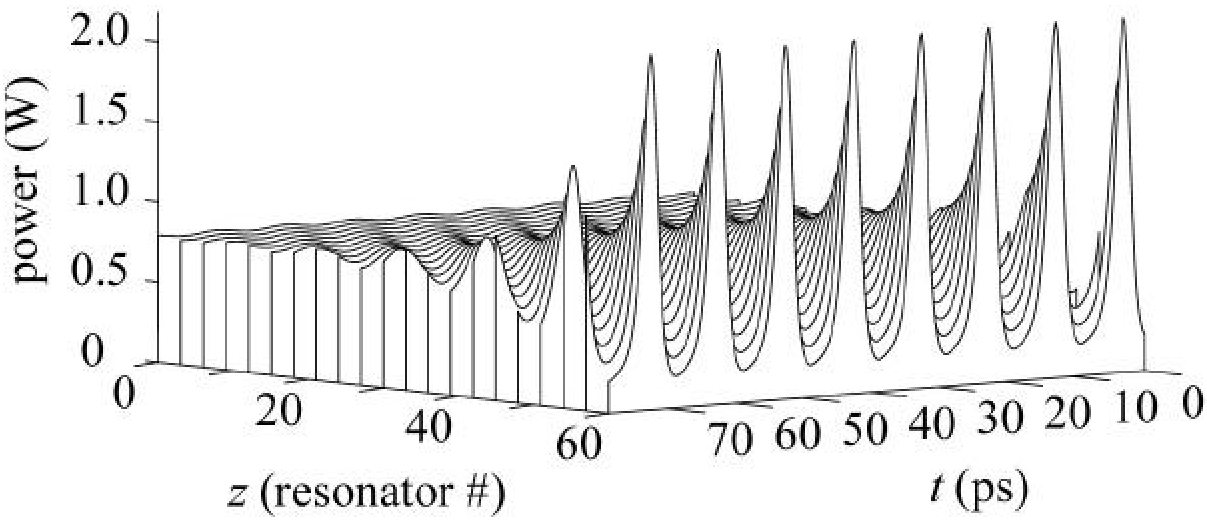


dark solitons
non-dispersing
intensity dips

modulation instability

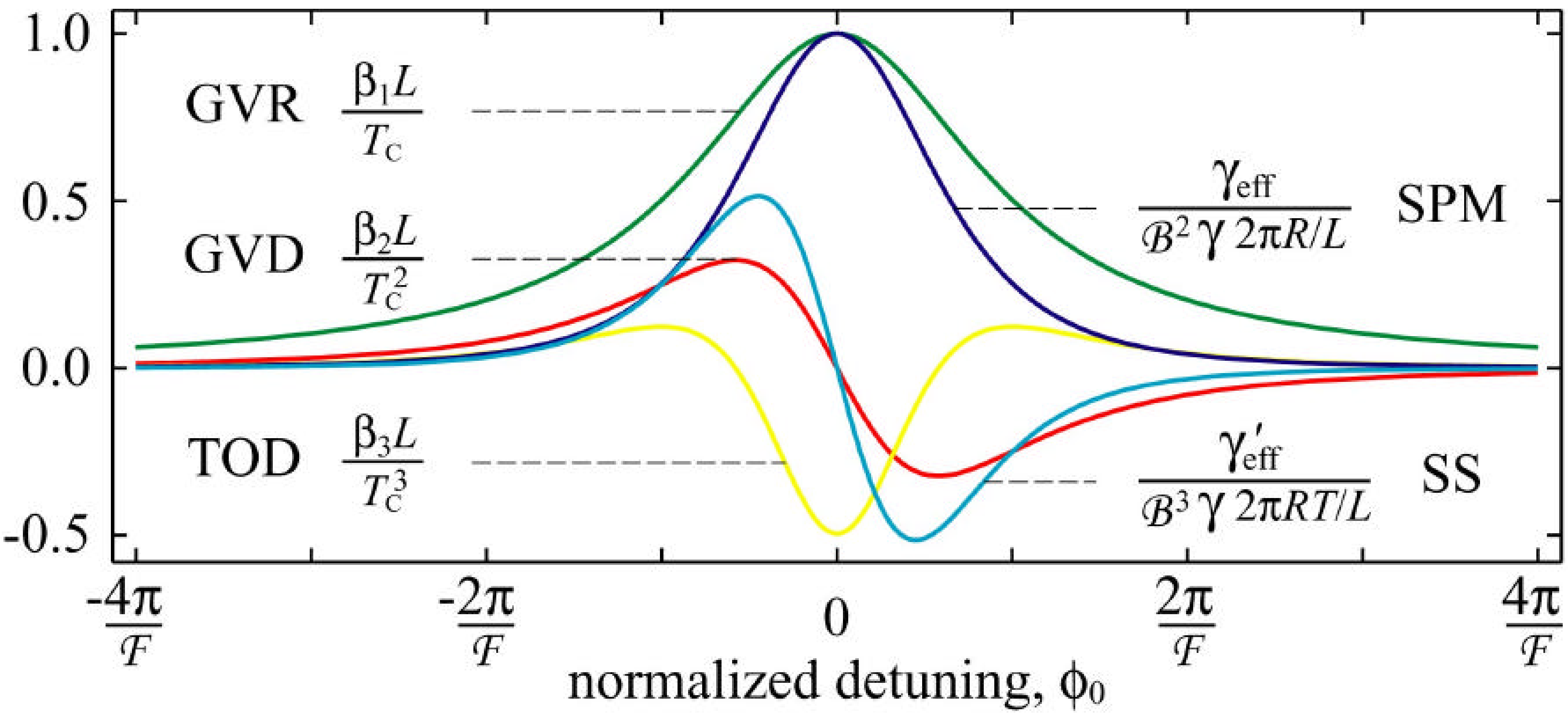
or

four wave mixing



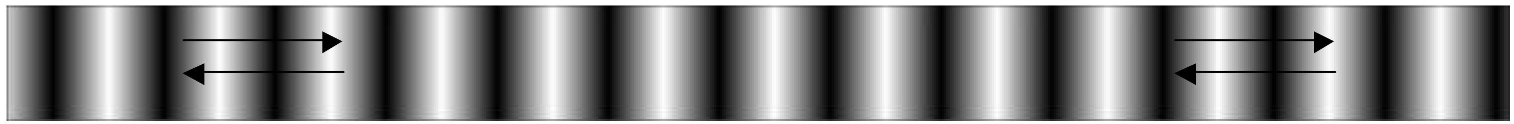
Engineerable Parameters

- The dispersive and nonlinear behavior of microresonator-modified waveguides can be engineered and/or even