

# Induced focusing and spatial wave breaking from cross-phase modulation in a self-defocusing medium

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The spatial effects of cross-phase modulation on a weak probe beam as it copropagates with an intense pump beam through a self-defocusing medium are investigated. Experimental results are presented that demonstrate induced focusing, beam deflection, and the spatial analog of optical wave breaking. The experimental results are in good qualitative agreement with theoretical predictions.

The nonlinear Schrödinger equation (NLSE) is a powerful tool for describing nonlinear pulse propagation.<sup>1-4</sup> In the temporal domain, the NLSE has been successfully used in describing the evolution of pulses in optical fibers. Pulse compression,<sup>5,6</sup> soliton formation,<sup>3,7</sup> soliton dragging,<sup>8</sup> and optical wave breaking<sup>9,10</sup> are some of the diverse physical phenomena that have been investigated. In the spatial domain, the NLSE describes an equally rich set of physical phenomena that have not been so thoroughly investigated. Of interest in the spatial domain is the evolution of the transverse intensity profiles of the interacting beams as they copropagate through a nonlinear medium. A beam-deflection technique for measuring the optical nonlinearities of materials<sup>11-13</sup> is based on this nonlinear evolution. It has recently been predicted theoretically<sup>14</sup> that the presence of a strong pump beam can induce focusing and deflection of a weak probe beam even in a self-defocusing medium. In this Letter we report our experimental observation of induced focusing, beam deflection, and the spatial analog of optical wave breaking. The experimental results are found to be in good qualitative agreement with the theory of Agrawal.<sup>14</sup>

A schematic illustration of the experimental arrangement is shown in Fig. 1. Intersecting within a nonlinear medium are a strong pump beam and a weak probe beam. The centers of the two beams are coincident at the entrance face of the medium. In this experiment, sodium vapor is used as the nonlinear medium. The sodium cell is 5 cm long and is heated to a temperature of approximately 250°C, creating a number density of approximately  $2 \times 10^{13} \text{ cm}^{-3}$ . An excimer-pumped dye laser with a pulse width of approximately 10 ns is detuned by several Doppler widths to the defocusing side of the  $D_2$  resonance. The output of the dye laser is spatially filtered to produce a smooth spatial beam profile. The beam is then split into a strong pump and a weak probe beam. The energy of the pump beam is in the range of 10–100  $\mu\text{J}$ , and the probe beam is kept at least 1000 times weaker. The beams are gently focused with a 700-mm focal-length lens so that the beam waist is slightly behind the cell. At

the front window of the cell, the beams have approximately a 1-mm spot size and an angular separation of approximately 2°. The intensity profile of the probe beam is recorded by an optical multichannel analyzer placed approximately 1.3 m after the sodium cell.

Typical experimental results are shown in Fig. 2. In this case, the frequency of the beams is detuned by 28 GHz to the defocusing side of the sodium resonance. The energy of the pump pulse is approximately 20  $\mu\text{J}$ . In Fig. 2, the transverse intensity profile of the probe beam is shown for the cases in which the probe beam has propagated through the sodium cell without the pump beam [Fig. 2(a)] and with the pump beam [Fig. 2(b)]. Figure 2(b) illustrates that (1) most of the energy of the probe beam is induced to focus to approximately one half of its original spot size, (2) the focused lobe is deflected away from the pump beam by approximately 2.5 times its original beam radius, corresponding to a deflection angle of approximately 0.3°, and (3) the profile of the probe beam has developed an oscillatory wing. This oscillatory wing is a manifestation of the spatial analog of optical wave breaking.

Under our experimental conditions, the sodium vapor can be modeled as a Kerr medium where the evolution of the pump and probe beams is described by the following coupled nonlinear Schrödinger equations:

$$\frac{\partial A_1}{\partial z} - \frac{i}{2k} \frac{\partial^2 A_1}{\partial x^2} = \frac{ikn_2}{n_0} (|A_1|^2 + 2|A_2|^2) A_1, \quad (1a)$$

$$\frac{\partial A_2}{\partial z} - \frac{i}{2k} \frac{\partial^2 A_2}{\partial x^2} = \frac{ikn_2}{n_0} (|A_2|^2 + 2|A_1|^2) A_2, \quad (1b)$$

where  $A_1$  and  $A_2$  are the slowly varying envelope amplitudes of the probe and pump waves, respectively;  $k = 2\pi n_0/\lambda$ ;  $n_2$  is the nonlinear refractive index; and  $x$  and  $z$  are the transverse and longitudinal coordinates, respectively. For simplicity, the model treats the evolution in only one transverse spatial dimension. The forward four-wave-mixing terms have been neglected since they are not phase matched for our experimental arrangement. The nonlinear refractive index of the sodium vapor for

