

## Excess noise acquired by a laser beam after propagating through an atomic-potassium vapor

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We have found that an intense shot-noise-limited laser beam tuned near the  $4^2S_{1/2} \leftrightarrow 4^2P_{3/2}$  potassium resonance transition acquires excess noise after passing through an atomic-potassium vapor cell. The noise is maximum for laser detunings of approximately  $\pm 1$  GHz and falls to nearly the shot-noise limit for detunings greater than  $\pm 3$  GHz. We describe the production of this noise in terms of a forward four-wave mixing process involving the laser field and its side modes, which are initially in the vacuum state. We present a fully quantum-mechanical theory of forward four-wave mixing in a system of two-level atoms and use it to predict the noise properties of the transmitted laser beam. The predictions of this theory are in good agreement with the experimental data.

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### I. INTRODUCTION

In this paper, we present the results of our experimental and theoretical investigation of the noise characteristics of a laser beam that has passed through a resonant atomic vapor. In particular, we have found that even if the incident laser beam is free of noise in the sense that its photodetection statistics are shot-noise limited, the transmitted beam will contain significant amounts of excess noise (see Fig. 1). The excess noise observed in our experiments shares a common origin with that observed by Kauranen *et al.* [1]. In their experiments, the fluctuations of a weak probe beam were observed to increase above the shot-noise limit after passing through an atomic vapor where it intersected a strong degenerate pump beam.

The source of the excess noise can be understood by considering not only the incident laser beam with a frequency  $\omega_0$  but also its side modes, which are initially in the vacuum state. The situation is depicted in Fig. 2. When the intensity of the incident laser beam is

sufficiently large, the interaction of this beam with the potassium atoms can produce real photons at the side modes through the following processes. The first process is the spontaneous scattering of photons into the side modes due to the quantum fluctuations of the atomic medium [2,3]. The other process is connected with the semiclassical gain experienced by one or both of the side modes due to the nonlinear interaction between the carrier mode and the atomic medium. The statistical properties of the output field are a direct consequence of the quantum-mechanical commutation relations that must be obeyed by any optical field [4–9]. Since the side modes are now populated, the incident field and the side modes beat together at the photodetector to produce fluctuations in the detector photocurrent. We describe the production of this noise in terms of a forward four-wave mixing process involving the carrier mode of the field and the side modes which are initially in the vacuum state. The predictions for the quantum-noise properties of the detector photocurrent as determined from a fully quantum-mechanical treatment of forward four-wave mixing are presented in Sec. III.

It should be noted that the excess noise observed in our

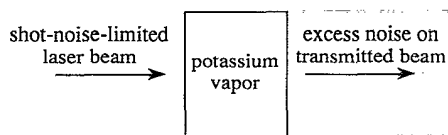


FIG. 1. Noise properties of a beam after passing through an atomic vapor.

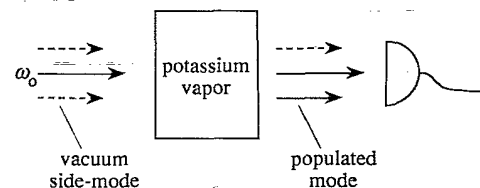


FIG. 2. Laser beam at a frequency  $\omega_0$  with its vacuum side modes pass through atomic-potassium vapor. Real photons are produced at one or both of the vacuum side modes through forward four-wave mixing which in turn produces excess noise in the detector photocurrent.

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experiments differs in origin from that observed previously in the context of FM spectroscopy. In FM spectroscopy, phase fluctuations on an incident laser beam are converted to amplitude fluctuations by the response of an atomic vapor [10–12]. For example, McIntyre and co-workers [12] observed that the frequency noise inherent in a beam from a broad-bandwidth diode laser is converted into amplitude fluctuations after it passes through an atomic vapor. Their measurements were performed at spectrum analyzer frequencies that were much lower than the diode-laser linewidth and for laser intensities that were smaller than the saturation intensity. When the laser linewidth was reduced, the intensity fluctuations they observed decreased dramatically. Their experiment differs from those presented in this paper in that the linewidth of our laser was much smaller than the spectrum analyzer frequencies used to perform the measurements and in that our laser intensity was much larger than the saturation intensity.

The fact that an intense beam of light acquires large amounts of noise in passing through an atomic medium has important consequences for any nonlinear optical process that utilizes the response of free atoms. In particular, the implications of this process on the possibility of producing squeezed light by forward four-wave mixing has been noted earlier by Maeda and co-workers [13–15].