controlled in real-time via electro-optic / thermo-optic means.
- Linear: a) group velocity, b) group velocity dispersion, c) third order dispersion
- Nonlinear: d) self-phase modulation, e) self-steepening



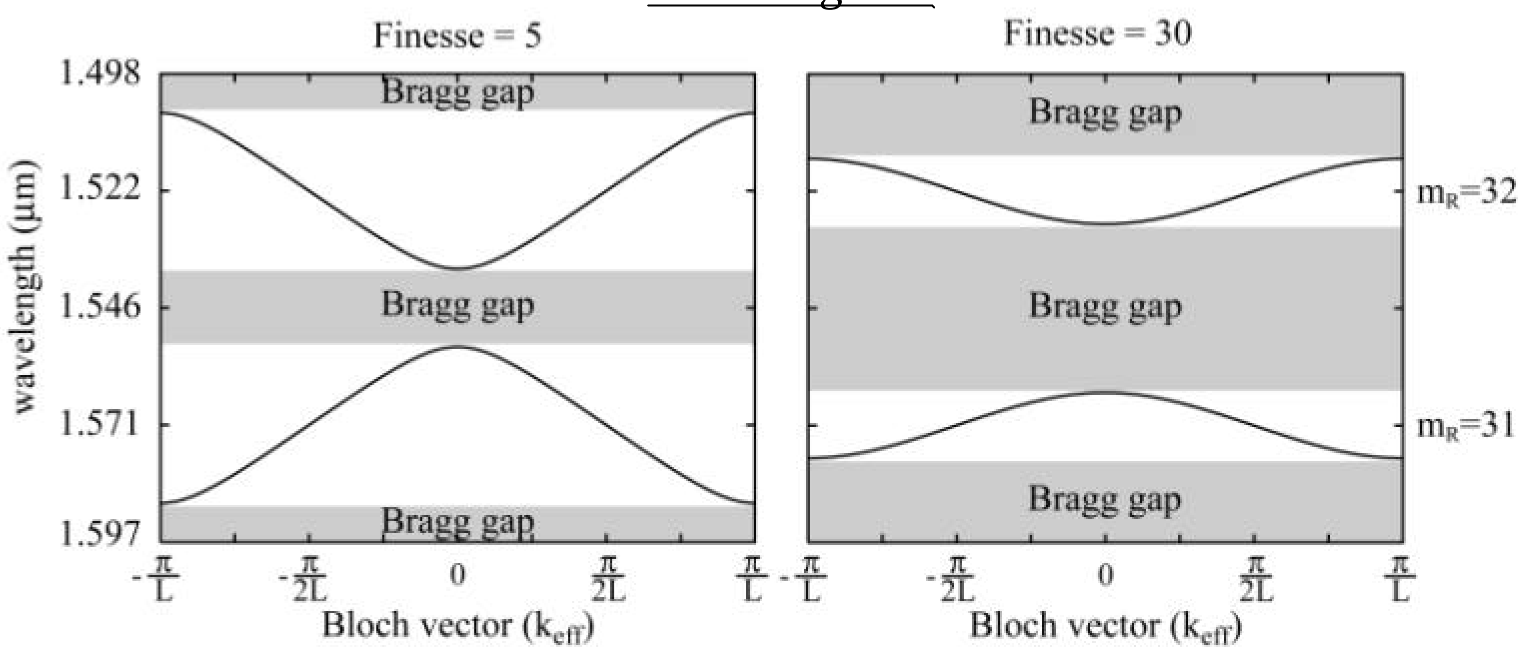
Bragg Stacks and CROWs

Bragg grating / multi-layer stack



1-D Coupled Resonator Optical Waveguide (Yariv)

