Slow light, induced dispersion, enhanced nonlinearity, and optical solitons in a resonator-array waveguide

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We describe an optical transmission line that consists of an array of wavelength-scale optical disk resonators coupled to an optical waveguide. Such a structure leads to exotic optical characteristics, including ultraslow group velocities of propagation, enhanced optical nonlinearities, and large dispersion with a controllable magnitude and sign. This device supports soliton propagation, which can be described by a generalized nonlinear Schrodinger equation.

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Nanocomposite materials display useful optical properties that can be qualitatively dissimilar [1] from those of their underlying constituents. Nanocomposite materials are especially well suited for photonics applications, because they can be constructed in such a manner as to produce enhanced nonlinear optical response. Some such materials are formed by a random association of the underlying constituents [2,3], whereas others are formed with deterministic properties through various fabrication methods [4,5].

Nanofabrication techniques are capable of forming structures with specially tailored optical properties. One approach leads to the creation of structures, such as photonic crystals [6–8]. In these materials, the refractive index is modulated periodically on the distance scale of an optical wavelength. Such structures necessarily produce a strong coupling between counterpropagating optical waves; for a sufficiently strong index modulation such structures produce a photonic band gap, that is, a range of frequencies over which light cannot propagate.

In this paper, we describe a different sort of structured optical medium that leads to exotic optical properties without necessarily producing a strong back reflection. This structure consists of a side-coupled integrated spaced sequence of resonators (SCISSOR), as illustrated in Fig. 1. Although the light interacts strongly with the optical resonators, there is no mechanism for the creation of a strong reflected wave, and thus the device is totally transmissive at all frequencies. However, the device displays strong nonlinear and dispersive effects in transmission. For a densely packed collection of high-finesse resonators, a light wave spends much more time circulating within each resonator than in propagating between resonators. Thus the group velocity of propagation can become very low. Because the time delay acquired in interacting with each resonator depends critically on the detuning of the optical wave from the resonance frequency, this device displays tailorable dispersion with a magnitude much larger than that of conventional materials. Also, owing to the buildup of intensity within each resonator, the nonlinear response of this structure is greatly enhanced (Heebner and Boyd show that the enhancement scales as the square of the resonator finesse [9]) with respect to propagation through a bulk nonlinear material. Under appropriate conditions, these dispersive and nonlinear effects can precisely balance one another, leading to the propagation of optical solitons. Our mathematical analysis of this structure proceeds as follows.

Figure 1 labels the fields associated with the coupling of a microresonator to a waveguide. The coupling is described mathematically by a transfer matrix in the frequency domain as

$$\begin{pmatrix} \tilde{E}_4(\omega) \\ \tilde{E}_2(\omega) \end{pmatrix} = \begin{pmatrix} r & it \\ it & r \end{pmatrix} \begin{pmatrix} \tilde{E}_3(\omega) \\ \tilde{E}_1(\omega) \end{pmatrix}, \tag{1}$$

where we assume that the self- and cross-coupling coefficients r and t are independent of the frequency and intensity and that the coupling matrix is unitary such that $r^2+t^2=1$. The field $\tilde{E}_4$ is fed back into $\tilde{E}_3$ via a mode of the microresonator, which may take the form of a mode of a ring waveguide or a whispering gallery mode of a disk or sphere [10]. We first assume that the internal attenuation and nonlinear behavior are negligible such that, after one pass, the field simply acquires an *internal phase shift* ϕ such that

$$\tilde{E}_3(\omega) = e^{i\phi(\omega)}\tilde{E}_4(\omega). \tag{2}$$

Furthermore, assuming negligible material dispersion, the internal phase shift may be linearly expanded in radian frequency ω about a resonance frequency ω_R of the resonator. Thus $\phi=(\omega-\omega_R)T$ can be understood as a *normalized* detuning, where the normalizing time of interest is the circumferential transit time ($T=n2\pi R/c$). Here, n is the refractive index, R is the effective radius, and c is the speed of light. It is easy to deduce from Eqs. (1) and (2) that the output field is related to the input field as

$$\tilde{E}_2(\omega) = e^{i\Phi(\omega)}\tilde{E}_1(\omega), \tag{3}$$

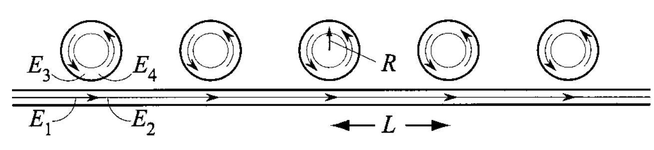

FIG. 1. A side-coupled integrated spaced sequence of resonators.