## II. EXPERIMENTAL RESULTS

The setup used to measure the noise properties of the transmitted laser beam is shown in Fig. 3. The output of a Coherent 699-21 frequency-stabilized ring dye laser operating at a wavelength of 767 nm with a bandwidth of 500 kHz was focused into a 5-mm-long potassium cell. The potassium number density in the cell was approximately  $10^{13}$  atoms/cm<sup>3</sup>, which corresponds to a temperature of approximately 150°C. A low number density was

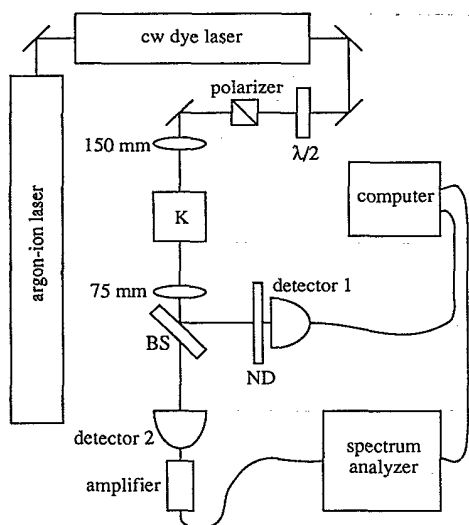


FIG. 3. Experimental setup used to measure the noise properties of the transmitted laser beam.

chosen to ensure that homogeneous broadening was dominated by radiative effects and not the effects of atomic collisions. Theoretical analysis has shown that noise processes should be minimized under such conditions [16]. The laser beam was linearly polarized and its intensity was controlled with a half-wave plate and a polarizer. The beam was focused into the cell with a 150-mm focal length lens. The transmitted light was collected by a 75-mm focal length lens placed 140 mm behind the cell and was focused onto the two photodiode detectors. The average power was measured using detector 1 and the noise properties of the transmitted laser beam were measured using detector 2 and a rf-spectrum analyzer. The detection system was shot-noise limited for laser powers of up to 10 mW falling on detector 2 and was able to measure the rms noise for spectrum analyzer frequencies between 10 and 100 MHz.

Our experimental results are shown in Fig. 4. These plots show the measured rms intensity noise in a narrow spectral band centered on 10 MHz and the noise level of a shot-noise-limited beam with an intensity equal to that of the transmitted laser beam plotted as functions of the

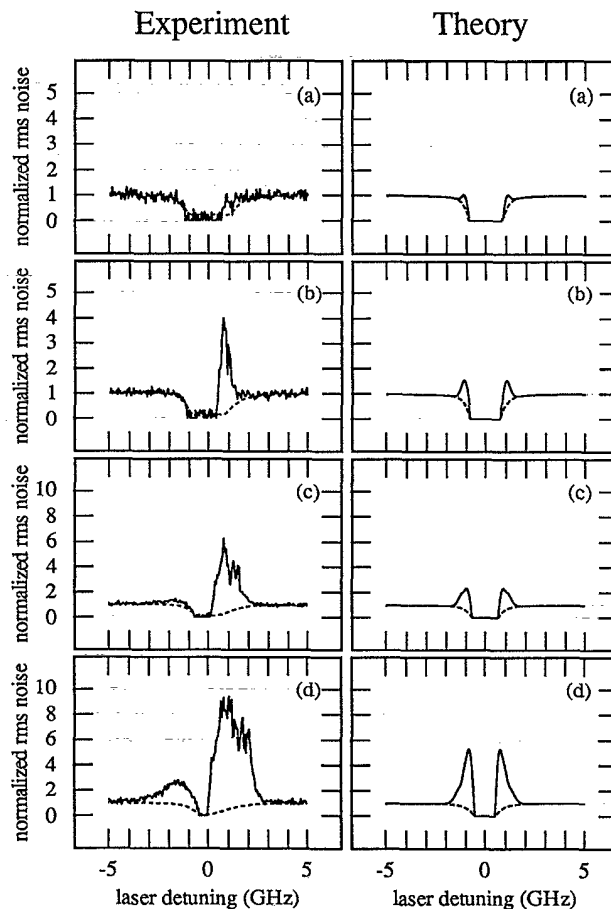


FIG. 4. Experimental measurements and theoretical predictions of the transmitted laser beam shot-noise limit (---) and normalized rms noise (—) at 10 MHz for laser intensities of (a) 1.9 W/cm<sup>2</sup>, (b) 3.8 W/cm<sup>2</sup>, (c) 7.5 W/cm<sup>2</sup>, and (d) 15 W/cm<sup>2</sup>.

laser detuning from resonance. The shot-noise limit is calculated from the transmitted laser power measured with detector 1. The measured rms noise is normalized to the shot-noise level when the frequency of the laser beam is turned far off resonance. The measurements were performed for four laser intensities. At an intensity of 1.9 W/cm<sup>2</sup>, the rms noise is equal to the shot noise for all detunings except at approximately 1 GHz on the blue side of resonance where a small amount of excess noise appears. As the intensity of the laser beam increases, this excess noise feature also increases. When the intensity of the laser reaches 7.5 W/cm<sup>2</sup>, a small amount of excess noise begins to appear at a detuning of approximately 1 GHz on the red side of resonance. This excess noise feature continues to increase as the intensity of the laser beam increases. The noise is maximum for laser detunings of approximately  $\pm 1$  GHz and approaches the shot-noise level for detunings greater than  $\pm 3$  GHz.

The amount of excess noise on the red side of resonance is less than that on the blue side. This difference is due presumably to self-focusing effects in the potassium vapor. On the red side of resonance, the laser beam is self-defocused, whereas on the blue side of resonance, the laser beam is self-focused. Thus, the excess noise is larger on the blue side of resonance because the laser-beam intensity is larger. It was also observed that for laser intensities greater than 5 W/cm<sup>2</sup>, the beam undergoes catastrophic self-focusing on the blue side of resonance causing the transmitted laser beam to break into as many as a dozen filaments. The structure of the excess noise peak on the blue side of resonance is due to this filamentation process. In fact, this noise peak would be expected to be much larger if it were not for filamentation.