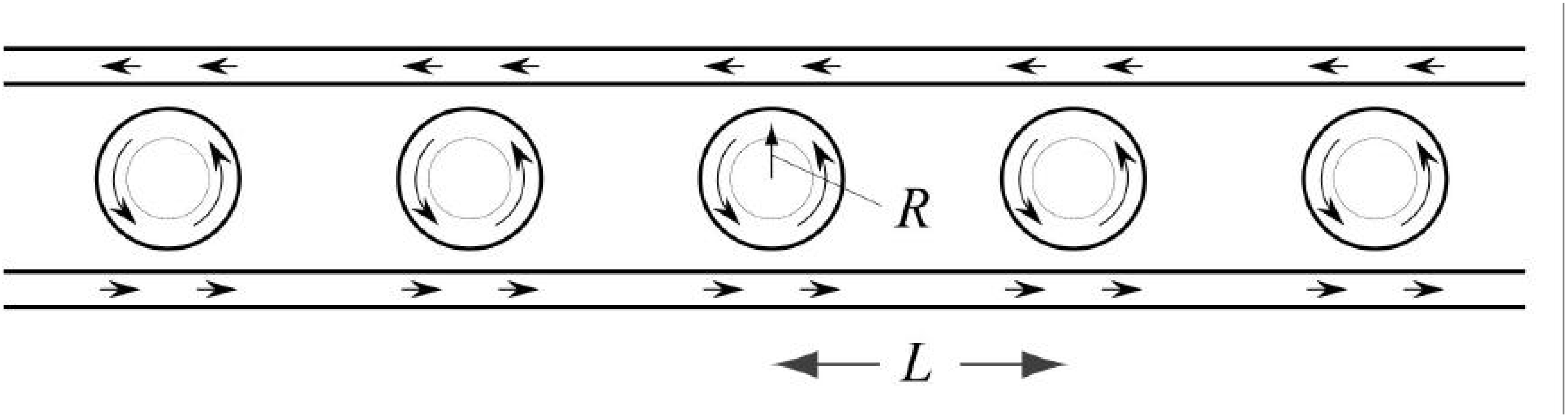
Band diagrams



mathematically equivalent structures
- *no new physics is introduced*

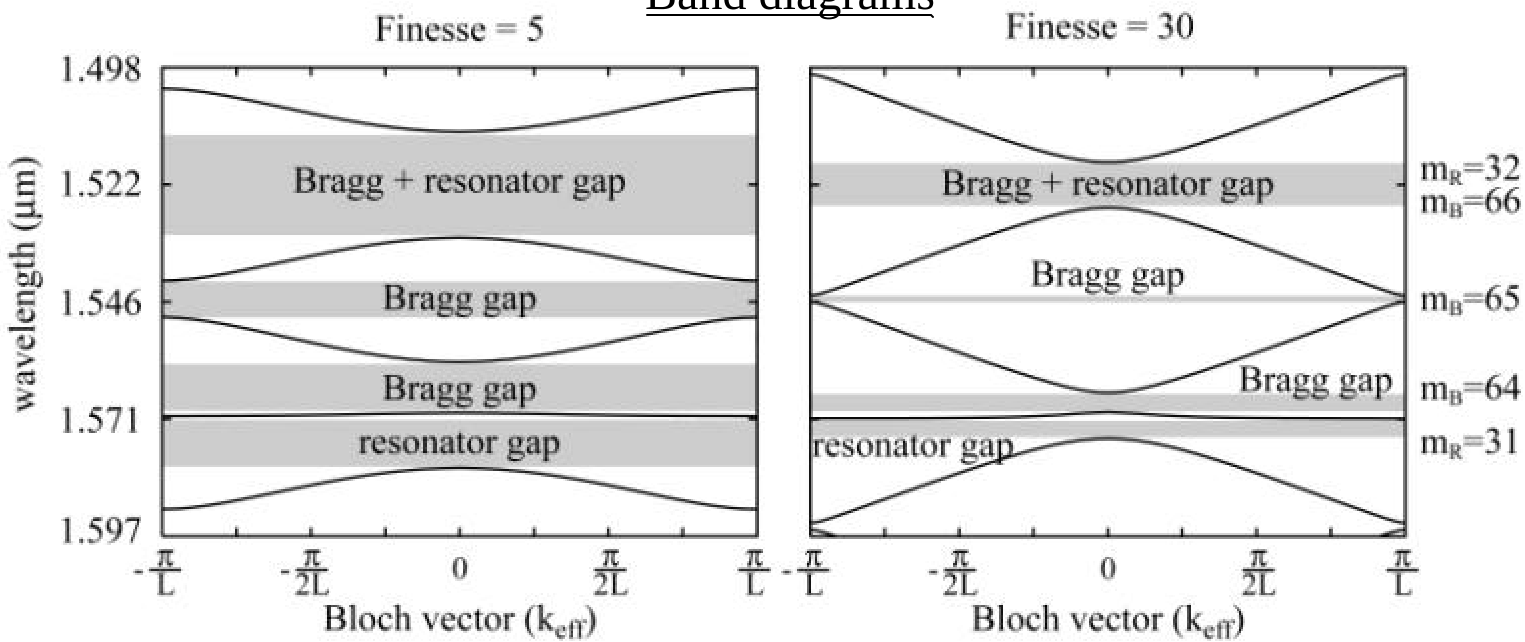
Double-Channel SCISSORs

- The addition of a second waveguide fundamentally and qualitatively alters the guiding properties of a single-guide SCISSOR
- The possibility for inter-resonator feedback and contradirectional coupling is introduced
- This structure can possess a photonic bandgap (PBG) with controllable parameters



Double-Channel SCISSORs

Band diagrams



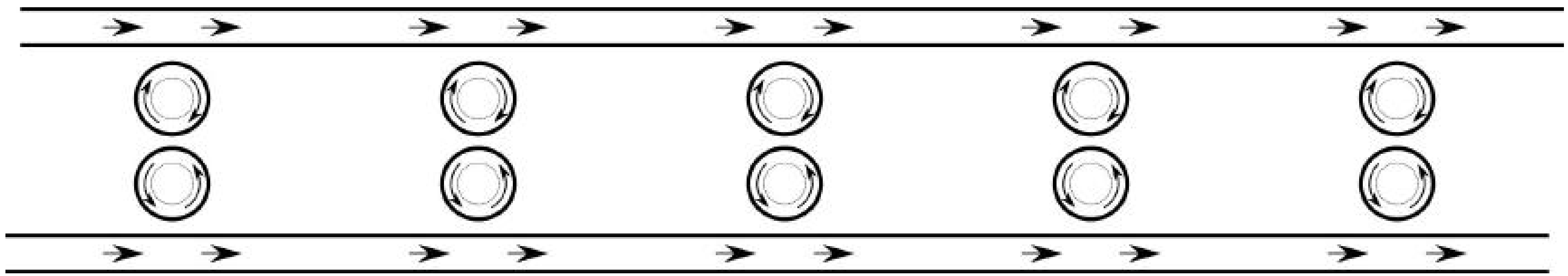
Photonic band-gaps:
– correspond to dropped channels
– resonator gaps due to intra-resonances
– Bragg gaps due to inter-resonances