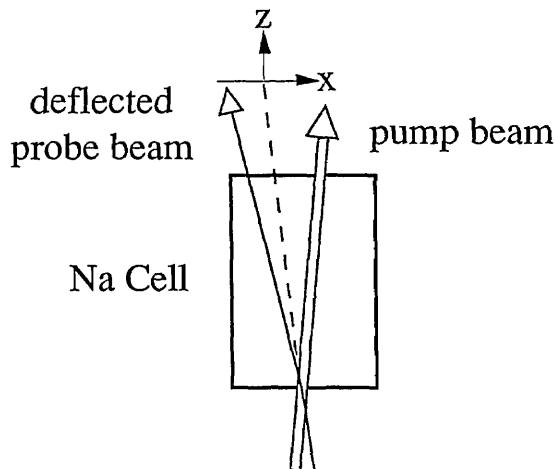


Fig. 1. Schematic illustration of the experimental setup. The dashed line is the path of the probe beam when the pump beam is blocked.

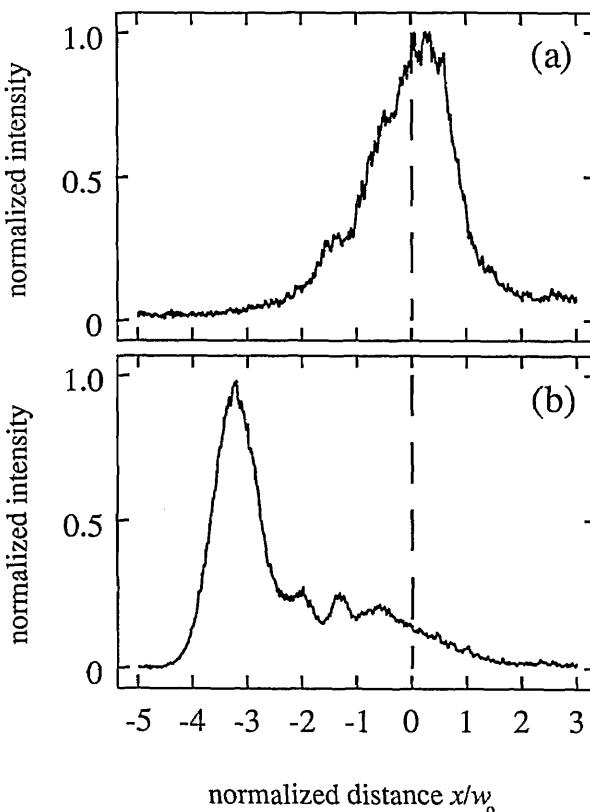


Fig. 2. Experimental realization of the transverse intensity profile of the probe beam (a) before and (b) after it copropagates with the pump beam through the sodium cell.

large detunings is given by<sup>15</sup>

$$n_2 = \frac{8\pi^2 N \mu^4}{n_0^2 c \hbar^3 (\omega - \omega_0)^3}, \quad (2)$$

where  $N$  is the number density,  $\mu$  is the dipole moment, and  $\omega_0$  is the resonance frequency. For the results shown in Fig. 2, the value of the nonlinear refractive index is approximately  $n_2 = 1.5 \times 10^{-10} \text{ cm}^2 \text{ W}^{-1}$ . The results of a numerical simulation based on Eqs. (1a) and (1b) are shown in Fig. 3. For this simulation, the value of  $n_2$  is taken as

$6 \times 10^{-10} \text{ cm}^2 \text{ W}^{-1}$ , which is well within the range of estimated experimental values. In order to achieve the best agreement between the experimental and the theoretical results, the initial spot size is assumed to be three times smaller than the best estimate of the spot size for the results in Fig. 2. This difference is most likely due to the one-dimensional nature of the model. The theoretical predictions are in good qualitative agreement with the experimental results. In particular, the theory predicts beam focusing and deflection as well as an oscillatory wing in the transverse intensity profile of the probe beam.

These three features can be understood intuitively. One might naïvely expect the pump beam to enhance the defocusing of the probe beam since the sign of  $n_2$  is negative for our experimental conditions. However, more important than the absolute sign of  $n_2$  is the sign of  $n_2$  relative to the sign of the curvature of the pump profile. Whenever the probe beam interacts with a section of the pump beam where these two signs are opposite, the pump beam will act as a positive lens and will hence induce focusing of the probe beam. For our experimental arrangement, where the beams are incident at a finite crossing angle, the probe beam moves in the negative  $x$  direction relative to the pump beam and therefore interacts mainly with the wing of the pump beam. Since this wing has a positive curvature, and because the sign of  $n_2$  is negative, the probe beam is induced to focus.