Excess noise was also observed for spectrum analyzer frequencies greater than 10 MHz. The amount of excess noise we observed was greatest at 10 MHz and decreased monotonically to the shot-noise level at approximately 60 MHz. Except for the magnitude of the noise, the characteristics of the excess noise as a function of laser detuning at these higher spectrum analyzer frequencies were similar to those observed at 10 MHz.

### III. QUANTUM THEORY OF FORWARD FOUR-WAVE MIXING

In this section, we present a quantum theory of vacuum side-mode amplification due to forward four-wave mixing in a two-level atomic system [2,3]. We compare the predictions of this theory with the experimental results. The interaction between the two-level atoms (with

an energy level separation of  $\hbar\omega_{ba}$ ) and the fields are treated in the electric-dipole approximation. All fields are assumed to propagate in the positive  $x$  direction and to be polarized in the  $z$  direction. The interaction Hamiltonian  $\hat{V}$  is given by

$$\hat{V} = - \int \hat{\mathbf{P}}(\mathbf{r}) \cdot \mathbf{E}_0(\mathbf{r}, t) d^3r - \int \hat{\mathbf{P}}(\mathbf{r}) \cdot \hat{\mathbf{E}}_{\text{vac}}(\mathbf{r}, t) d^3r, \quad (1a)$$

where the incident field  $\mathbf{E}_0$  at frequency  $\omega_0 = \omega_{ba} + \Delta$  given by

$$\mathbf{E}_0(\mathbf{r}, t) = \hat{\mathbf{z}} E_0 e^{ik_0 x - i\omega_0 t} + \text{c.c.} \quad (1b)$$

is treated classically and the field  $\hat{\mathbf{E}}_{\text{vac}}$  for a single pair of vacuum side modes given by

$$\hat{\mathbf{E}}_{\text{vac}}(\mathbf{r}, t) = \hat{\mathbf{z}} (L_x L_y L_z)^{-1/2} [\beta_s \hat{a} e^{ik_s x - i\omega_s t} + \beta_c \hat{b} e^{ik_c x - i\omega_c t}] + \text{H.a.} \quad (1c)$$

is quantized in the volume  $L_x L_y L_z$ . The annihilation operators  $\hat{a}$  and  $\hat{b}$  are arbitrarily called the signal and conjugate, respectively, and have frequencies  $\omega_s = \omega_0 + \delta_s$  and  $\omega_c = \omega_0 - \delta_c$ . The frequencies of the signal and conjugate are related by  $\omega_c = 2\omega_0 - \omega_s$  (i.e., conservation of energy) or in terms of the signal and conjugate detunings by  $\delta_c = \delta_s$ . The normalization constants  $\beta_\alpha$  are given by  $\beta_\alpha = i\sqrt{2\pi\hbar\omega_\alpha}$  where  $\alpha = s, c$ . The polarization operator  $\hat{\mathbf{P}}(\mathbf{r})$  is related to the dipole moment operator  $\hat{\boldsymbol{\mu}}^{(q)}$  for an atom at the position  $\mathbf{R}^{(q)}$  through the expression

$$\hat{\mathbf{P}}(\mathbf{r}) = \sum_q \delta(\mathbf{r} - \mathbf{R}^{(q)}) \hat{\boldsymbol{\mu}}^{(q)} \quad (2)$$

with the summation extending over all of the atoms in the interaction region.

The total Hamiltonian is given by  $\hat{H} = \hat{H}_A + \hat{H}_F + \hat{V}$  where  $\hat{H}_A$  and  $\hat{H}_F$  are the unperturbed Hamiltonians for the atomic system and the field, respectively. The equation of motion for the total density operator  $\hat{\rho}$  is determined from the Heisenberg equation

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}] + \left[ \frac{\partial \hat{\rho}}{\partial t} \right]_{\text{relax}}, \quad (3)$$

where the effects of atomic relaxation are contained in the quantity  $(\partial \hat{\rho} / \partial t)_{\text{relax}}$ . The equation of motion for the field density operator  $\hat{\rho}_F$  is determined from the total density operator by tracing  $\hat{\rho}$  over the atomic variables, that is,  $\hat{\rho}_F = \text{Tr}_A \hat{\rho}$ . The details of this calculation are given in Refs. [2 and 3]. The master equation for the fields is given by