Flat bands:
– low group velocity
– low dispersion
Ideal for delay lines

"Gap solitons in a two-channel SCISSOR structure"
S. Pereira, J. E. Sipe, J. E. Heebner, and R. W. Boyd, Optics Letters, 27 (2002)

“Twisted” Double-Channel SCISSORs

Simple forward-only coupling between guides
No photonic bandgaps
Has analogies with vector solitons



Structure behaves like a resonator-enhanced directional coupler

Loss-Limited Finesse

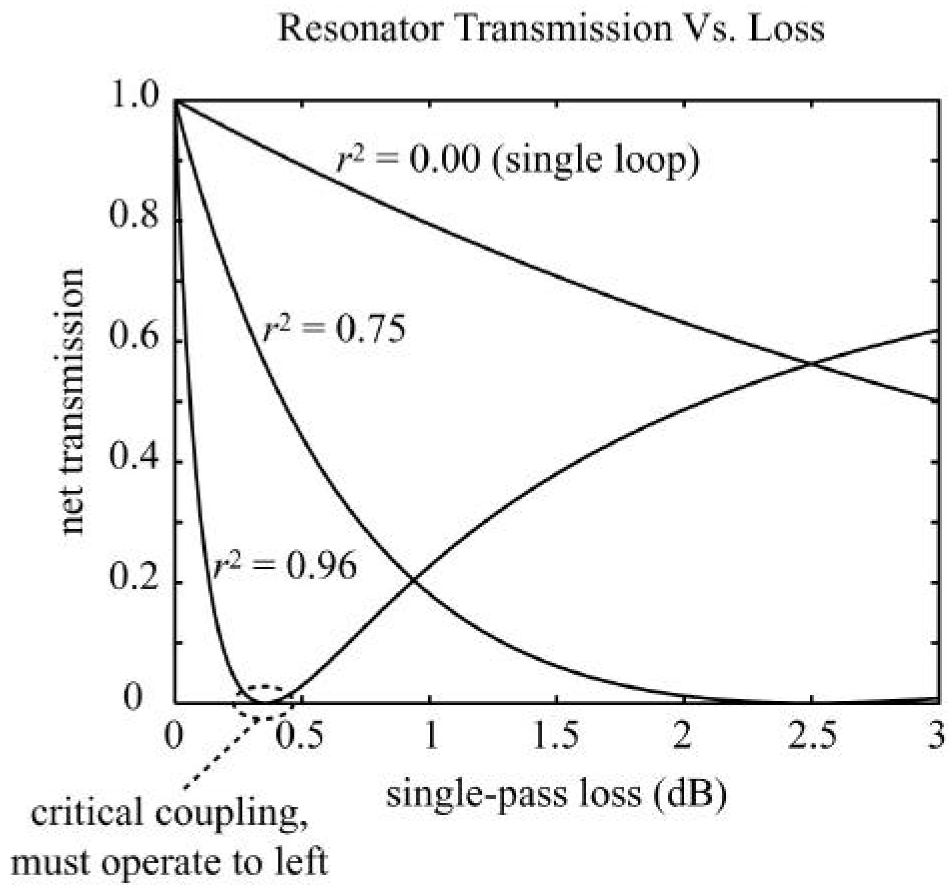
When the single-pass loss, $\alpha 2\pi R$ is high enough to be nearly equal to the cross-coupling coefficient, t^2 , the net transmission through the resonator is poor.
When the two quantities are equal, net transmission is zero (critically-coupled).
In general, a resonator based switch design requires over-coupling ($\alpha 2\pi R< t^2$)

Silica SMF	$\alpha \sim$ 0.2 dB/km
Air-clad AlGaAs	$\alpha \sim$ 1 dB/mm

This translates to an upper boundary on the finesse:

$$\mathcal{F} < \frac{10}{\ln 10\, a_{\mathrm{dB}} R}$$

For a 5 micron diameter high-contrast AlGaAs resonator, finesse limit ~1000

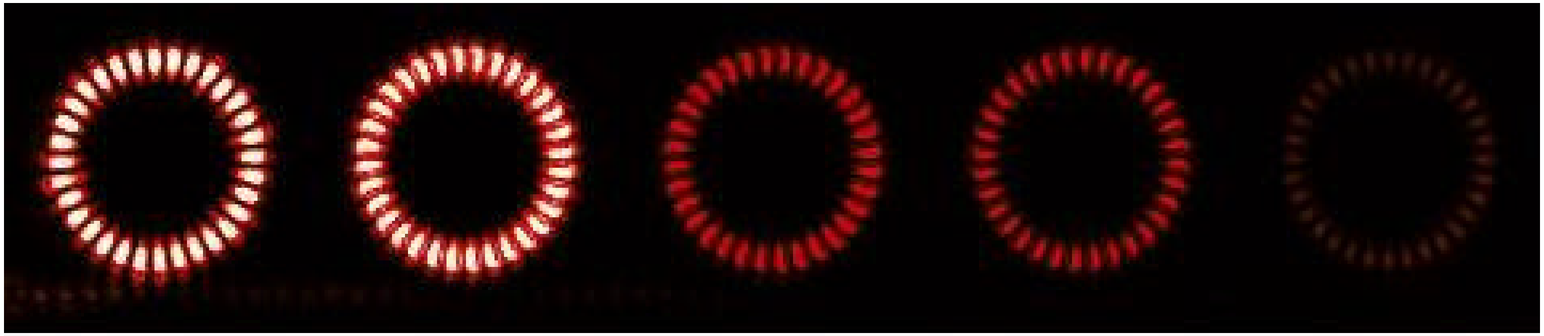


Scattering Losses in a SCISSOR

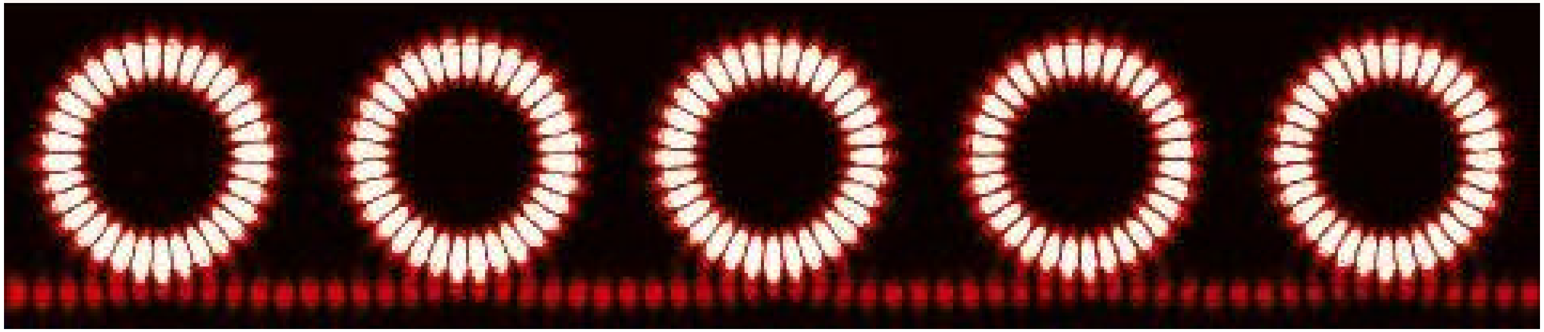
Attenuation in high index contrast waveguides is typically dominated by scattering due to edge roughness resulting from etch processes which in practice cannot produce perfectly smooth sidewalls.