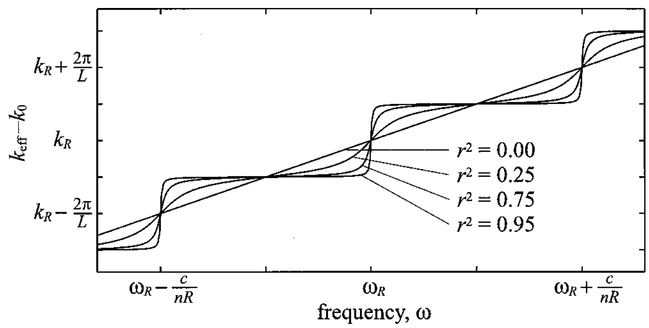

FIG. 2. Dispersion relation for light propagation in a SCISSOR with differing values of the self-coupling coefficient r. $k_{\rm eff}-k_0$ is the resonator contribution to the propagation constant.

with a *transmitted phase shift* Φ that exhibits the following detuning dependence:

$$\Phi=\pi+\phi+2\arctan\frac{r\sin\phi}{1-r\cos\phi}. \tag{4}$$

For a sequence of microresonators with unit spacing L, the additional phase imparted by the microresonators modifies the propagation constant of the unloaded waveguide, which becomes $k_{\rm eff}=n\omega/c+\Phi/L$. The dispersion relation ($k_{\rm eff}$ vs ω) is thus altered periodically as shown in Fig. 2. Near resonance, the resonator contribution Φ/L to the propagation constant becomes sensitively dependent frequency, leading to a reduced group velocity. The inverse of the group velocity for a given detuning $\phi_0=(\omega_0-\omega_R)T$, is specifically given by

$$\begin{aligned}\frac{1}{v_g}&=k'_{\rm eff}\equiv\frac{dk_{\rm eff}}{d\omega}=\frac{n}{c}+\frac{1}{L}\frac{d\Phi}{d\omega}\\&=\frac{n}{c}\left[1+\frac{2\pi R}{L}\left(\frac{1-r^2}{1-2r\cos\phi_0+r^2}\right)\right]\\&\xrightarrow[\phi_0=0,r\approx1]{}\frac{n}{c}\left(1+\frac{4R}{L}\mathfrak{F}\right),\end{aligned} \tag{5}$$

where the last form of this result refers to the resonant excitation of a sequence of high finesse microresonators. A resonant pulse propagating through a sequence of resonators would effectively travel with a group velocity that decreases with the finesse $\mathfrak{F}$ [11].

Higher-order derivatives of the transmitted phase shift with respect to normalized detuning $d^m\Phi/d\phi^m$ describe the frequency dependence of the group velocity that distorts the envelope of a pulse upon propagation. The transmitted phase shift may be expanded in a Taylor's series about the normalized carrier frequency, ϕ_0 as

$$\Phi=\Phi_0+\sum_{m=1}^{\infty}\frac{1}{m!}\left(\frac{d^m\Phi}{d\phi^m}\right)_{\phi=\phi_0}(\phi-\phi_0)^m. \tag{6}$$

In particular, the second derivative of the transmitted phase shift gives rise to the lowest-order effective group velocity dispersion (GVD)

$$\begin{aligned}k''_{\rm eff}&\equiv\frac{1}{L}\frac{d^2\Phi}{d\omega^2}\\&=\frac{T^2}{L}\left[\frac{-2r(1-r^2)\sin\phi_0}{(1-2r\cos\phi_0+r^2)^2}\right]\\&\xrightarrow[\phi_0=\pm\pi/\sqrt{3}\mathfrak{F},r\approx1]{}\mp\frac{3\sqrt{3}T^2}{16L}\frac{4}{\pi^2}\mathfrak{F}^2.\end{aligned} \tag{7}$$