$$\begin{aligned} \frac{\partial \hat{\rho}_F}{\partial t} = & -\frac{\beta_s^* \beta_c^* N}{2\hbar^2} (Q_{sc}^{++} (-i\delta_c) [\hat{a}^\dagger, \{\hat{b}^\dagger, \hat{\rho}_F\}] + Q_{cs}^{++} (-i\delta_s) [\hat{b}^\dagger, \{\hat{a}^\dagger, \hat{\rho}_F\}]) \\ & + C_{sc}^{++} (-i\delta_c) [\hat{a}^\dagger, \{\hat{b}^\dagger, \hat{\rho}_F\}] + C_{cs}^{++} (-i\delta_s) [\hat{b}^\dagger, \{\hat{a}^\dagger, \hat{\rho}_F\}] e^{i\Delta k x} \\ & -\frac{|\beta_s|^2 N}{2\hbar^2} (Q_{ss}^{+-} (i\delta_s) [\hat{a}^\dagger, \{\hat{a}, \hat{\rho}_F\}] + C_{ss}^{+-} (i\delta_s) [\hat{a}^\dagger, \{\hat{a}, \hat{\rho}_F\}]) \\ & -\frac{|\beta_c|^2 N}{2\hbar^2} (Q_{cc}^{+-} (i\delta_c) [\hat{b}^\dagger, \{\hat{b}, \hat{\rho}_F\}] + C_{cc}^{+-} (i\delta_c) [\hat{b}^\dagger, \{\hat{b}, \hat{\rho}_F\}]) + \text{H.a.}, \end{aligned} \quad (4)$$

where  $N$  is the atomic number density and  $\Delta k = 2k_0 - k_s - k_c$  is the wave-vector mismatch. The quantities  $C_{\alpha\beta}^{\pm\pm}(i\delta)$  and  $Q_{\alpha\beta}^{\pm\pm}(i\delta)$  are the Laplace transforms of specific linear combinations of two-time atomic-polarization correlation functions [3]. Physically,  $C_{\alpha\beta}^{\pm\pm}(i\delta)$  is proportional to the nonlinear susceptibility that appears in semiclassical theories. The quantity  $Q_{\alpha\beta}^{\pm\pm}(i\delta)$ , however, has no counterpart in semiclassical theories and represents the quantum fluctuations of the atomic system.

For the case of a two-level atom, the correlation functions  $C_{\alpha\beta}^{\pm\pm}(i\delta)$  and  $Q_{\alpha\beta}^{\pm\pm}(i\delta)$  are determined from the optical Bloch equations. First, the polarization operator is rewritten in the form

$$\hat{P}(\mathbf{r}) = \mu \hat{S}^+ \delta(\mathbf{r} - \mathbf{R}) + \text{H.a.}, \quad (5)$$

where  $\hat{S}^+$ , its adjoint  $\hat{S}^-$ , and  $S^z = \frac{1}{2}[\hat{S}^+, \hat{S}^-]$  obey the commutation relations for a spin- $\frac{1}{2}$  system. The quantity  $\mu$  is defined by  $\mu \equiv \langle \mu \rangle$ . Then, the matrix form of the Bloch equations for an atom located at position  $\mathbf{R}$  interacting with the pump field is given by

$$\frac{\partial \Phi}{\partial t} = \mathbf{M} \Phi + \Gamma, \quad (6a)$$

where the components of the Bloch vector  $\Phi$  and the vector  $\Gamma$  are

$$\Phi_1 = \langle S^+ \rangle e^{i(\mathbf{k}_0 \cdot \mathbf{R} - \omega_0 t)}, \quad \Phi_2 = \Phi_1^*, \quad \Phi_3 = \langle S^z \rangle, \quad (6b)$$

and

$$\Gamma_1 = \Gamma_2 = 0, \quad \Gamma_3 = -\frac{1}{2T_1}, \quad (6c)$$

respectively. The matrix  $\mathbf{M}$  is equal to

$$\mathbf{M} = \begin{bmatrix} -1/T_2 - i\Delta & 0 & i\Omega^* \\ 0 & -1/T_2 + i\Delta & -i\Omega \\ i\Omega/2 & i\Omega^*/2 & -1/T_1 \end{bmatrix}, \quad (7)$$

where  $\Delta \equiv \omega_0 - \omega_{ba}$  is the detuning of the pump laser from the atomic resonance, and the Rabi frequency  $\Omega$  is defined by  $\Omega \equiv 2\mu_z E_0(\mathbf{r})/\hbar$ . The correlation functions  $C_{\alpha\beta}^{\pm\pm}(i\delta)$  and  $Q_{\alpha\beta}^{\pm\pm}(i\delta)$  can then be calculated in terms of the solution of Eq. (6a) by using the quantum regression theorem. The results given in terms of the steady-state solution of the Bloch equations and the matrix  $\mathbf{U}$  are

$$C_{sc}^{++}(i\delta) = C_{cs}^{++}(i\delta) = |\mu|^2 T_2 \sum_l U_l(i\delta) (2\Phi_3, 0, -\Phi_2)_l, \quad (8a)$$

$$C_{ss}^{+-}(i\delta) = C_{cc}^{+-}(i\delta) = |\mu|^2 T_2 \sum_l U_l(i\delta) (0, -2\Phi_3, \Phi_1)_l, \quad (8b)$$

$$Q_{sc}^{++}(i\delta) = Q_{cs}^{++}(i\delta) = |\mu|^2 T_2 \sum_l U_l(i\delta) \times (1 - 2\Phi_1\Phi_2, -2\Phi_2\Phi_2, -2\Phi_2\Phi_3)_l, \quad (8c)$$