RMS <u>roughness:</u>

60 nm

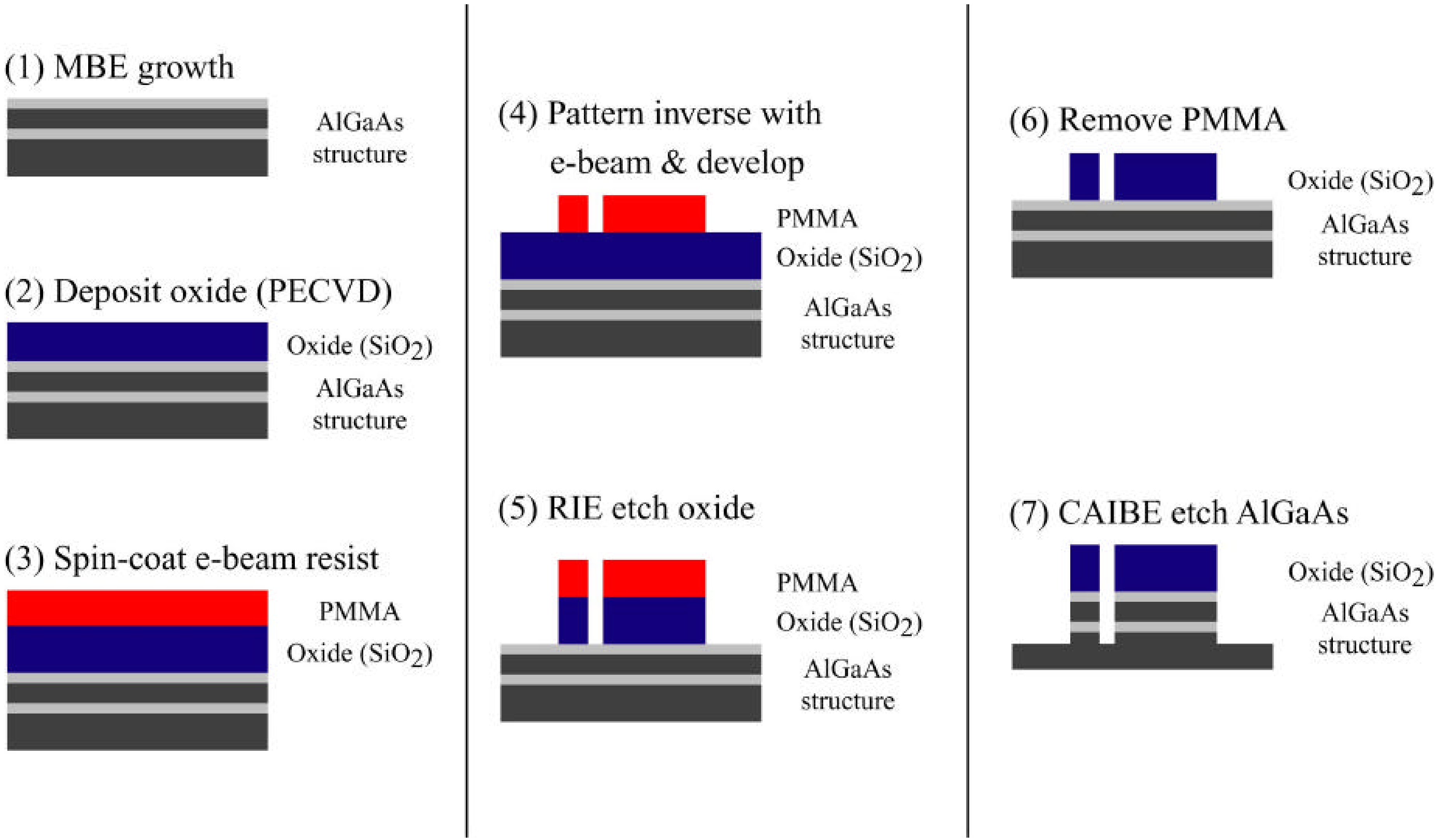


30 nm



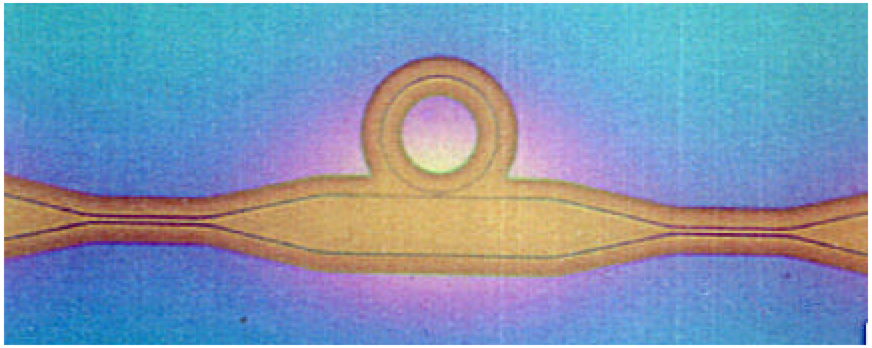
Attenuation in an N-resonator SCISSOR $\alpha_{eff} \sim \alpha \, N \, \mathcal{F} \, 2\pi R/L$

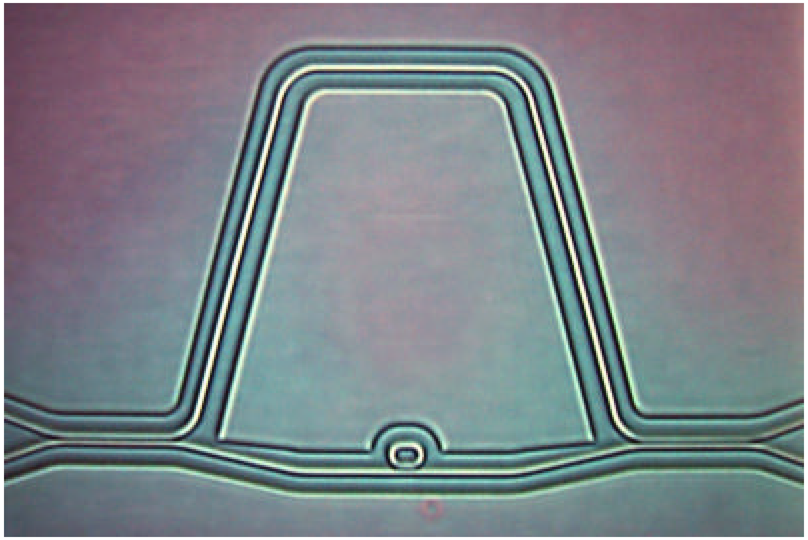
Nanofabrication Process

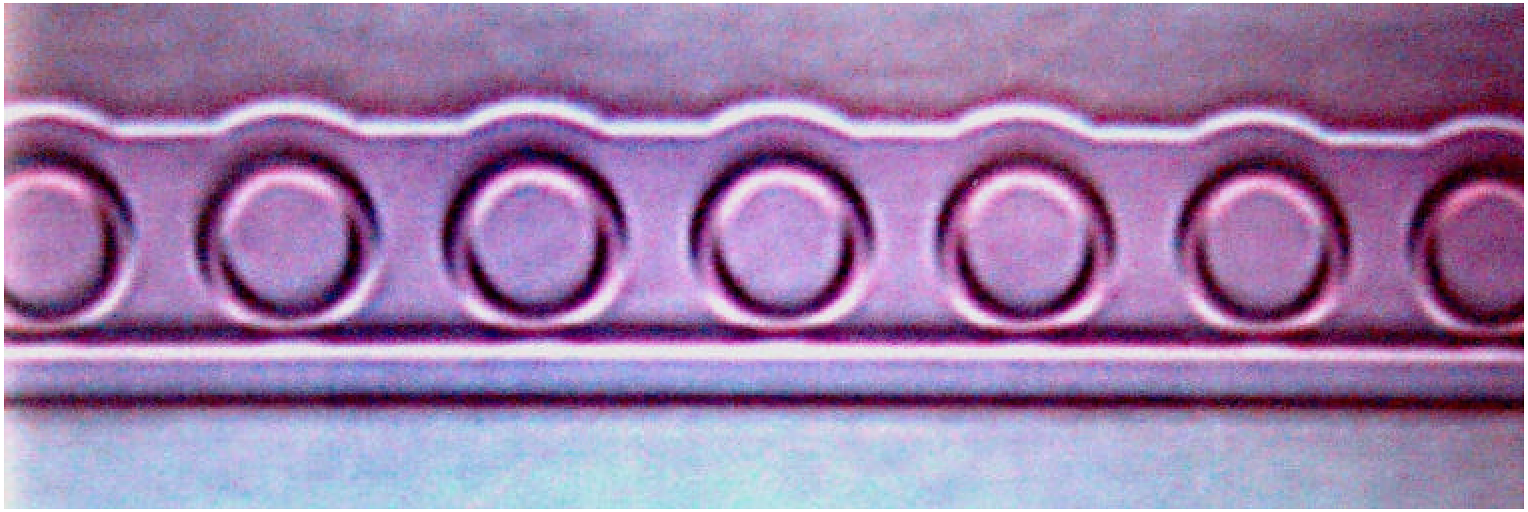


- MBE vertical growth done in Rochester (Dr. Gary Wicks)
- Lateral patterning processes done at Cornell Nanofabrication Facility (CNF)

