Stochastic dynamics of stimulated Brillouin scattering in an optical fiber

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We have investigated the statistical properties of stimulated Brillouin scattering (SBS) in a single-mode optical fiber. Gain narrowing of the Stokes spectrum is observed as the input laser power is increased, and large stochastic fluctuations are observed in the Stokes output intensity. The experimental results can be described well by a theoretical model that includes the spontaneous nature of the initiation of SBS.

Stimulated Brillouin scattering (SBS) is a nonlinear-optical process that leads to the generation of Stokes-shifted radiation. This generated light can display large fluctuations in intensity due either to deterministic or to stochastic processes. For example, deterministic fluctuations in the Stokes intensity can occur following the rapid turn-on of the pump field as the result of relaxation oscillations [1]. In addition, several workers have shown that, in the presence of external feedback, the Stokes intensity can fluctuate in a periodic [2,3] or a chaotic manner [4]. For the case in which laser beams counterpropagate in a Brillouin-active medium, instabilities [5] and chaos [6] in the temporal evolution of the transmitted waves have been predicted and observed [7].

Fluctuations of a stochastic nature can also occur in normal SBS (i.e., SBS excited by a single pump wave in the absence of external feedback). The origin of these fluctuations is the noise in the spontaneous scattering process that initiates SBS. Recent theoretical work [8–10] has shown that the Stokes output wave can show large stochastic fluctuations even when SBS is excited by a continuous-wave single-mode pump field. In this paper, we examine the noise properties of SBS excited within a single-mode optical fiber. Optical fibers are well suited for this purpose since SBS can be excited in optical fibers using continuous-wave lasers [11]. Most previous studies of SBS in optical fibers have concentrated on measurements of the Stokes spectrum [12] including measurements of the Brillouin linewidth [13] and Brillouin frequency shift [11,14]. In contrast, our work has concentrated on studies of the dynamical and statistical behavior of SBS.

Specifically, in this paper we describe our experimental investigation of the statistical properties of SBS under continuous-wave excitation, and we compare our experimental results with theoretical predictions. The Stokes spectrum is found to display gain narrowing as the input laser power is increased. The temporal behavior of the Stokes output intensity is also studied, and the intensity is found to exhibit large stochastic fluctuations. The statistical properties of the output are found to be in good agreement with the predictions of a theoretical model that accounts for the dynamics associated with the spontaneous initiation of the SBS process.

The following equations are use to model stimulated Brillouin scattering including its spontaneous initiation from noise [8–10]:

$$\frac{\partial E_l}{\partial z}+\frac{n}{c}\frac{\partial E_l}{\partial t}=-\alpha E_l+i\kappa\rho E_s\ , \tag{1a}$$

$$\frac{\partial E_s}{\partial z}-\frac{n}{c}\frac{\partial E_s}{\partial t}=\alpha E_s-i\kappa\rho^* E_l\ , \tag{1b}$$

and

$$\frac{\partial\rho}{\partial t}+\frac{\Gamma}{2}\rho=i\Lambda E_l E_s^*+f\ , \tag{1c}$$

where $E_l(z,t)$ and $E_s(z,t)$ are the laser and the Stokes field amplitudes, respectively, α is the linear absorption coefficient, $\rho(z,t)$ is the complex amplitude of the density variation, Γ is the phonon decay rate, κ and Λ are Brillouin coupling constants, and the Langevin noise source $f(z,t)$ describes the thermal fluctuations in the density of the medium that lead to spontaneous Brillouin scattering. We assume that $f(z,t)$ is a Gaussian random process with zero mean and is δ correlated in space and time in the sense that $\langle f(z,t)f^*(z',t')\rangle=Q\delta(z-z')\delta(t-t')$. The value of Q can be derived using thermodynamic arguments [10] and is given by $Q=2k_BT\rho_0\Gamma/v^2A$, where k_B is Boltzmann's constant, T is the temperature, ρ_0 is the mean density of the material, v is the velocity of sound in the material, and A is the cross-sectional area of the interaction region. Equations (1) allow us to treat properly the stochastic initiation of SBS and thus to make predictions regarding the Stokes spectrum and the temporal evolution of the Stokes wave.