and

$$Q_{ss}^{+-}(i\delta) = Q_{cc}^{+-}(i\delta) = |\mu|^2 T_2 \sum_l U_l(i\delta) \times (-2\Phi_1\Phi_1, 1 - 2\Phi_1\Phi_2, -2\Phi_1\Phi_3)_l, \quad (8d)$$

where  $\mu \equiv \mu_z$ . The form of  $\Phi_l$  and  $U_l(i\delta)$  are given by

$$\Phi_1 = -\frac{\Omega^* T_2 (\Delta T_2 + i)}{2P(0)}, \quad (9a)$$

$$\Phi_2 = \Phi_1^* = -\frac{\Omega T_2 (\Delta T_2 - i)}{2P(0)}, \quad (9b)$$

$$\Phi_3 = -\frac{[1 + (\Delta T_2)^2]}{2P(0)}, \quad (9c)$$

and

$$U_1(i\sigma) = \frac{\Omega^2 T_1 T_2}{2P(\sigma)}, \quad (9d)$$

$$U_2(i\sigma) = -\frac{2(\gamma\sigma T_2 + i)((\sigma - \Delta)T_2 + i) - |\Omega|^2 T_1 T_2}{2P(\sigma)}, \quad (9e)$$

$$U_3(i\sigma) = -\frac{\Omega T_1 [(\sigma - \Delta)T_2 + i]}{P(\sigma)}, \quad (9f)$$

where

$$P(\sigma) = (1 - i\gamma\sigma T_2)[(1 - i\sigma T_2)^2 + (\Delta T_2)^2] + (1 - i\sigma T_2)|\Omega|^2 T_1 T_2, \quad (9g)$$

$\Omega = 2\mu E_0/\hbar$  is the Rabi frequency,  $T_1$  is the population decay time,  $T_2$  is the dipole dephasing time, and  $\gamma \equiv T_1/T_2$ .

The master equation [Eq. (4)] can be used to derive the equation of motion for the expectation values of the side-mode operators. In the Schrödinger picture, it can be shown that the time derivative of the expectation value of a field operator  $\hat{G}$  is given by [17]

$$\frac{d}{dt} \langle \hat{G} \rangle = \frac{d}{dt} \text{Tr}(\hat{G} \hat{\rho}_F) = \text{Tr} \left[ \hat{G} \frac{\partial \hat{\rho}_F}{\partial t} \right]. \quad (10)$$

For the case of a propagating field, the equation of motion determined using Eq. (10) can be replaced with an equation describing the spatial evolution by setting the time  $t$  equal to  $nx/c$ , where  $n$  is the index of refraction. This method was used by Agarwal and Boyd to determine the equation for the spatial evolution of the expectation value of various combinations of side-mode operators up to second order [3]. These equations can be written in matrix form by introducing the vector  $\langle \Psi \rangle$  and the matrix  $\langle \Psi \Psi^\dagger \rangle$  given by

$$\langle \Psi \rangle = \begin{bmatrix} \langle \hat{a} \rangle \\ \langle \hat{b} \rangle \end{bmatrix} \quad (11a)$$

and

$$\langle \Psi \Psi^\dagger \rangle = \begin{bmatrix} \langle \hat{a}^\dagger \hat{a} \rangle + \frac{1}{2} & \langle \hat{a} \hat{b} \rangle \\ \langle \hat{a}^\dagger \hat{b}^\dagger \rangle & \langle \hat{b}^\dagger \hat{b} \rangle + \frac{1}{2} \end{bmatrix}. \quad (11b)$$

and

$$\frac{d}{dx} \langle \Psi \Psi^\dagger \rangle = \frac{1}{2} n \alpha_0 \mathbf{A} \langle \Psi \Psi^\dagger \rangle + \frac{1}{2} n \alpha_0 \langle \Psi \Psi^\dagger \rangle \mathbf{A}^\dagger + \frac{1}{2} n \alpha_0 (2\mathbf{D}), \quad (12b)$$

The equations for the expectation values are

$$\frac{d}{dx} \langle \Psi \rangle = \frac{1}{2} n \alpha_0 \mathbf{A} \langle \Psi \rangle \quad (12a)$$

where  $\alpha_0 = 4\pi N \omega_{ba} |\mu|^2 T_2 / \hbar c$  is the weak-field line-center absorption coefficient. The matrix  $\mathbf{A}$  given by

$$\mathbf{A} = (\omega_{ba} |\mu|^2 T_2)^{-1} \begin{bmatrix} -\omega_s C_{ss}^{+-}(i\delta_s) & \sqrt{\omega_s \omega_c} C_{sc}^{++}(-i\delta_c) \\ \sqrt{\omega_s \omega_c} \{C_{cs}^{++}(-i\delta_s)\}^* & -\omega_c \{C_{cc}^{+-}(i\delta_c)\}^* + 2iP \end{bmatrix} \quad (13a)$$

depends on the side-mode susceptibilities and the matrix  $\mathbf{D}$  given by

$$\mathbf{D} = (2\omega_{ba} |\mu|^2 T_2)^{-1} \times \begin{bmatrix} \omega_s \{Q_{ss}^{+-}(i\delta_s) + \text{c.c.}\} & -\sqrt{\omega_s \omega_c} \{Q_{sc}^{++}(-i\delta_c) + Q_{cs}^{++}(-i\delta_s)\} \\ -\sqrt{\omega_s \omega_c} \{Q_{sc}^{++}(-i\delta_c) + Q_{cs}^{++}(-i\delta_s)\}^* & \omega_c \{Q_{cc}^{+-}(i\delta_c) + \text{c.c.}\} \end{bmatrix} \quad (13b)$$