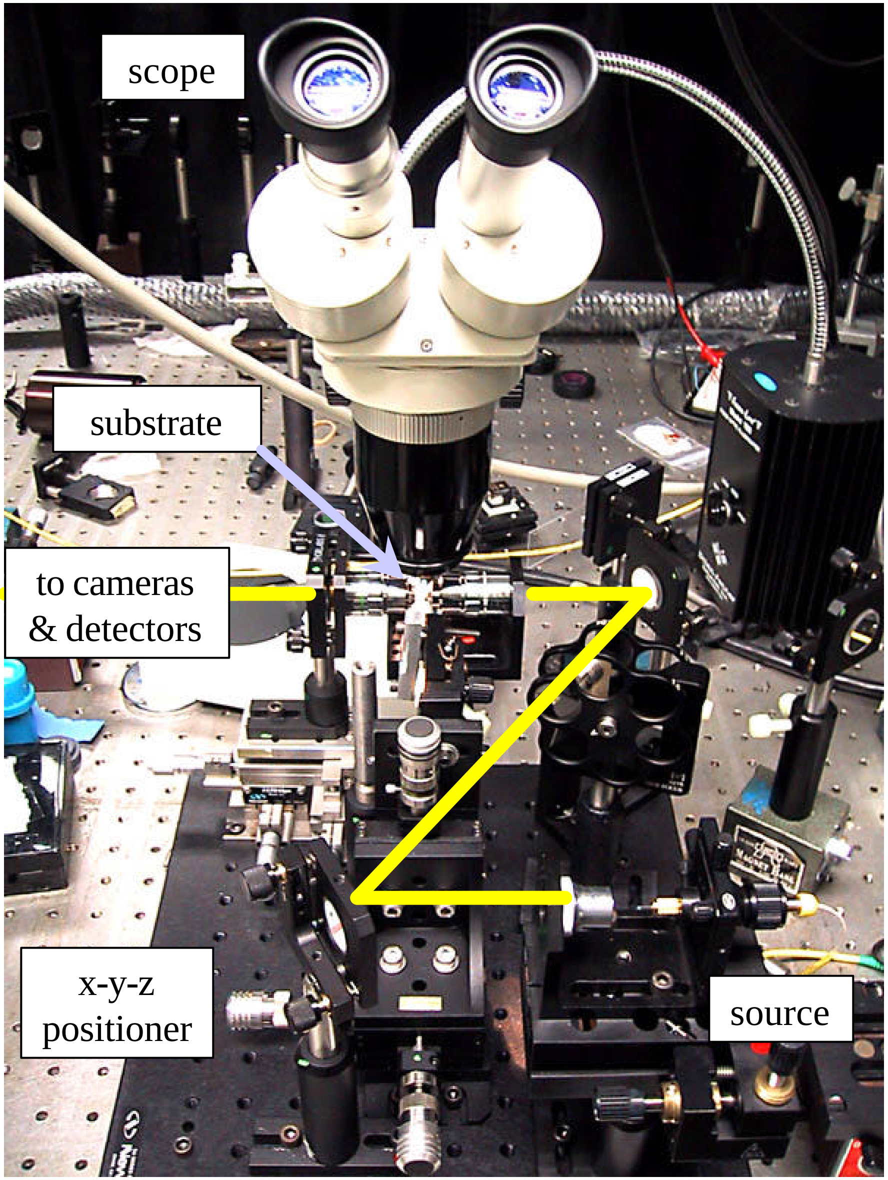
Patterned Structures







Waveguide Coupling Setup

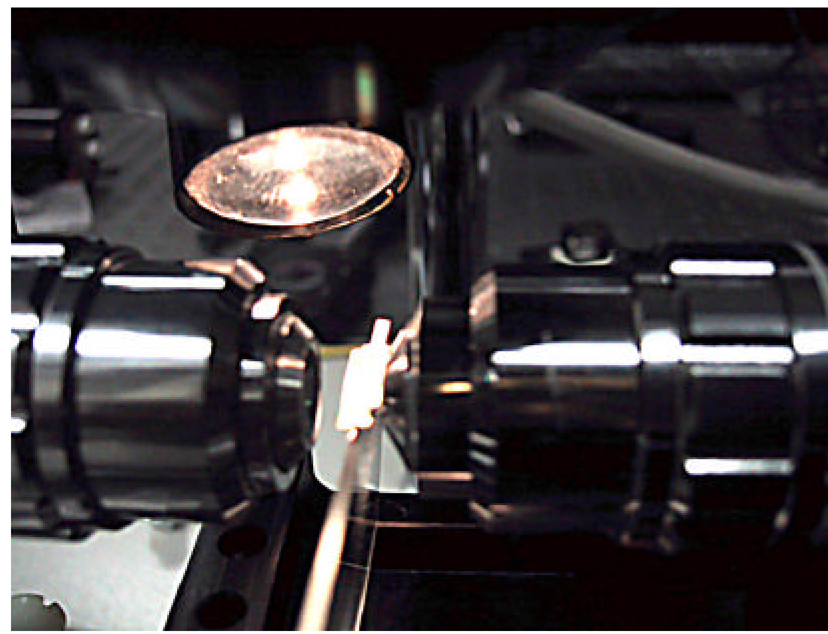


<u>Sources:</u>

Tunable (1530-1570nm) Modelocked Fiber Laser
1ps, 10 kW peak power

Tunable (400-1800nm) Nd YAG Pumped OPG
25ps, 1 MW peak power

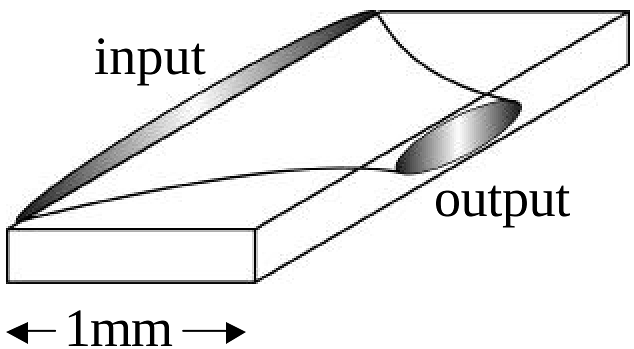
High index-contrast guides with N.A.>1 require high N.A. objectives to mode-match the free-space spot size to the mode field



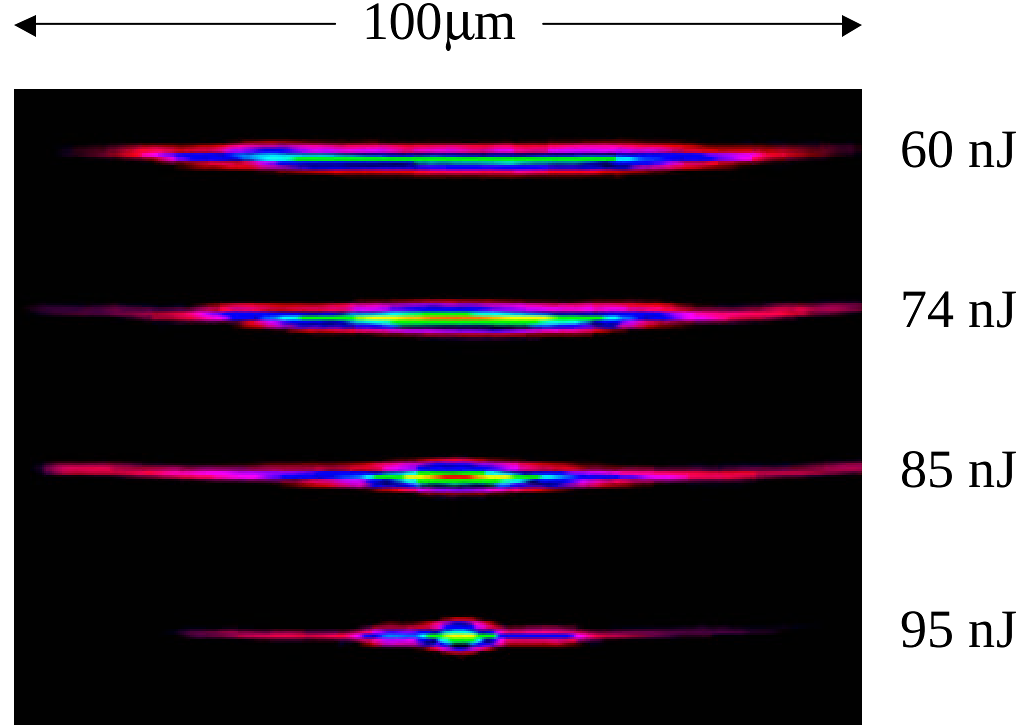
Nonlinear Transverse Self-Focusing

for characterizing the nonlinearity

geometry:



AlGaAs planar waveguide, λ=1.51μm



exiting intensity profile for increasing pulse energies (τ = 25ps)

Conclusions

- Studied the nonlinear phase transfer characteristics of microresonators
- All-optical switching thresholds may be reduced without compromising bandwidth by shrinking resonator size
- Demonstrated numerically, the propagation of SCISSOR solitons based on a balance between resonator enhanced nonlinearities and resonator induced group-velocity dispersion.
- SCISSOR structures allow the possibility for controllable nonlinear pulse evolution on a chip
  - Pulse compression in an integrated device