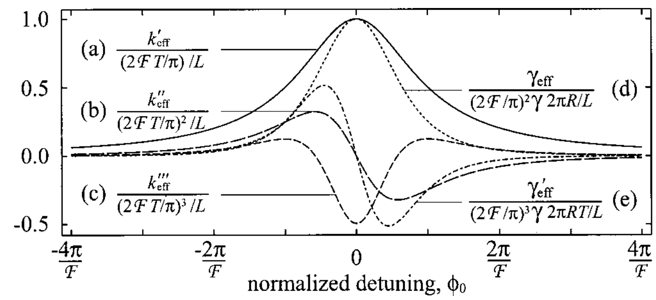

FIG. 3. Functional dependence of (a) the group velocity reduction, (b) group velocity dispersion, (c) third-order dispersion, (d) Kerr coefficient, and (e) self-steepening coefficient on the detuning for a SCISSOR. The parameters have been scaled such that the curves are universal and fit within the same plot limits.

The effective GVD is zero on resonance; however, a small positive or negative detuning can lead to a large anomalous or normal dispersion, respectively (see Fig. 3). The extremum values are, in fact, proportional to the square of the finesse, occurring slightly off-resonance at $\phi_0=\pm\pi/(\sqrt{3}\mathfrak{F})$. Still higher-order derivatives of the transmitted phase shift correspond to higher-order contributions to the resonator dispersion. This dispersion, induced by the structure, may be many orders of magnitude greater than the material dispersion of conventional material systems [12]. For example, a 10-ps optical pulse propagating in a sequence of resonators with a finesse of 10π, free-spectral range of 10 THz ($\sim5\ \mu$m diameter), and a spacing of 10 μm experiences a group velocity dispersion coefficient $k''_{\rm eff}$ of roughly 100 ps^2/mm. In general, this structural dispersion can be four to eight orders of magnitude greater than material dispersion in conventional materials such as silica fiber (20 ps^2/km).

In addition to inducing a strong group delay and dispersion, a resonator may enhance a weak nonlinearity. The ratio of circulating intensity $|\tilde{E}_3|^2$ to incident intensity $|\tilde{E}_1|^2$, known as the build-up factor, is easily derived from Eqs. (1) and (2) and is given by

$$\frac{|\tilde{E}_3|^2}{|\tilde{E}_1|^2}=\frac{1-r^2}{1-2r\cos\phi_0+r^2}\xrightarrow[\phi_0=0,r\approx1]{}\frac{2}{\pi}\mathfrak{F}. \tag{8}$$

The last form of this result shows that for a high finesse resonator, the maximum build-up factor is $2/\pi$ times the finesse. If the resonator material possesses a Kerr nonlinearity, then the resonance frequency will be intensity dependent, which may be modeled by adding a contribution $\gamma|\tilde{E}_3|^2 2\pi R$ to the normalized detuning, where γ represents the strength of the intrinsic material nonlinearity. The strength of the enhanced nonlinearity is calculated from the derivative of the transmitted phase shift with respect to the input intensity as

$$\gamma_{\mathrm{eff}} \equiv \frac{1}{L}\frac{d\Phi}{d|\tilde{E}_1|^2} = \frac{1}{L}\frac{d\Phi}{d\phi}\frac{d\phi}{d|\tilde{E}_3|^2}\frac{d|\tilde{E}_3|^2}{d|\tilde{E}_1|^2} \underset{\phi_0=0, r\approx 1}{\rightarrow} \gamma\frac{8R}{\pi L}\mathfrak{F}^2. \tag{9}$$

As can be seen from the equation, it is the combined action of the group velocity reduction and the buildup of the intensity, that yields an overall nonlinear response, which is *quadratically* enhanced by the finesse at resonance [9]. Quadratic finesse enhancements in ring resonators have also been shown for other $\chi^{(3)}$ processes, such as four-wave mixing [13,14].