depends on the quantum fluctuations of the atomic system. The effects of the phase mismatch are included in the theory through the quantity  $P$  in the matrix  $\mathbf{A}$

$$P = -\frac{\Delta T_2}{1 + (\Delta T_2)^2 + |\Omega|^2 T_1 T_2}. \quad (14)$$

The initial conditions at  $x=0$  are  $\langle \Psi(0) \rangle = 0$  and  $\langle \Psi(0) \Psi^\dagger(0) \rangle = \frac{1}{2} \mathbf{I}$  where  $\mathbf{I}$  is the identity matrix. These conditions reflect the fact that the side modes are assumed to be initially unpopulated.

Equations (12a) and (12b) can be easily integrated to determine the expectation values at  $x = \ell$ . The solution to Eqs. (12a) and (12b) are

$$\langle \Psi(\ell) \rangle = 0 \quad (15a)$$

and

$$\langle \Psi(\ell) \Psi^\dagger(\ell) \rangle = \frac{1}{2} e^{n\alpha_0 \ell \mathbf{A}/2} (e^{n\alpha_0 \ell \mathbf{A}/2})^\dagger + \frac{1}{2} n \alpha_0 \ell \int_0^1 d\xi e^{n\alpha_0 \ell \mathbf{A} \xi/2} (2\mathbf{D}) \times (e^{n\alpha_0 \ell \mathbf{A} \xi/2})^\dagger, \quad (15b)$$

respectively. Equation (15b) can be cast into a more useful form by introducing a matrix  $\mathbf{S}$  which diagonalizes  $\mathbf{A}$

$$\mathbf{K} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \mathbf{S}^{-1} \mathbf{A} \mathbf{S}. \quad (16)$$

The eigenvalues  $\lambda_1$  and  $\lambda_2$  are determined from  $|\mathbf{A} - \lambda \mathbf{I}| = 0$ . Using the matrix  $\mathbf{K}$  and the rules of matrix multiplication, the exponential factors in Eq. (15b) can be rewritten in the form

$$(e^{n\alpha_0 \ell \mathbf{A}/2})_{ij} = \sum_{l=1}^2 (\mathbf{S})_{il} e^{n\alpha_0 \ell \lambda_l/2} (\mathbf{S}^{-1})_{lj} \quad (17a)$$

and

$$([e^{n\alpha_0 \ell \mathbf{A}/2}]^\dagger)_{jk} = \sum_{l=1}^2 (\mathbf{S}^{-1})_{ij}^* e^{n\alpha_0 \ell \lambda_l^*/2} (\mathbf{S})_{kl}^*. \quad (17b)$$

Substituting Eqs. (17) into Eq. (15b) and performing the integration, the nonzero expectation values of the side-mode operators at  $x = \ell$  can be determined from

$$\langle \langle \Psi(\ell) \Psi^\dagger(\ell) \rangle \rangle_{ij} = \left[ \frac{1}{2} \right] \sum_{k,l,m=1}^2 (\mathbf{S})_{il} (\mathbf{S}^{-1})_{lk} (\mathbf{S}^{-1})_{mk}^* (\mathbf{S})_{jm}^* e^{n\alpha_0 \ell (\lambda_l + \lambda_m^*)/2} + \left[ \frac{n\alpha_0 \ell}{2} \right] \sum_{k,l,m,n=1}^2 (2\mathbf{D})_{kl} (\mathbf{S})_{im} (\mathbf{S}^{-1})_{mk} (\mathbf{S}^{-1})_{nl}^* (\mathbf{S})_{jn}^* \left[ \frac{e^{n\alpha_0 \ell (\lambda_m + \lambda_n^*)/2} - 1}{n\alpha_0 \ell (\lambda_m + \lambda_n^*)/2} \right] \quad (18a)$$

where

$$\langle\langle\Psi(\ell)\Psi^\dagger(\ell)\rangle\rangle_{11}=\langle\hat{a}^\dagger\hat{a}\rangle+\frac{1}{2}, \quad (18b)$$

$$\langle\langle\Psi(\ell)\Psi^\dagger(\ell)\rangle\rangle_{12}=\langle\hat{a}\hat{b}\rangle, \quad (18c)$$

$$\langle\langle\Psi(\ell)\Psi^\dagger(\ell)\rangle\rangle_{21}=\langle\hat{a}^\dagger\hat{b}^\dagger\rangle, \quad (18d)$$

and

$$\langle\langle\Psi(\ell)\Psi^\dagger(\ell)\rangle\rangle_{22}=\langle\hat{b}^\dagger\hat{b}\rangle+\frac{1}{2}. \quad (18e)$$

The effects of atomic motion need to be introduced into the theory since atomic motion will produce Doppler shifts in the frequency of the fields. Grating washout effects, however, are unimportant in forward four-wave mixing when all of the fields propagate in the same direction and have frequency differences on the order of tens of megahertz since for this case the period of the grating is much greater than the length of the interaction region. Therefore, the effects of atomic motion can be included in the theory by performing a one-dimensional Doppler average of the side-mode expectation values.

