Improved measurement of multimode squeezed light via an eigenmode approach

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We analyze the output of a degenerate optical parametric amplifier using a different approach to understanding and characterizing multimode squeezed light. This approach is based on the concept of eigenmodes of the squeezing and predicts the mode structure of the local oscillator that should be used in order to measure the smallest quadrature noise. Although the importance of mode matching is well known, we find that typical experimental setups are suboptimal and we predict that in such cases squeezed light measurements stand to be improved noticeably with appropriate shaping (in space and/or time) of the local oscillator field in accordance with the results of the theory. Under conditions of negligible or small phase mismatch, the optimal local oscillator pulse duration is found to be approximately equal to the geometric mean of the pump pulse duration and nonlinear response time. Also, we find that the effective number of squeezed modes can be altered by varying the pump pulse duration and/or phase-matching parameters.

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It is well established that optical parametric amplification produces nonclassical light. In the limit of very small parametric gain, the quantum state of the output light approximates an entangled two-photon state and has prompted exciting avenues of research including quantum imaging [1], quantum teleportation [2], and quantum lithography [3]. In the regime of modest or large parametric gain, the superposition of the signal and idler fields shows reduced variance (compared to the standard quantum limit) along one axis of its phase space [4,5]. Such light [6,7] has come to be known as squeezed light [8] and has shown promise in applications such as ultra-low-noise spectroscopy [9,10] and microscopy [11] and noiseless image amplification [12,13]. Most studies of multimode squeezed light (see Ref. [14] for a review) have implicitly assumed that in the parametric process the signal and idler photons are emitted into separate pairs of modes. This assumption, however, is generally invalid in conventional bases. For example, pump fields are often pulsed and (or) focused in order to achieve desired downconversion efficiencies. The spread of frequencies and transverse momenta in the pump field cause each signal mode of definite frequency and transverse momentum to be entangled with many idler modes, and vice versa. In the small gain (entangled photon) regime, Law *et al.* [15] have found that this cross-modal coupling affects the entropy of entanglement of the output light. In the context of squeezing, it has been recognized that the cross-modal coupling affects the measured amount of squeezing. La Porta and Slusher [16] showed that when the squeezed light produced by a monochromatic, focused Gaussian beam is measured with a Gaussian local oscillator, the measurable amount of squeezing is bounded. Kim and Kumar [17] demonstrated that measurements of squeezing can be improved by using a local oscillator generated in the same medium as the squeezed field rather than a Gaussian local oscillator. Others (e.g., Refs. [18,19]) have proposed the use of spectral filters or temporal modulation of particular functional forms and determined the parameters which yield the best squeezing measurements.

In this paper we present a general, systematic approach to the analysis of multimode squeezed light produced by parametric downconversion. This approach involves finding the eigenvectors of a matrix (or the eigenfunctions of a continuous two-argument function) which is directly related to the parametric gain. We note that similar approaches based on the use of eigenbases have been developed by Shapiro and Shakeel [20] and, most recently, by Opatrný *et al.* [21]. While our approach identifies the optimal local oscillator mode, it also (i) gives a physical interpretation to the eigenmodes, (ii) provides insight into the correlations which exist in multimode squeezed fields, (iii) simplifies the calculation of the field statistics, and (iv) indicates how the pump field parameters and phase-matching conditions of the parametric interaction affect the number of squeezed modes and their degrees of squeezing. After developing a general formalism, we will present analytical and numerical results for the case of a single-spatial mode optical parametric amplifier (OPA) pumped by a short pulse. We find that typical setups for squeezed light measurement (Fig. 1) do not optimally probe the squeezed field and that shaping the local oscillator in accordance with the predictions of our theory could increase the measured amount of squeezing in such experiments by at least several dB.

We model the action of an OPA on a set of (orthonormal) quantized field modes with slowly-varying operators $\hat{a}_p(z)$ by the Hamiltonian

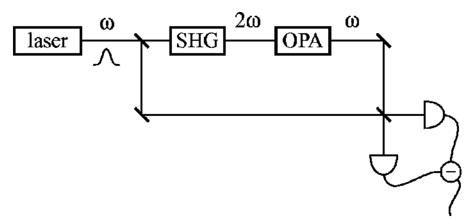

FIG. 1. A typical experimental setup to measure squeezing produced by parametric downconversion.