We next derive a pulse-envelope evolution equation that retains the lowest-order dispersive and nonlinear terms. A modulated field can be decomposed into a slowly varying envelope $A(t)$ and a carrier wave with frequency ω_0 as $E(t) \equiv \frac{1}{2}A(t)\exp ik_{\mathrm{eff}}(\omega_0)z(-i\omega_0 t) + \mathrm{c.c.}$ By taking the Fourier transform of the transfer function $e^{i\Phi}$ and adding the nonlinear term as a perturbation along with the normalized modulation frequency, $(\omega-\omega_0)T$, we obtain an impulse response function for pulse envelopes passing through a single resonator. By convolving the impulse response (up to second order in finesse) with the incident pulse envelope $A(z,t)$, then time shifting to a retarded time coordinate $(\tau = t - k'_{\mathrm{eff}}z)$, and replacing differences for a single resonator with differentials for a continuum of distributed resonators with density $1/L$, an evolution equation emerges for pulses propagating in a sequence of resonators. We find that, in this limit, the pulse evolution is governed by a nonlinear Schrödinger equation (NLSE) with the effective parameters introduced above,

$$\frac{\partial}{\partial z}A = -i\frac{1}{2}k''_{\mathrm{eff}}\frac{\partial^2}{\partial \tau^2}A + i\gamma_{\mathrm{eff}}|A|^2 A. \tag{10}$$

Soliton solutions exist provided that the enhanced nonlinearity and induced dispersion are of opposite signs. While the sign of the enhanced nonlinearity is predetermined by the material properties, the sign of the structurally induced dispersion is, as previously shown in Eq. (7), determined by the sign of the detuning from resonance as illustrated in Fig. 3(b). The fundamental soliton solution for this equation is given by

$$A(z,\tau) = A_0 \,\mathrm{sech}(\tau/T_p)e^{i(1/2)\gamma_{\mathrm{eff}}|A_0|^2 z}, \tag{11}$$

where the amplitude and pulse width are related according to $|A_0|^2 = |k''_{\mathrm{eff}}|/\gamma_{\mathrm{eff}}T_p^2$. The finite response time of the resonator places a lower bound on the pulse width T_p. We define a scaling factor B as the ratio of the pulse bandwidth $(2\,\mathrm{arcsech}(1/\sqrt{2})/\pi^2 T_p)$ to the resonator bandwidth $(1/\mathfrak{F}T)$, such that $B = [2\,\mathrm{arcsech}(1/\sqrt{2})/\pi^2]\mathfrak{F}T/T_p$. We also define a nonlinear strength parameter as $\Gamma = (4/\pi^2)\mathfrak{F}^2\gamma|A_0|^2 R$. With these definitions, a simple relation holds between Γ and B for the fundamental soliton operating at the peak of the dispersion maxima [15]

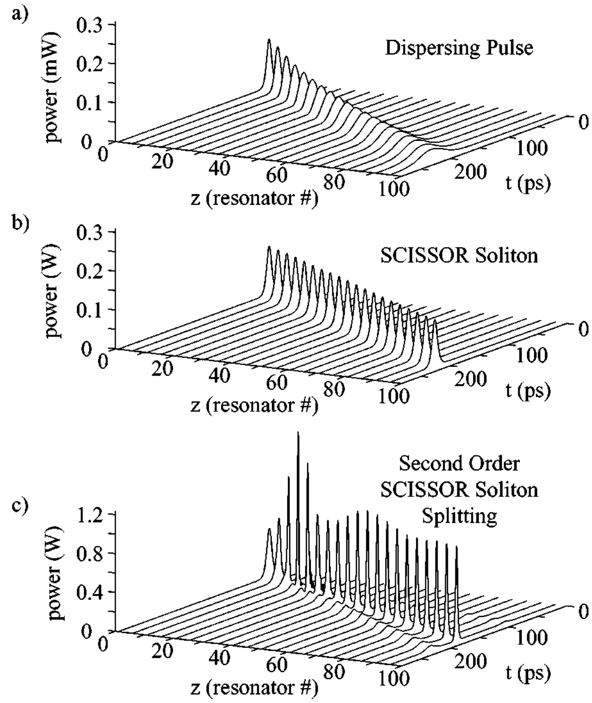

FIG. 4. (a) A weak pulse propagating in a SCISSOR disperses. (b) A pulse with amplitude corresponding to the fundamental soliton propagates without dispersing. (c) A higher-order breathing soliton is unstable under the influence the resonator-induced intensity-dependent group velocity (self-steepening).