In the limit in which the laser field is undepleted and in which absorption can be neglected (i.e., $\alpha=0$), the following solution for Eqs. (1) for the Stokes field amplitude can be found [10,15]:

$$E_s(0,t)=i\kappa E_l\int_{-\infty}^{t}dt'\int_{L}^{0}dz' e^{-(\Gamma/2)(t-t')}f^*(z',t')\times I_0(\sqrt{G\Gamma z'(t'-t)/L}\,)\ , \tag{2}$$

where I_0 is the zeroth-order modified Bessel function and where we assume that no Stokes field is incident at $z=L$. Here $G=gI_lL$ is the single-pass intensity gain of the

Stokes field where $g=32\pi\kappa\Lambda/nc\Gamma$ is the SBS gain factor, $I_l=(nc/8\pi)|E_l(0)|^2$ is the input laser intensity, and L is the length of the interaction region. Two important conclusions regarding the Stokes output can be deduced from an analysis of this solution: (i) The spectrum of the Stokes field evolves from a Lorentzian to a Gaussian line shape and narrows as the input laser intensity is increased. For $G\gg 1$, the gain-narrowed spectrum has a linewidth [full width at half maximum (FWHM)] that is equal to $\Delta\omega=\Gamma(\ln 2/G)^{1/2}$. (ii) The intensity of the Stokes output $I_s=(nc/8\pi)|E_s(0,t)|^2$ exhibits 100% temporal fluctuations in the sense that the normalized standard deviation of the intensity $\Delta I_s=(\langle I_s^2\rangle-\langle I_s\rangle^2)^{1/2}/\langle I_s\rangle$ is equal to unity. This result is to be expected because, in this limit in which the laser field is undepleted, the SBS medium behaves as a linear amplifier for the thermal noise that initiates the SBS process. Numerical integration of Eq. (1) demonstrates that the Stokes output intensity can exhibit these large fluctuations even under conditions of reasonably strong pump depletion.

The experimental setup that we used to verify these predictions is shown in Fig. 1. The single-mode Corning optical fibers are step-indexed with a 3 mol % GeO_2-doped core of 3.6 μm diameter. Both 100-m and 500-m-long fibers are used, and similar results are obtained for each length. The value of the SBS gain factor g was measured [16] using a pump-probe arrangement and found to be $g=2.5\times10^{-11}$ m/W, which is close to the theoretically predicted [17] value of 5×10^{-11} m/W for bulk silica. In all our experiments the light from the laser is passed through a Faraday isolator to prevent coupling between the fiber and the laser. Under conditions of high Brillouin gain, even small reflections from the endfaces of the fiber can lead to the formation of a Brillouin oscillator [3,9,16,18]. The reflection from each end of the fiber is minimized by cleaving the end at an oblique angle and by placing the back end of the fiber into an index matching fluid. Reflections from the pellicle beamsplitter BS1 are used to monitor both the input laser light and the output Stokes light. In all cases the output Stokes light contained approximately 70% of its power in the polarization direction parallel to that of the incident laser. Thus, for our experimental estimates of G we use the expression

$$G=0.7gP_lL/A\ , \tag{3}$$

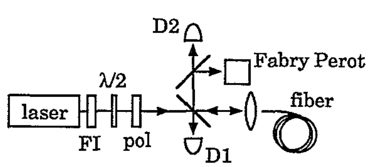

FIG. 1. Experimental setup used to measure the spectrum and the temporal evolution of the intensity of the Stokes radiation. A Faraday isolator (FI) is used to isolate the optical fiber from the laser.

where P_l is the laser power at the front end of the fiber, L is the length of the fiber, $A=10^{-11}$ m^2 is the core area, and the factor of 0.7 accounts for the depolarization [19] of the laser light within the fiber.