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$$\hat{H}=\frac{i\hbar c}{2}\sum_{pq} G^{*}_{pq}\hat{a}_p\hat{a}_q\exp(-iK_{pq}z)+\text{H.c.}. \tag{1}$$

Here G_{pq} is the parametric gain coefficient of the wave mixing between modes p and q, $K_{pq}=k(\omega_p+\omega_q)-k(\omega_p)-k(\omega_q)$ is the longitudinal wave vector mismatch, and ω_p is the frequency of mode p. When the modes are monochromatic and planar, G_{pq} is given (in the Gaussian system of units) by $G_{pq}=-i(4\pi/c)\sqrt{\omega_p\omega_q}\chi^{(2)}(\omega_p,\omega_q)E(\omega_p+\omega_q)\exp[ik(\omega_p+\omega_q)z]$ where $\chi^{(2)}$ is the second-order optical susceptibility and $E(z,t)=\Sigma_r E(\omega_r)\exp[ik(\omega_r)z-i\omega_r t]$ +c.c. is the real, classical pump field. More generally, the modes may have any spatial or temporal form, as long as they are eigenmodes of linear-optic propagation (not to be confused with the eigenmodes of the squeezing, introduced below). For convenience we have assumed that the pump field remains undepleted. In the slowly-varying amplitude approximation, the spatial evolution of the field operators is governed by

$$\frac{d}{dz}\hat{a}_p=\frac{i}{\hbar c}[\hat{H},\hat{a}_p]. \tag{2}$$

The field at the plane z can then be expressed as

$$\hat{a}_p(z)=\exp[i\hat{\Omega}(z)]\hat{a}_p(0)\exp[-i\hat{\Omega}(z)], \tag{3}$$

where $\hat{\Omega}(z)$ is in general a complicated function involving nested commutators of integrals of $\hat{H}$ [22]. Nevertheless, $\hat{\Omega}$ will be quadratic in the field operators and the transformation may be written [23] as

$$\exp(-i\hat{\Omega})=\hat{S}(\mathbf{\Gamma})\hat{F}(\mathbf{\Phi}) \tag{4}$$

for some symmetric matrix $\mathbf{\Gamma}$ and some Hermitian matrix $\mathbf{\Phi}$, where

$$\hat{S}(\mathbf{\Gamma})=\exp\left[\frac{1}{2}\sum_{pq}\left(\Gamma^{*}_{pq}\hat{a}_p(0)\hat{a}_q(0)-\Gamma_{pq}\hat{a}^{\dagger}_p(0)\hat{a}^{\dagger}_q(0)\right)\right], \tag{5}$$

$$\hat{F}(\mathbf{\Phi})=\exp\left(i\sum_{pq}\Phi_{pq}a^{\dagger}_p(0)a_q(0)\right). \tag{6}$$

$\hat{F}$ is a unitary operator which merely changes the input field basis, while $\hat{S}$ is a multimode squeezing operator which accounts for parametric coupling between all possible pairs of modes. The elements of the matrix $\mathbf{\Gamma}$ play the role that the squeeze parameter plays in single-mode squeezing, with the diagonal elements promoting single-mode squeezing and the off-diagonal elements promoting pairwise squeezing. In the case of perfect phase matching ($K_{pq}=0$) one has $\mathbf{\Phi}=\mathbf{0}$ and $\Gamma_{pq}=G_{pq}L$, where L is the length of the OPA. Since each mode is influenced by many different elements of $\mathbf{\Gamma}$, the effect of any single element on the statistics of the mode is usually not simple. However, we find that there always exists a canonical basis for the field which diagonalizes $\mathbf{\Gamma}$. In this basis the expressions for the output field operators reduce to the particularly simple form

$$\hat{a}''_p=\hat{a}'_p\cosh\gamma_p-\hat{a}'^{\dagger}_p\sinh\gamma_p, \tag{7}$$