$$\Gamma = \frac{\pi}{2\sqrt{3}\,\mathrm{arcsech}^2(1/\sqrt{2})}B^2 \approx B^2. \tag{12}$$

Higher-order dispersive and nonlinear terms become significant when either B and/or Γ approach unity. By ensuring that $B \lesssim 1$ and $\Gamma \lesssim 1$, one can ensure that propagation can be accurately modeled by the NLSE.

To test the validity of this approximation we have conducted rigorous time domain simulations. In Fig. 4(a) we show the pulse evolution of a low-power 10-ps full width at half maximum (FWHM) hyperbolic secant pulse detuned for maximum anomalous GVD ($B = 0.13$) in a chalcogenide glass-based system. The system consisted of 100 resonators spaced by 10 μm each with a 5 μm diameter and finesse of 10π. As can be seen, the temporal pulse profile is greatly dispersed. Figure 4(b) shows the pulse evolution for the same system, but with a peak power of 125 mW corresponding to the fundamental SCISSOR soliton ($\Gamma = 0.0196$). As can be seen, the pulse shape is well preserved upon propagation. Many of the familiar soliton characteristics, such as robustness, reshaping, and pulse compression or expansion have been observed to carry over from the continuous-medium case. In particular, higher-order breathing solitons, satisfying $\Gamma \approx N^2 B^2$, where N is an integer, are readily observed in the simulation, but are unstable due to of higher-order dispersive nonlinear effects present in this system [16]. Specifically, the frequency dependence of the nonlinear enhancement results in an intensity-dependent group velocity that leads to the phenomenon of soliton decay such that an *N*th-order breathing soliton is split into *N* fundamental solitons of differing pulse amplitudes and widths [17]. Figure 4(c) shows a situation in which a second-order soliton with a launched peak power of 500 mW undergoes decay and splits into two stable fundamental solitons. The solitons are well isolated in time and one of them possesses a higher peak power and narrower width than the original demonstrating the potential for optical pulse compression.

In many respects, the SCISSOR soliton is analogous to the gap [18] or Bragg [19] soliton, which results from nonlinear pulse propagation within the photonic band gap (PBG) of a distributed feedback structure. The structure itself bears similarity to a coupled resonator optical waveguide, which consists of a multidimensional array of intercoupled resonators and no side-coupled waveguide [8]. While resonators coupled to a guide display PBG-like enhancement of nonlinear effects arising from feedback, they do not restrict light from propagating at any frequency.

At present, many single microresonator systems with excellent optical properties have been constructed [20–26]. In many of these cases, extending the fabrication techniques to construct long sequences of such devices is achievable. Furthermore, via the application of thermal or electrical fields, it is possible to control the resonance frequency and/or coupling coefficients. We envision that such structures could be used as artificial media to study NLSE pulse propagation effects on an integrated chip with thermally or electrically controllable parameters $(k'_{\text{eff}}, k''_{\text{eff}}, \gamma_{\text{eff}}, \ldots)$. Other applications might include studies of slow-light phenomena, variable optical delay lines and clean pulse compression on a chip without pedestal formation via the soliton decay mechanism. Finally, soliton-based optical switching with low energy pulses perhaps even at the single-photon level might be feasible. While all of these concepts have been implemented in various geometries and material systems, the SCISSOR system has the potential for providing a highly compact, integrated optical structure for their study.

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$$\mathfrak{F} = \frac{(\text{FSR})}{(\text{FWHD})} = \frac{2\pi}{2\arccos\left(\dfrac{2r}{1+r^2}\right)} \underset{r\approx 1}{\rightarrow} \frac{\pi}{1-r}.$$

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