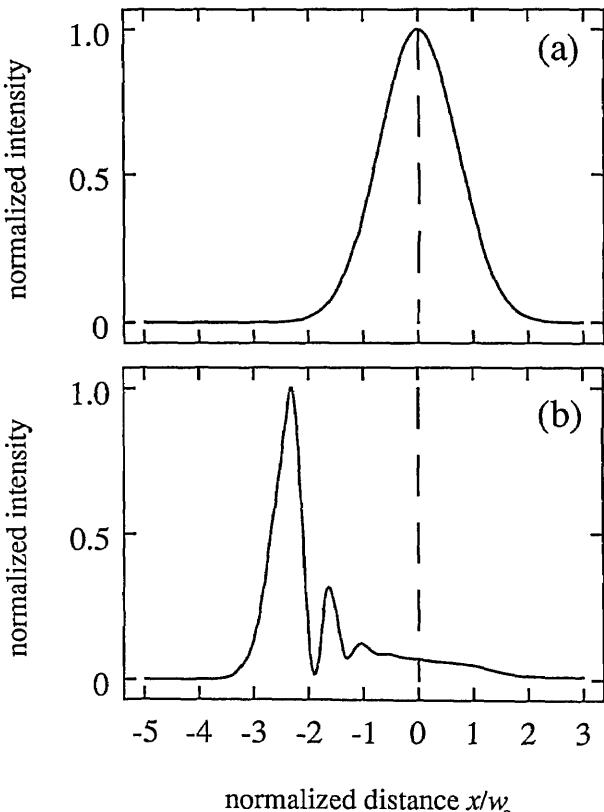


Fig. 3. Results of a numerical simulation of the transverse intensity profile of the probe beam (a) before and (b) after it copropagates with the pump beam through the sodium cell.

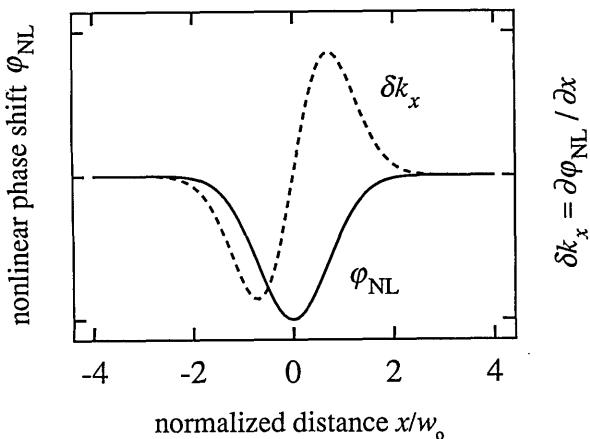


Fig. 4. Instantaneous nonlinear phase shift induced by a Gaussian beam in a self-defocusing nonlinear medium and the corresponding contribution to the  $x$  component of the propagation vector.

In order to understand the beam deflection<sup>16</sup> and the oscillatory wing, we need to consider the incremental nonlinear phase shift induced by the pump beam and the shift's contribution to the transverse propagation vector  $k_x$  of the probe beam as shown in Fig. 4. The centers of the two beams are initially coincident. Owing to the incident angular separation, the probe beam slides to the negative  $x$  direction across the pump beam on propagation through the cell. The rightmost wing of the probe beam interacts with nearly the entire intensity profile of the pump beam, and therefore the net phase shift and the beam deflection are small. The leftmost wing of the probe beam interacts with only the leftmost wing of the pump beam and therefore is deflected slightly to the left. The central portion of the probe beam interacts mainly with the left half of the pump beam and therefore experiences the most beam deflection. This central portion of the probe beam interferes with what was initially (at the entrance face) the leftmost wing of the probe beam, and their interference is responsible for the oscillatory nature of the wing of the deflected probe beam.

The differences between the theoretical and experimental results can probably be attributed to two causes. First, the theoretical model assumes that the incident pump and probe beams have Gaussian profiles. However, as can be seen from Fig. 2(a), the actual probe beam profile (as well as the pump beam profile) used in the experiment deviated somewhat from a Gaussian profile. Second, a more fundamental limitation is that in the theoretical model we treat the evolution of the beams in only one transverse dimension. (It would be computationally cumbersome to treat the full three-dimensional nature of the interaction.) For this reason, only qualitative agreement between the theory and the experiment is to be expected.

An interesting analogy can be drawn between the effects described above and those studied in an opti-

cal fiber. The spatial effects presented here result from the interplay of cross-phase modulation and diffraction, whereas the temporal effects seen in a fiber result from the interplay of cross-phase modulation and group-velocity dispersion. If the analogous fiber experiment were performed, the probe pulse would be compressed as our probe beam was focused, the probe pulse would be delayed as our beam was deflected, and the tail of the probe pulse would develop rapid oscillations because of optical wave breaking, analogous to the oscillatory wing of our probe beam.

In conclusion, we have investigated the spatial analogs of some well-known temporal effects in the propagation of short pulses in optical fibers. Our experimental results show that the presence of an intense pump beam can induce focusing, deflection, and the spatial analog of wave breaking in the transverse intensity profile of a weak probe beam as the two beams copropagate through a self-defocusing medium. The experimental results are in good qualitative agreement with the predictions of our theoretical model.

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