where γ_p is the pth eigenvalue of $\mathbf{\Gamma}$. If $\mathbf{U}$ is the unitary matrix that transforms the field to this eigenbasis [24], i.e., the matrix whose columns are the eigenvectors of $\mathbf{\Gamma}$, then $(\hat{a}'_1,\hat{a}'_2,\dots)^T=\mathbf{U}^{\dagger}e^{i\mathbf{\Phi}}(\hat{a}_1(0),\hat{a}_2(0),\dots)^T$ and $(\hat{a}''_1,\hat{a}''_2,\dots)^T=\mathbf{U}^{\dagger}(\hat{a}_1(z),\hat{a}_2(z),\dots)^T$. When the input field operators $\hat{a}_p(0)$ refer to coherent states (such as the vacuum state), the form of Eq. (7) guarantees that the output operators $\hat{a}''_p$ describe canonical single-mode squeezed states. That is to say, *the eigenvectors of* $\mathbf{\Gamma}$ *define the squeezed modes of the field and the corresponding eigenvalues are the squeeze parameters*. We call such modes the eigenmodes of the squeezing, and the eigenvalues of the squeezing.

The squeeze eigenmodes are fundamental in that they define the basis in which the squeezing is maximum. Let $\psi(s)$ be the field of a classical local oscillator, where s represents the relevant spatial and (or) spectral coordinates, and let $\phi_p(s)$ be the field of the pth eigenmode with $\int\phi^{*}_p\phi_q\,ds=\delta_{pq}$. In balanced homodyne or heterodyne detection schemes the difference photocurrent $\hat{i}_d$ contains only the interference terms and can be written as

$$\hat{i}_d\propto\sum_p|O_p|\hat{E}_p(\arg O_p), \tag{8}$$

where $O_p=\int\psi^{*}\phi_p ds$ is the overlap between the local oscillator and mode p over the domain of the detector and $\hat{E}_p(\theta)=e^{i\theta}\hat{a}''_p+e^{-i\theta}\hat{a}''^{\dagger}_p$ is the projection of $\hat{a}''_p$ onto the quadrature with phase θ. The photocurrent variance is

$$\langle\Delta\hat{i}^2_d\rangle\propto\sum_p|O_p|^2[e^{-2\gamma_p}\cos^2(\arg O_p)+e^{2\gamma_p}\sin^2(\arg O_p)]. \tag{9}$$

That is, the photocurrent variance is a weighted sum of quadrature variances (some perhaps squeezed, some perhaps antisqueezed) of all modes which overlap the local oscillator. If the local oscillator overlaps multiple eigenmodes the photocurrent may not show variance below the standard quantum limit for any phase of the local oscillator. The smallest photocurrent variance is obtained when the mode of the local oscillator is chosen to match the eigenmode with the largest squeeze parameter.

If the squeezing has more than one nonzero eigenvalue, then in principle multiple photocurrents with reduced noise may be measured simultaneously. Ideally, each eigenmode is projected onto a different detector and exhibits a photocurrent noise determined by the corresponding eigenvalue. To accomplish this one must be able to physically separate the various eigenmodes (via lossless spatial, temporal, or spectral filtering) and direct them to separate detectors. The photocurrent at the jth detector will be given by Eq. (9), where

O_p is replaced by O_{jp}, the overlap between eigenmode p and the local oscillator field over the jth detector.

As the eigenvalues of a matrix are unchanged by linear unitary transformations, the squeeze eigenvalues are unchanged either by propagation through a lossless linear optical system or by paraxial diffraction. For this reason, together with the fact that the statistics of any field mode can be expressed in terms of the squeeze eigenvalues, we view the distribution of eigenvalues as a fundamental property of a multimode squeezed field. Together, the squeeze eigenvalues and eigenmodes contain all there is to know about the quantum statistics of the field. A significant advantage of this approach is that it separates the quantum aspects of the field (the squeezing) from the classical aspects (diffraction and imaging).

To illustrate this approach, we have performed an eigenmode analysis of the light produced by a frequency-degenerate OPA. For clarity of discussion, the field modes of Eq. (1) will be assumed to differ only in frequency. (This assumption is applicable either if the pump and medium have no transverse spatial dependence, or if the medium is a waveguide which allows only a single-spatial mode at each frequency.) We consider the pump field to be of the form $E_{\mathrm{pump}}(t)=E_{\mathrm{pump}}(0)\exp[-\frac{1}{2}(t/T_{\mathrm{pump}})^2]$ where $E_{\mathrm{pump}}(0)$ is such that the parametric (amplitude) gain at the peak of the pulse is g. For the case of arbitrary phase mismatch and nonlinear response function, numerical methods must be employed to determine the structure of the squeeze matrix; the results of such computations will follow. But first, an analytical study of a slightly more restricted system will provide considerable insight.