  - Optical Time Division Multiplexing (OTDM)
- In the process of testing several resonator-enhanced Kerr switches and SCISSORs grown in AlGaAs

Acknowledgements & Publications

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Dr. Q-Han Park

Dr. Nick Lepeshkin

Aaron Schweinsberg

- **"SCISSOR Solitons & other propagation effects in microresonator modified waveguides"**
  J. E. Heebner, R. W. Boyd, and Q. Park, JOSA B, 19 (2002)
- **"Slow light, induced dispersion, enhanced nonlinearity, and optical solitons in a resonator-array waveguide"**
  J. E. Heebner, R. W. Boyd, and Q. Park, Phys. Rev. E, 65 (2002)
- **"Gap solitons in a two-channel SCISSOR structure"**
  S. Pereira, J. E. Sipe, J. E. Heebner, and R. W. Boyd, Optics Letters, 27 (2002)
- **"Beyond the absorption-limited nonlinear phase shift with microring resonators"**
  S. Blair, J. E. Heebner, and R. W. Boyd, Optics Letters, 27 (2002)
- **"Sensitive disk resonator photonic biosensor"**
  R. W. Boyd and J. E. Heebner, Applied Optics, 40, pp. 5742-5747, (2001)
- **"Enhanced All-Optical Switching Using a Nonlinear Fiber Ring Resonator"**
  J. E. Heebner and R. W. Boyd, Optics Letters, 24, pp.847-849, (1999)