For our measurements of the Stokes spectrum we used a Coherent 699-21 frequency-stabilized (less than 2 MHz linewidth) dye laser operating at a wavelength of 0.57 μm. As much as 70 mW of power could be launched into the fiber. The Stokes light reflected from the pellicle beamsplitter BS1 is sent into a "supercavity" scanning Fabry-Pérot interferometer which possesses a free spectral range of 6 GHz and a finesse greater than 10^4. The light transmitted by the interferometer is detected by a cooled photomultiplier. In Fig. 2(a) Stokes spectra obtained through the use of the 500-m-long fiber are plotted for both high (66 mW) and low (5.8 mW) pump powers. Gain narrowing at the higher laser power is evident. Figure 2(b) is a plot of the normalized FWHM linewidth $\Delta\omega$ of the Stokes spectrum as a function of the normalized input intensity G. The experimental data are given by the solid circles. The solid line gives the theoretical prediction for the linewidth obtained by numerical integration of Eqs. (1) using the manufacturer's quoted value ($\alpha=1.7$ km^{-1}) of the absorption coefficient of the fiber at $\lambda_l=0.57$ μm. The measured value of the spontaneous linewidth Γ is 135 MHz, which is slightly larger than the value obtained by Thomas *et al.* [13] in a fiber with a pure silica core. However, the Stokes spectrum is known to broaden as a result of doping with GeO_2 [14]. Near and below the SBS threshold ($G<20$), the linewidth is seen to vary as $G^{-1/2}$, as expected, but for larger values the linewidth saturates to a value approximately equal to

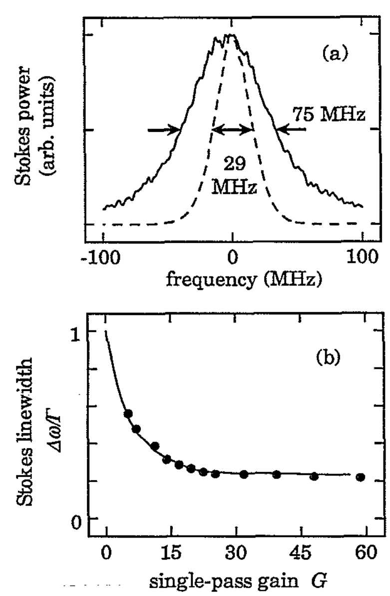

FIG. 2. (a) Stokes spectrum for low input laser power (5.8 mW, solid line) and high input laser power (66 mW, dotted line). (b) Dependence of the stokes linewidth on the single-pass gain. The solid circles show the measured values and the solid line shows the prediction of theory.

one-fifth of the spontaneous linewidth. The suppression of gain narrowing at the larger values of G is a result of the depletion of the laser field, which prevents the Stokes field from experiencing exponential gain throughout the entire medium.

The temporal evolution of the Stokes intensity is investigated under excitation by a single-mode argon-ion laser operating at 0.5145 μm and capable of launching as much as 400 mW of power into the fiber. The large area photodetector $D1$ monitors the input laser power, and the signal from a fast photodiode $D2$ (Fig. 1) is sent to a Tektronix RTD710 transient digitizer with the sampling interval set to 5 nsec. Figure 3 shows plots of the time evolution of the Stokes output power from the 100-m-long fiber for an input laser power just above the threshold for SBS [$G=23$, Fig. 3(a)] and approximately three times above threshold [$G=70$, Fig. 3(b)]. In both cases large fluctuations are observed in the output power. The results of numerically integrating Eqs. (1) with the inclusion of linear absorption ($\alpha=2.2$ km^{-1} at 0.5145 μm) for the Stokes output for the experimental conditions of Figs. 3(a) and 3(b) are shown in Figs. 3(c) and 3(d), and fluctuations comparable to those observed in the experiment are seen. The corresponding time-averaged SBS reflectivity (I_s/I_l) at the higher input laser powers [Figs. 3(b) and 3(d)] is approximately equal to 0.5. This result demonstrates that, even under conditions in which the laser field is strongly depleted by the interaction, the fluctuations associated with the initiation of the SBS process are still present in the output Stokes wave. Previous workers [9,20] have observed fluctuations in the Stokes intensity, but under conditions of pulsed excitation.

We have also calculated the normalized correlation function $C(\tau)=[\langle I_s(t)I_s(t+\tau)\rangle/\langle I_s(t)\rangle^2-1]$ for each of the time series associated with the plots in Figs. 3(a)–3(d), and these results are plotted in Figs. 4(a)–4(d), respectively. At the lower input powers [4(a) and 4(c)] the value of $C(\tau)$ decays monotonically to zero with a