We assume for the moment that the nonlinear susceptibility has a Gaussian temporal response and that the pump and downconverted pulses are phasematched to all orders of dispersion. Analysis is performed in the time domain using a continuous parametric gain function $G(t,t')$ in favor of a discrete matrix. $G(t,t')$ is the parametric coupling between the downconverted fields at times t and t'. Under the conditions stated above, the squeeze function $\Gamma(t,t')$ (the continuous analog of the squeeze matrix $\mathbf{\Gamma}$) has the form

$$\Gamma(t,t')=G(t,t')L \tag{10}$$

$$=gL\frac{\exp\left[-\frac{(t+t')^2}{4T_{\mathrm{pump}}^2}-\frac{(t-t')^2}{4T_\chi^2}\right]}{T_\chi\sqrt{2\pi}}, \tag{11}$$

where L is the length of the nonlinear medium and T_χ is the response time of the nonlinearity. The temporal function $\phi_n(t)$ of the nth eigenmode of the squeezing obeys the integral eigenvalue equation

$$\int \Gamma(t,t')\phi_n(t')dt'=\gamma_n\phi_n(t) \tag{12}$$

which is satisfied by

$$\phi_n(t)=H_n(t/\tau), \quad \gamma_n=gL\frac{r}{r+r^{-1}}\left(\frac{r-r^{-1}}{r+r^{-1}}\right)^n, \tag{13}$$

where $H_n(t)$ is the Hermite-Gauss function of order n, $\tau=\sqrt{2T_{\mathrm{pump}}T_\chi}$, and $r=\sqrt{T_{\mathrm{pump}}/T_\chi}$. That is, the eigenmodes of the squeezing are Hermite-Gauss functions of time with a characteristic width equal (within a small numerical factor) to the geometric mean of the pump pulse duration and nonlinear response time. The eigenvalues form a geometric sequence and we may define the effective number of squeezed modes as

$$N_\lambda\equiv\left(1-\frac{r-r^{-1}}{r+r^{-1}}\right)^{-1}=\frac{T_{\mathrm{pump}}+T_\chi}{2T_\chi}. \tag{14}$$

The lowest-order mode $(n=0)$ is Gaussian and has the largest squeeze parameter. In the limit of either instantaneous material response $(T_\chi\to0)$ or monochromatic pump $T_{\mathrm{pump}}\to\infty)$, the maximum squeeze eigenvalue γ_0 approaches the net peak parametric gain gL and the effective number of squeezed modes becomes infinite. For instantaneous material response, $\tau=0$ and the eigenmodes are pointlike in time, while for a monochromatic pump $\tau=\infty$ and the eigenmodes have sinusoidal envelopes (bichromatic modes). In the limit in which the pump pulse duration is as short as the nonlinear response time, $\tau\to T_{\mathrm{pump}}\sqrt{2}$ and $r\to1$. In this limit the first mode $(n=0)$, whose squeeze parameter is $gL/2$, is the only squeezed mode $(N_\lambda=1)$.

The presence of phase velocity mismatch (i.e., wave vector mismatch at the carrier frequencies of the pump and downconverted pulses) does not alter the eigenmodes. This can be seen by noting that if $K_{pq}=\Delta k=\text{const}$, then the transformation $\hat{a}_p\to\hat{a}_pe^{-i\Delta k/2}$ transforms the Hamiltonian (1) to

$$\hat{H}\to\hbar c\frac{\Delta k}{2}\sum_p \hat{a}_p^\dagger\hat{a}_p+\frac{i\hbar c}{2}\sum_{pq}(G_{pq}^*\hat{a}_p\hat{a}_q-G_{pq}\hat{a}_p^\dagger\hat{a}_q^\dagger)$$

which is diagonalized by the same transformation that diagonalizes $\boldsymbol{G}$. The output field operators for the eigenmodes become

$$\hat{a}_p''\to\hat{a}_p'\left(\cosh\zeta_p-\frac{i\Delta kL}{\zeta_p}\sinh\zeta_p\right)-\hat{a}_p'^\dagger\frac{\gamma_p}{\zeta_p}\sinh\zeta_p, \tag{15}$$