The results given above can be easily generalized to the situation in which an infinite number of pairs of vacuum side modes ( $\hat{a}_i$  and  $\hat{b}_i$  where  $i=1$  to  $\infty$ ) interact with the laser field when it is assumed that there is no coupling between different pairs of side-mode operators [18]. In this case, the expectation values for each pair of side-mode operators are determined from Eqs. (18).

#### IV. PHOTOCURRENT FLUCTUATIONS

The expectation values of the side-mode operators can be used to calculate the photodetector current fluctuations produced by the transmitted laser beam after propa-

gation through a two-level atomic system. Note that the Doppler-averaged expectation values are used in this calculation. The detection operator is [19]

$$\hat{F}^{(+)}(t)=v_0e^{-i\omega_0t}+\sum_{i=1}^{\infty}[\hat{a}_i(t)+\hat{b}_i(t)], \quad (19a)$$

where  $v_0$  is the complex amplitude of the coherent-state laser field and the side-mode operators are given by

$$\hat{a}_i(t)=\hat{a}_ie^{-i(\omega_0+\delta_i)t} \quad (19b)$$

and

$$\hat{b}_i(t)=\hat{b}_ie^{-i(\omega_0-\delta_i)t} \quad (19c)$$

with the definitions  $\hat{a}_i\equiv\hat{a}(\omega_0+\delta_i)$  and  $\hat{b}_i\equiv\hat{b}(\omega_0-\delta_i)$ .

The two-time photocurrent correlation function [20] is calculated from the expression

$$C(\tau)=e\langle\hat{I}\rangle\delta(\tau)+\langle\hat{I}\rangle^2g^{(2)}(\tau), \quad (20a)$$

where the average photocurrent  $\langle I \rangle$  (which is constant) is given in terms of the detection operator by

$$\langle\hat{I}\rangle=\frac{ec}{L}\langle\hat{F}^{(-)}(t)\hat{F}^{(+)}(t)\rangle, \quad (20b)$$

with  $L\equiv L_x$  and where the degree of second-order temporal coherence  $g^{(2)}(\tau)$  is given by

$$g^{(2)}(\tau)=\frac{\langle\hat{F}^{(-)}(t)\hat{F}^{(-)}(t+\tau)\hat{F}^{(+)}(t+\tau)\hat{F}^{(+)}(t)\rangle}{\langle\hat{F}^{(-)}(t)\hat{F}^{(+)}(t)\rangle^2}. \quad (20c)$$

The quantity  $\langle\hat{F}^{(-)}(t)\hat{F}^{(-)}(t+\tau)\hat{F}^{(+)}(t+\tau)\hat{F}^{(+)}(t)\rangle$  is calculated keeping terms only up to second order in the side-mode operators. The result for the photocurrent correlation function is given by

$$\begin{aligned} C(\tau)= & \left[\frac{ec}{L}\right]^2\left[|v_0|^4+2|v_0|^2\sum_{i=1}^{\infty}\{\langle\hat{a}^\dagger\hat{a}_i\rangle+\langle\hat{b}^\dagger\hat{b}_i\rangle\}+\left[\frac{e^2c}{L}\right]\left[|v_0|^2+\sum_{i=1}^{\infty}\{\langle\hat{a}^\dagger\hat{a}_i\rangle+\langle\hat{b}^\dagger\hat{b}_i\rangle\}\right]\delta(\tau)\right. \\ & +|v_0|^2\sum_{i=1}^{\infty}\{\langle\hat{a}^\dagger\hat{a}_i\rangle+\langle\hat{b}^\dagger\hat{b}_i\rangle\}[e^{i\delta_i\tau}+e^{-i\delta_i\tau}] \\ & \left.+(v_0^*)^2\sum_{i=1}^{\infty}\{\langle\hat{a}_i\hat{b}_i\rangle e^{-i\delta_i\tau}+\langle\hat{b}_i\hat{a}_i\rangle e^{i\delta_i\tau}\}+v_0^2\sum_{i=1}^{\infty}\{\langle\hat{a}^\dagger\hat{b}^\dagger\rangle e^{-i\delta_i\tau}+\langle\hat{b}_i^\dagger\hat{a}_i^\dagger\rangle e^{i\delta_i\tau}\}\right]. \quad (21) \end{aligned}$$