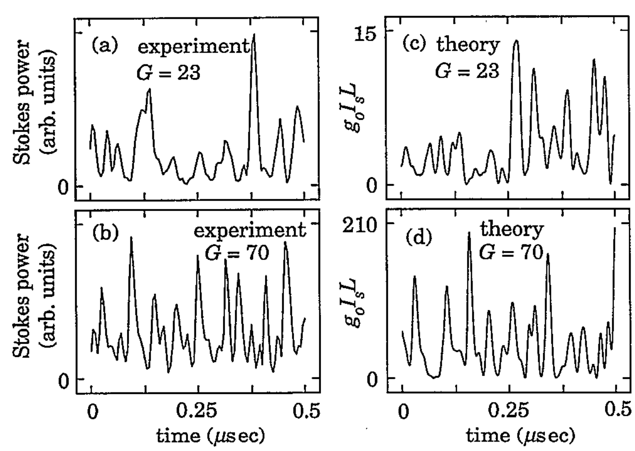

FIG. 3. Temporal evolution of the Stokes output power for the case of the 100-m-long fiber for an input laser power that corresponds to (a) $G=23$ and (b) $G=70$. Plots (c) and (d) are the theoretical simulations for the experimental conditions.

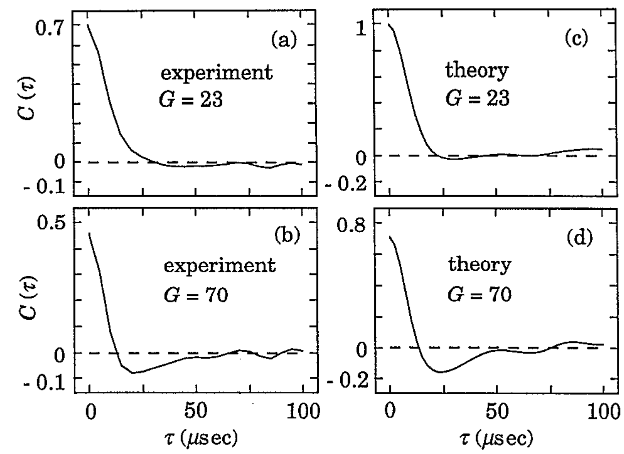

FIG. 4. (a)–(d) The correlation function $C(\tau)=[\langle I_s(t)I_s(t+\tau)\rangle/\langle I_s(t)\rangle^2-1]$ is plotted as a function of τ for the corresponding time series shown in Figs. 3(a)–3(d), respectively.

shape that is nearly Gaussian. Since the spectrum of the amplified Stokes field is predicted to be Gaussian [10], this result for $C(\tau)$ can be understood from the fact that for a thermal field with a Gaussian frequency distribution $C(\tau)$ is itself Gaussian [21]. However, at the higher input laser intensities [Figs. 4(b) and 4(d)], $C(\tau)$ initially decays more rapidly, becomes negative, and then rises back to zero. The shapes of the curves for $C(\tau)$ for theory and experiment are in good agreement; we believe that the disagreement in the range between maximum and minimum values of $C(\tau)$ can be attributed to the depolarization of the Stokes light within the fiber.

Figure 5 is a plot of the normalized standard deviation ΔI_s of the Stokes intensity as a function of the single-pass gain G. We see that the fluctuations are quenched slightly at higher input powers and that in all cases the value of ΔI_s is slightly less than unity. The solid curve shows the theoretical prediction with the inclusion of depolarization of the Stokes field. The effect of the depolarization was taken into account by assuming that the statistical nature of the Stokes field is similar to that of thermal light. The normalized standard deviation $\Delta I_{\rm PP}$ for partially polar-

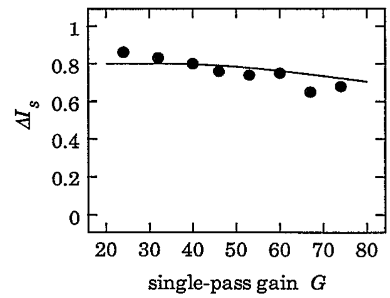

FIG. 5. Normalized standard deviation of the Stokes output intensity as a function of the normalized input intensity G. The solid curve represents the predictions of theory.

ized light from a thermal source is given by [21] $\Delta I_{PP} = (\langle I_x \rangle^2 + \langle I_y \rangle^2)^{1/2} / \langle I \rangle$, where I_x and I_y are the intensities of the x and y components of the thermal field, respectively, and where $I = I_x + I_y$ is the total intensity. For the case in which 70% of the light is polarized along the x direction, the resulting value for ΔI_{PP} is 0.78. This factor of 0.78 was used to scale the values of ΔI_s obtained from numerical simulations (which assume that all optical waves are linearly polarized in the same direction), and the theoretical curve is found to be in good agreement with experimental data.