where $\zeta_p=\sqrt{\gamma_p^2-(\Delta kL)^2}$ and γ_p is the pth eigenvalue in the absence of the phase mismatch. By comparing Eq. (15) with Eq. (7), the new squeeze eigenvalue of the pth eigenmode is seen to be $\sinh^{-1}[(\gamma_p/\zeta_p)\sinh\zeta_p]$, which is smaller than γ_p. The smaller γ_p, the larger the relative decrease in the squeezing. Thus the net result of phase velocity mismatch between the pump and downconverted pulses is to reduce the squeeze parameters of all eigenmodes, but especially of the higher-order eigenmodes, so that the effective number of squeezed modes is also reduced.

We now present the results of numerical studies of more realistic cases involving a causal nonlinear susceptibility and the possibility of group-velocity mismatch as well as

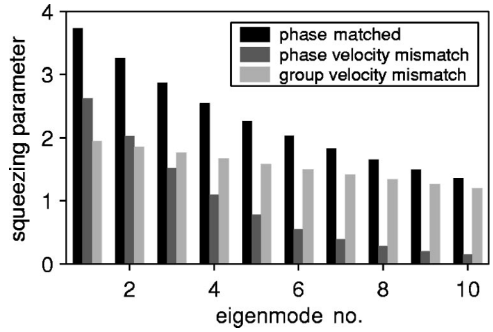

FIG. 2. The ten largest eigenvalues of the squeezing for a pump pulse long compared to the nonlinear response time ($T_{\rm pump}/T_\chi = 10$), and with an intensity such that the gain-length product at the peak of the pulse is 4. For the case of phase velocity mismatch, $\Delta kL = 2\pi$ and $\Delta T_{\rm delay} = 0$. For the case of group-velocity mismatch, $\Delta kL = 0$ and $\Delta T_{\rm delay} = 5T_{\rm pump}$.

phase velocity mismatch. For these studies the squeeze eigenmodes were obtained by expressing the parametric gain as a function of frequency, discretizing this function, integrating Eq. (2) numerically, and diagonalizing the resulting squeeze matrix. The parametric gain was taken to have the form

$$G(\delta\omega,\delta\omega') = \frac{g}{T_{\rm pump}\sqrt{2\pi}} \frac{\exp\left[-\frac{1}{2}T_{\rm pump}^2(\delta\omega+\delta\omega')^2\right]}{(1-iT_\chi\delta\omega)(1-iT_\chi\delta\omega')}, \tag{16}$$

where $\delta\omega,\delta\omega'$ are detunings from the center frequency of the downconverted field. The spectral continuum was approximated by 300 modes spanning a bandwidth of $8T_\chi^{-1}$. Phase mismatch was incorporated by expanding $k(\omega)$ in two power series about the carrier frequencies of the pump and downconverted fields and retaining the two lowest terms in each series. The phase mismatch was then written in terms of the mode detunings $\delta\omega_p,\delta\omega_q$ as $\Delta K_{pq}L = \Delta kL - \Delta T_{\rm delay}(\delta\omega_p+\delta\omega_q)$, where Δk is the longitudinal wave vector mismatch at the carrier frequencies and $\Delta T_{\rm delay}$ is the difference in group delay between the pump and downconverted pulses.

Figure 2 shows the ten largest squeeze eigenvalues for a moderately short pump ($T_{\rm pump}/T_\chi = 10$) and moderate gain ($gL = 4$). Comparison of the phase-matched and phase-mismatched cases confirms that phase velocity mismatch both reduces the maximum degree of squeezing and the effective number of squeezed modes. Group-velocity mismatch also decreases the maximum degree of squeezing, but tends to increase the number of squeezed modes. The eigenmodes were found to be very similar to the Hermite-Gauss modes predicted by the analytical study above (cf. Fig. 3), even though a very different response function was used for the nonlinear susceptibility. As expected, the eigenmodes were unaffected by the addition of phase velocity mismatch. Surprisingly, the eigenmodes were also not changed significantly by a moderately large group-velocity mismatch: the overlap of the first eigenmode in this case with that of the phase-matched case was 0.99, even though the difference in group delay between the pump and downconverted fields was five times the pump duration.