In the limit of continuously distributed side-mode frequencies (i.e.,  $L\rightarrow\infty$ ) and in the limit of an intense input beam (i.e.,  $|v_0|\gg 1$ ), the power spectrum (which is equal to the Fourier transform of the photocurrent correlation function) is given by [21]

$$\begin{aligned} S(\omega)= & \langle\hat{I}\rangle^2\delta(\omega)+\left[\frac{e}{2\pi}\right]\langle\hat{I}\rangle[1+\langle\hat{a}^\dagger(\omega_0+\omega)\hat{a}(\omega_0+\omega)\rangle+\langle\hat{a}^\dagger(\omega_0-\omega)\hat{a}(\omega_0-\omega)\rangle+\langle\hat{b}^\dagger(\omega_0+\omega)\hat{b}(\omega_0+\omega)\rangle \\ & +\langle\hat{b}^\dagger(\omega_0-\omega)\hat{b}(\omega_0-\omega)\rangle+\langle\hat{a}(\omega_0+\omega)\hat{b}(\omega_0-\omega)\rangle+\langle\hat{b}(\omega_0+\omega)\hat{a}(\omega_0-\omega)\rangle \\ & +\langle\hat{a}^\dagger(\omega_0+\omega)\hat{b}^\dagger(\omega_0-\omega)\rangle+\langle\hat{b}^\dagger(\omega_0+\omega)\hat{a}^\dagger(\omega_0-\omega)\rangle], \quad (22) \end{aligned}$$

where  $v_0$  has been chosen to be real. The first term represents the average value of the photocurrent and the remaining terms are the power fluctuations in the current at angular frequency  $\omega$ .

The variance of the photocurrent is calculated from the fluctuating part of the power spectrum by integrating it over

the electrical bandwidth of the detection system. The resulting current fluctuations at a spectrum analyzer frequency  $f$  are

$$\langle (\Delta \hat{I})^2 \rangle_f = 2eB \langle \hat{I} \rangle Z(f), \quad (23a)$$

where

$$Z(f) = [1 + \langle \hat{a}^\dagger(f_0 + f) \hat{a}(f_0 + f) \rangle + \langle \hat{a}^\dagger(f_0 - f) \hat{a}(f_0 - f) \rangle + \langle \hat{b}^\dagger(f_0 + f) \hat{b}(f_0 + f) \rangle + \langle \hat{b}^\dagger(f_0 - f) \hat{b}(f_0 - f) \rangle + \langle \hat{a}(f_0 + f) \hat{b}(f_0 - f) \rangle + \langle \hat{b}(f_0 + f) \hat{a}(f_0 - f) \rangle + \langle \hat{a}^\dagger(f_0 + f) \hat{b}^\dagger(f_0 - f) \rangle + \langle \hat{b}^\dagger(f_0 + f) \hat{a}^\dagger(f_0 - f) \rangle], \quad (23b)$$

$f_0$  is the laser frequency, and  $B$  is the electrical bandwidth of the detection system. The first term in the square bracket of  $Z(f)$  comes from the shot noise. The remaining terms in Eq. (23b) lead to the excess current fluctuations that result from the amplification of the vacuum side modes due to the forward four-wave mixing process. The quantity  $Z(f)$  can be interpreted as the Fano number which by definition is the ratio of the power fluctuations to the shot noise [22].

## V. NUMERICAL RESULTS

In order to compare the predictions of the theoretical model developed in the last section with the results of our experimental investigation, we have performed numerical evaluations of the formulas derived above. In the numerical simulations, the values of the atomic parameters used were  $\mu = -6.14 \times 10^{-18}$ ,  $T_1 = 27$  ns, and  $T_2 = 2T_1$  [23]. The expectation values [such as  $\langle \hat{a}^\dagger(f_0 + f) \hat{a}(f_0 + f) \rangle$ ] needed to calculate the statistical-noise properties of the photocurrent were calculated numerically. The Rabi frequency used in evaluating these expectation values was determined from the laser intensity at the center of the cell which was calculated from the propagation equation for a saturable absorber. The effects of atomic motion were then included by performing a Doppler average. The Fano number of the field after propagating through the cell was calculated from Eq. (23b). Finally, the normalized rms noise was calculated by taking the square root of the product of the Fano number at the detector and the laser transmittance through the cell, i.e., we plot  $R \equiv \sqrt{Z(f)T}$  versus detuning.

The results of the calculation are shown in Fig. 4 along with the experimental data. As can be seen from the plots, the excess noise grows at the correct detunings and continues to increase as the intensity of the laser beam increases. Notice also that the detuning where the peak excess noise occurs decreases with increasing intensity due to saturation of the absorption, as seen in the experiment. Thus, the theory is able to predict the correct qualitative

features of the experimental data. The unequal amounts of noise on the two sides of resonance, however, cannot be explained by this model since the effects of self-focusing were not included.

## VI. CONCLUSIONS

In conclusion, a significant amount of excess noise was acquired by an intense, nearly resonant laser beam after propagating through atomic-potassium vapor. The noise was maximum at laser detunings of approximately  $\pm 1$  GHz and approaches the shot-noise level for detunings greater than  $\pm 3$  GHz. A quantum-mechanical theory of vacuum side-mode amplification due to forward four-wave mixing is able to predict the qualitative behavior of the experimental data. However, the theory cannot account for the different amount of excess noise on the two sides of resonance since the effects of self-focusing were not included in the treatment.

An understanding of the noise features reported here is important in that these features are general in nature and presumably will add noise to any process that involves the interaction of laser light with an atomic vapor. For example, it is likely that the amount of squeezing that can be achieved in noise-reduction experiments utilizing atomic vapors is limited by such noise. In fact, noise processes of this sort may explain why noise-reduction experiments using parametric mixing crystals and optical fibers have been much more successful than those carried out in atomic vapors.

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