The occurrence of large fluctuations in the Stokes intensity even under conditions in which the laser field is, on the average, strongly depleted by the interaction can be explained by the following argument. Let us assume that a spontaneous Stokes noise spike of temporal duration Γ^{-1} is generated near the back end of the interaction region. This pulse experiences exponential amplification (with a gain coefficient that is roughly equal to gI_{th}, where I_{th} is the laser intensity at the threshold for SBS [22]) until it reaches the region near the front of the interaction volume where most of the pump depletion occurs. The length of this depletion region is roughly equal to the gain length $1/gI_l$ ($=L/G$), and thus the energy of the laser field contained in this region is equal to $I_l AnL/Gc$. Since the peak intensity of the Stokes noise pulse will be roughly equal to the input laser intensity I_l (because the SBS reflectivity is nearly equal to unity under the assumed conditions), then the amount of energy contained in the Stokes pulse is equal to $I_l A/\Gamma$. The pulse will be amplified appreciably in passing through the depletion region only if its energy is less than the laser energy contained in this region, that is, if $\Gamma T_t > G$ (where $T_t = nL/c$ is the transit time). When this inequality is fulfilled, Stokes fluctuations will not be suppressed due to gain saturation, and the output Stokes intensity will display large fluctuations. We have verified this conclusion that Stokes fluctuations are suppressed only when G exceeds ΓT_t by performing numerical integrations of the SBS equations [Eqs. (1)] for various values of the parameter ΓT_t. In our experiment with the 100-m-long fiber, the value of ΓT_t is equal to 450, and thus only a small reduction in the value of ΔI_s is observed even at the largest values of G (≈ 80) that we could attain. For input intensities such that $G > \Gamma T_t$, fluctuations in the output wave are predicted to diminish except for the infrequent occurrence of an intensity spike that results from an abrupt phase change in the Stokes field; these spikes are analogous to the phase waves that have been predicted [23] and observed [24] in stimulated Raman scattering. Phase waves in SBS have been observed in an optical fiber with 250-nsec laser pulses [9,25]. For values of G that are much greater that ΓT_t, the fluctuations in the Stokes wave are predicted to be suppressed completely. For the case of SBS in focused geometries, the value of ΓT_t is typically less than 0.1, which explains why fluctuations are not usually observed.

Fluctuations in the Stokes intensity have been observed recently by Harrison *et al.* [18] under experimental conditions very similar to our own. However, those authors attribute the source of the fluctuations to deterministic chaos resulting from the combined action of Brillouin gain and the nonlinear Kerr effect. By integrating Eqs. (1) with the inclusion of a Kerr nonlinearity and in the absence of a fluctuating noise source, we have verified their conjecture that the combined action of Brillouin gain and Kerr nonlinearity can lead to chaotic behavior. However, we find that, for realistic values of the laser intensity, chaotic behavior occurs only if the dimensionless ratio n_2k/g_0 (where n_2 is the nonlinear Kerr coefficient and $k = 2\pi/\lambda$) is of the order of unity or greater. The measured value of the nonlinear index coefficient of a silica-core fiber is given by $n_2 = 3.2 \times 10^{-20}$ m^2/W [26], and hence for our fiber the ratio of the Kerr to Brillouin nonlinearities is $n_2k/g_0 = 0.015$. Thus chaotic behavior is not expected to occur under the conditions of either experiment. In fact, we find the theoretical predictions of our model (with the inclusion of a noise source) are not modified by a Kerr nonlinearity so small that n_2k/g_0 is equal to 0.015. In addition, we have analyzed various experimental Stokes time series of 16 384 points using the method of Grassberger and Procaccia [27], and for all cases we find that the dynamic evolution does not correspond to low-dimensional chaos. For these reasons and the fact that our experimental results are in very good agreement with the predictions of our theoretical model, which contains no free parameters, we feel that the fluctuations observed in our experiment are of a purely stochastic nature.

In conclusion, we have studied the statistical behavior of the SBS in an optical fiber. We have observed gain narrowing of the Stokes spectrum and large fluctuations in the Stokes output intensity. Our results are consistent with the predictions of a theoretical model that includes the stochastic nature of the spontaneous Brillouin scattering that initiates the SBS process.

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