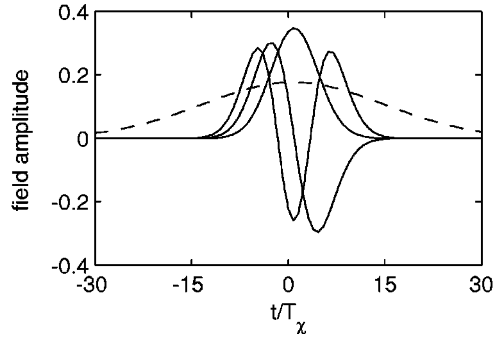

FIG. 3. The temporal envelopes of the first three eigenmodes of the squeezing (solid lines) and of a local oscillator derived from the same pulse as the pump (dashed line). The conditions are those of the phase-matched case of Fig. 2.

A "common sense" way to setup a squeezed light experiment is to derive the local oscillator from the same pulse as the pump (Fig. 1). Such a pulse, however, is not well matched to any of the eigenmodes of the squeezing (Fig. 3). For typical parameters the overlap of the (Gaussian) local oscillator with the most-squeezed eigenmode is only 70%, with the result that the measured amount of noise reduction could be many dB smaller than the squeezing of this eigenmode (Fig. 4). Another "common sense" experimental approach when the OPA is seeded with a coherent state input is to match the local oscillator to the mode of the coherent portion of the output, that is, to maximize the fringe contrast. But the shape of this mode is a function of the amplitudes of the input modes, whereas the shapes of the eigenmodes of the squeezing are determined only by the parameters of the Hamiltonian. Thus maximizing fringe contrast does not identify the optimum local oscillator, either. These findings suggest that, even though the importance of mode matching is widely recognized, previous measurements of squeezed light may not have used the optimal mode for the local oscillator, and that in some experimental setups the measured noise reduction stands to be improved significantly by shaping the

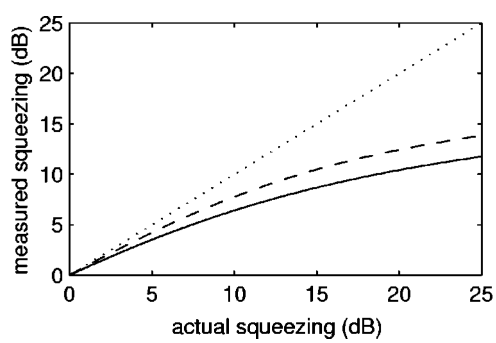

FIG. 4. The decrease in the amount of squeezing measured when the local oscillator of Fig. 3 is used instead of that which matches the most-squeezed eigenmode. Solid line, $T_{\rm pump}/T_\chi = 10$; dashed line, $T_{\rm pump}/T_\chi = 2$. The dotted line has a slope of unity and is included as a guide to the eye.

local oscillator to match the most-squeezed eigenmode. For the case shown in Fig. 3, the reshaping could be achieved by controlled spectral broadening of the seed pulse via self-phase modulation followed by passive linear filtering in the spectral domain (akin to what is commonly done in chirped pulse amplification) to shorten the local oscillator pulse. In the spatial analog of this problem, reshaping could be achieved with a telescope and combination of phase and amplitude masks.

In conclusion, we have presented an approach to analyzing multimode squeezed light which addresses the fact that squeezed light sources based on parametric downconversion produce fields which generally contain correlations between multiple, overlapping pairs of modes. This approach is based on our observation that the matrix which characterizes the squeezing can always be diagonalized by a change of the field basis. The eigenvectors of the squeeze matrix define the modes of the field which exhibit single-mode squeezed statistics and the eigenvalues are the corresponding squeeze parameters. To observe the most noise reduction, one should match the mode of the local oscillator to the eigenmode with the largest squeeze parameter. For a pump having a Gaussian temporal profile the squeeze eigenmodes are very nearly Hermite-Gauss modes in both the time and frequency domains; but in general none of these matches the mode of any coherent-state field produced in a typical squeezing experiment. Hence the levels of squeezing which have been observed to date may yet be increased by appropriate shaping of the local oscillator field based on the theory presented here.

ACKNOWLEDGMENTS

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